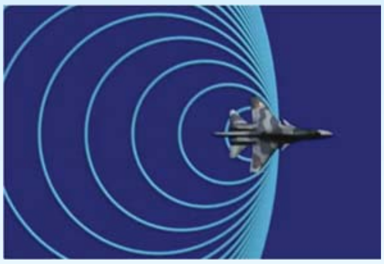
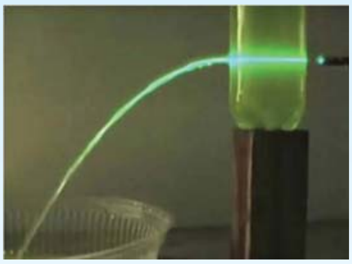
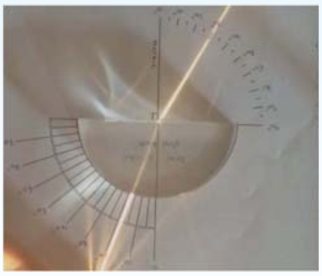
Vågrörelselära och optik



Kapitel 15 – Mekaniska vågor

Vågrörelselära och optik

Kurslitteratur: University Physics by Young & Friedman

Harmonisk oscillator:	Kapitel 14.1 – 14.4
Mekaniska vågor:	Kapitel 15.1 – 15.8
Ljud och hörande:	Kapitel 16.1 – 16.9
Elektromagnetiska vågor:	Kapitel 32.1 & 32.3 & 32.4
Ljusets natur:	Kapitel 33.1 – 33.4 & 33.7
Stråloptik:	Kapitel 34.1 – 34.8
Interferens:	Kapitel 35.1 – 35.5
Diffraktion:	kapitel 36.1 - 36.5 & 36.7

Vågrörelselära och optik

Tid	Må 02-nov	Ti 03-nov	On 04-nov	To 05-nov	Fr 06-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 14	Kvantfysik (A)	Våglära/optik (A)	Kvantfysik (A)
10-12	Intro period 2 (A) Informationssökning (A)	Kvantfysik (A)	Våglära/optik kap 14+15 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A) kap 15
13-15	Utvärdering (A) 12-13	Övningar Optik&Våg (L218-19)	SI gp6-10 (L219) ÄFYA11 (L218)	SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 09-nov	Ti 10-nov	On 11-nov	To 12-nov	Fr 13-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 16	Våglära/optik (A) kap 16+32	Kvantfysik (A)	Kvantfysik (A)
10-12	Våglära/optik kap 15+16 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A)	Våglära/optik (A) kap 32+33	Våglära/optik (A) kap 33
13-15	SI gp1-5 (L219) ÄFYA11 (L218)	Övningar Optik&Våg (L218-19)	ÄFYA11 (l218) SI gp6-10 (L219)	SI gp1-5 (L218) SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 16-nov	Ti 17-nov	On 18-nov	To 19-nov	Fr 20-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 35	Våglära/optik (A) kap 36
10-12	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 34+35	Våglära/optik (A) kap 36 ÄFYA11 (L218)	Kvantfysik (A)
13-15	SI gp6-10 (L219)	Övningar Optik&Våg (L218-19)	Seminar.gen.gång (A) 12-13 SI gp1-5 (L218) 13-15 SI gp11-15 (L219) 13-15	Labbintroduktion (A) O2-O3, K1-K3	Övningar Optik&Våg (L218-19)
15-17					

Mechanical waves: Transverse waves

Transverse waves

Mechanical waves: Transverse waves

A wave is when a system is disturbed from its equilibrium and the disturbance is moving.

A mechanical wave propagates in a medium.

An electromagnetic wave can propagate without a medium in vacuum.

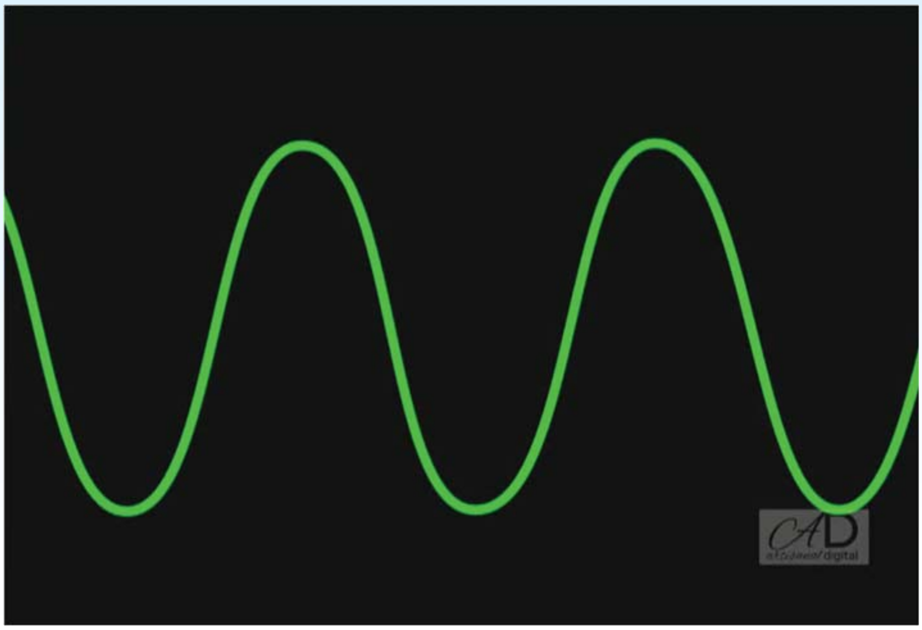
Waves transports energy but not matter.

Mechanical waves: Transverse waves

Transverse wave: The medium moves transverse to the wave direction.

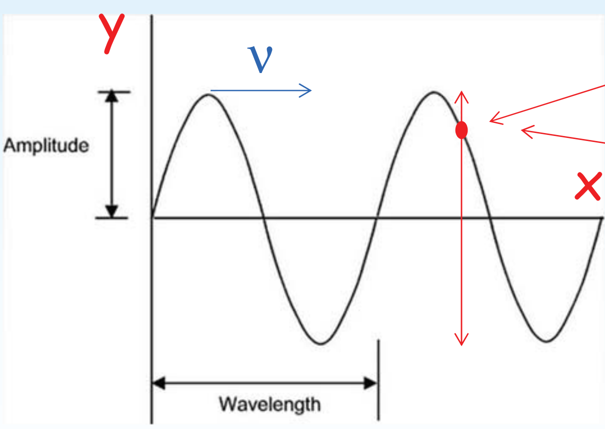
Mechanical waves: Transverse waves

A sinusoidal transverse wave is when the waves have a periodic sinus shape.

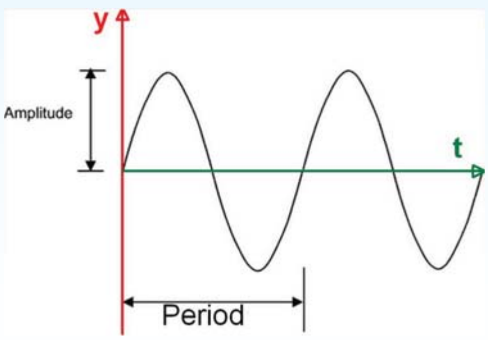


Mechanical waves: Transverse waves

Transversal sinusoidal wave:

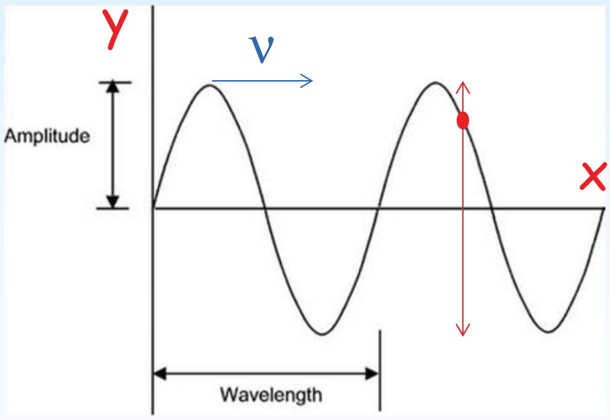


Every point on the wave moves up and down like an harmonic oscillator with the period T.



Mechanical waves: Transverse waves

Definitions:



A: Amplitude (m)

T: Period (s)

λ: Wavelength (m)

v: Wave speed (m/s) = λ / T

f: Frequency (Hz) = $1 / T$

ω: Angular frequency (radians/s) = $2 \pi f$

k: Wave number (radians/m) = $2 \pi / \lambda$

Mechanical waves: Longitudinal waves

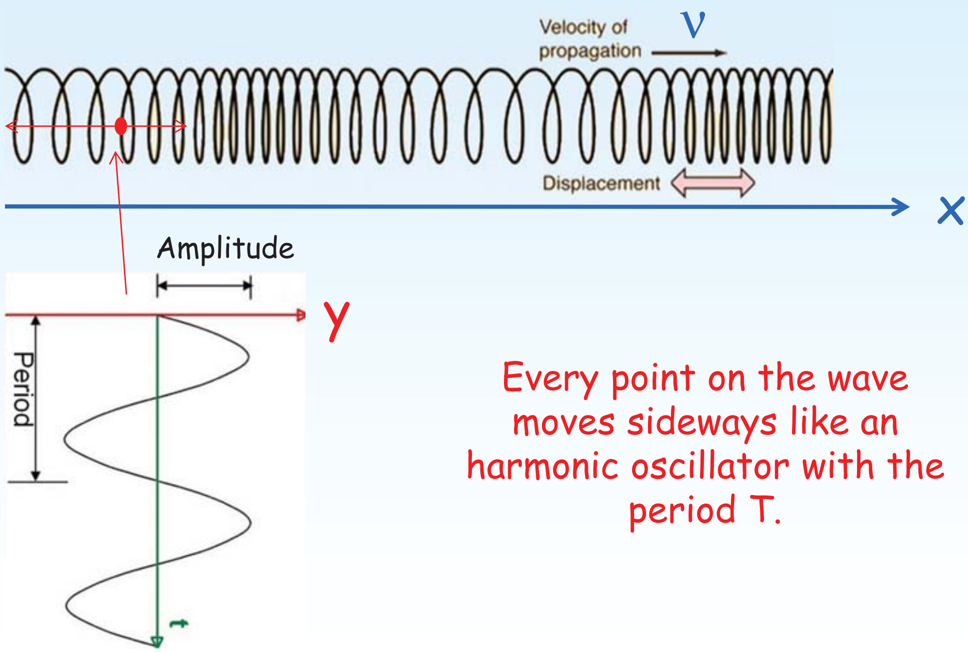
Longitudinal waves

Mechanical waves: Longitudinal waves

Longitudinal wave: The medium moves in the wave direction.

Mechanical waves

Longitudinal sinusoidal wave



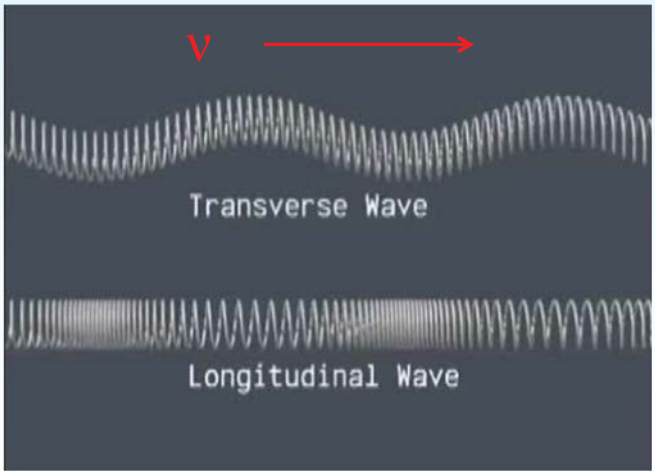
Every point on the wave moves sideways like an harmonic oscillator with the period T.

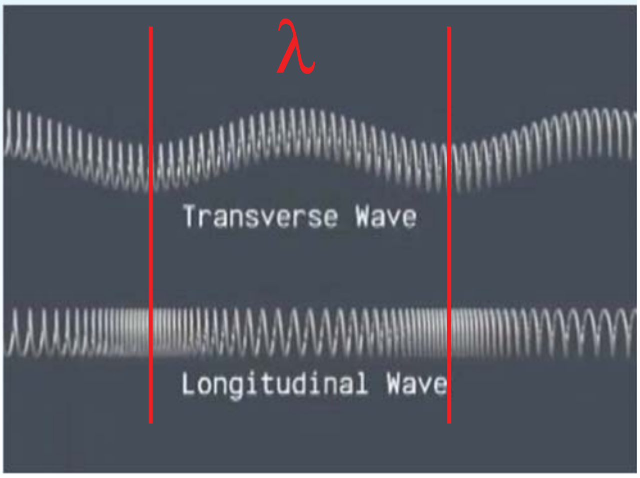
Mechanical waves: Longitudinal waves

What is the wavelength (λ) for a sinusoidal wave ?

What is the wave speed (ν) ?

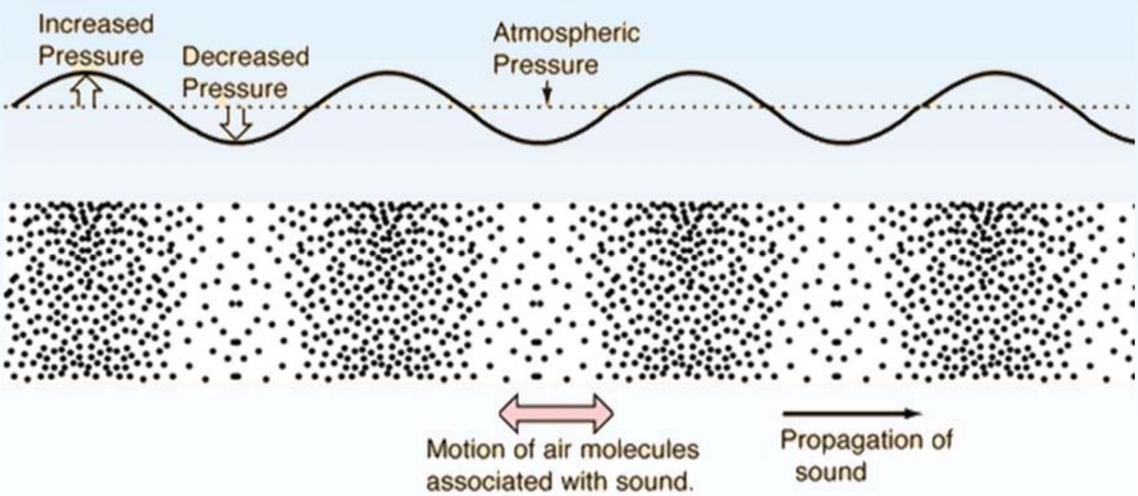
$$\nu = \lambda / T$$





Mechanical waves: Longitudinal waves

Sound is longitudinal waves in air



Mechanical waves: Problem

Problem solving

Mechanical waves: Problem

Sound waves are longitudinal waves in air. The speed of sound depends on temperature; at 20°C it is 344 m/s What is the wavelength of a sound wave in air at 20°C if the frequency is 262 Hz

$$v = \lambda / T$$
$$f = 1 / T$$

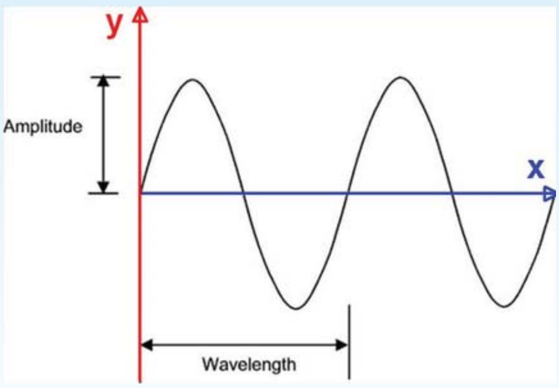
$$\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{262 \text{ Hz}} = \frac{344 \text{ m/s}}{262 \text{ s}^{-1}} = 1.31 \text{ m}$$

Mechanical waves: The wavefunction

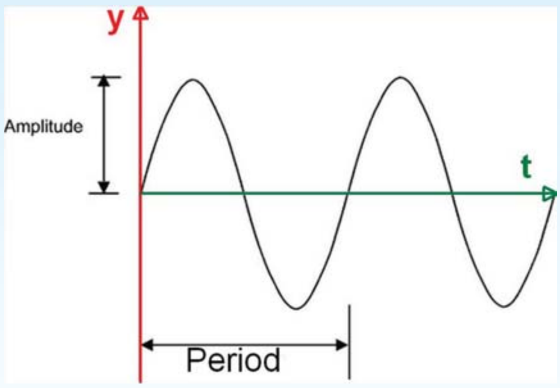
The wavefunction

Mechanical waves: The wavefunction

The height of the wave as a function of distance x



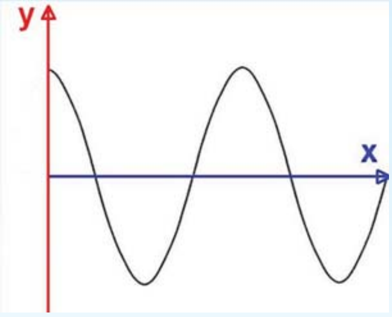
The height of the wave as a function of time t



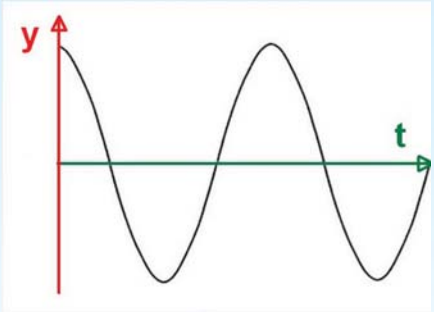
Wavefunction y(x,t):

Function that describs the height of the wave as a function of time and distance

Mechanical waves: The wavefunction



$y(x, t = 0) = A\cos kx$



$y(x = 0, t) = A\cos \omega t$

$$y(x, t) = A\cos(kx - \omega t)$$ (sinusoidal wave moving in $+x$-direction)

+ if moving in the –x direction

Mechanical waves: The wavefunction

$$y(x, t) = A\cos(kx - \omega t)$$ (sinusoidal wave moving in $+x$-direction)

Amplitude: A

$v = \lambda / T$
$f = 1 / T$

Wavenumber: $k = 2\pi/\lambda$

Angular frequency: $\omega = 2\pi/T$

$v = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k$

Mechanical waves: The wavefunction

The wavefunction:

$$y(x, t) = A\cos(kx - \omega t)$$

Velocity and acceleration up and down:

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)$$

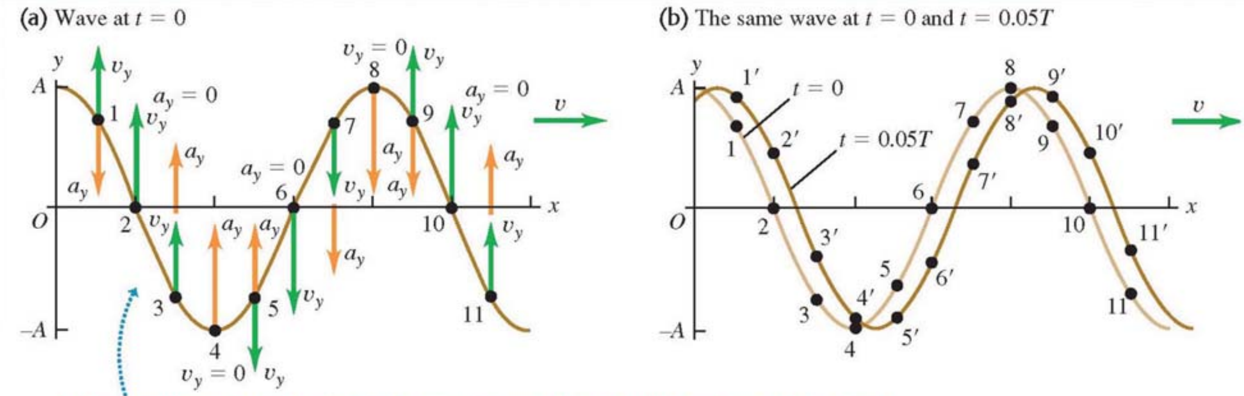
$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x, t)$$

Mechanical waves: The wavefunction

Velocity and acceleration up and down:

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x, t)$$



- Acceleration a_y at each point on the string is proportional to displacement y at that point.
- Acceleration is upward where string curves upward, downward where string curves downward.

Mechanical waves: The wave equation

The wave equation

Mechanical waves: The wave equation

The wavefunction: $y(x, t) = A\cos(kx - \omega t)$

Velocity and acceleration up and down:

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = -\omega^2 A \cos(kx - \omega t) = -\omega^2 y(x, t)$$

The curvature:

$$\frac{\partial^2 y(x, t)}{\partial x^2} = -k^2 A \cos(kx - \omega t) = -k^2 y(x, t)$$

$$\frac{\partial^2 y(x, t)/\partial t^2}{\partial^2 y(x, t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

The wave equation: $$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x, t)}{\partial t^2}$$

Mechanical waves: The wave equation

The wave equation: $$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

The wave equation describes also waves that are not sinusoidal !

It even describes waves that are not periodic !

And waves in three dimensions !

Mechanical waves: Problem

Problem solving

Mechanical waves: Problem

Cousin Throckmorton holds one end of the clothesline taut and wiggles it up and down sinusoidally with frequency 2.00 Hz and amplitude 0.075 m. The wave speed on the clothesline is $v = 12.0$ m/s. At $t = 0$ Throcky's end has maximum positive displacement and is instantaneously at rest. Assume that no wave bounces back from the far end. (a) Find the wave amplitude A, angular frequency ω, period T, wavelength λ, and wave number k.

Given in problem:

A: Amplitude = 0.075 m

f: Frequency = 1 / T = 2.00 Hz

v: Wave speed = λ / T = 12.0 m/s

To calculate:

T: Period = 1 / f = 0.5 s

λ: Wavelength = v T = 6.00 m

ω: Angular frequency = $2\pi f = 4\pi$

k: Wave number = $2\pi / \lambda = \frac{1}{3}\pi$

Mechanical waves: Problem

(b) Write a wave function describing the wave. (c) Write equations for the displacement, as a function of time, of Throcky's end of the clothesline and of a point 3.00 m from that end.

ω: Angular frequency = $2\pi f = 4\pi$

k: Wave number = $2\pi / \lambda = \frac{1}{3}\pi$

$$y(x,t) = A\cos(kx - \omega t) = 0.075\cos(\tfrac{1}{3}\pi x - 4\pi t)$$

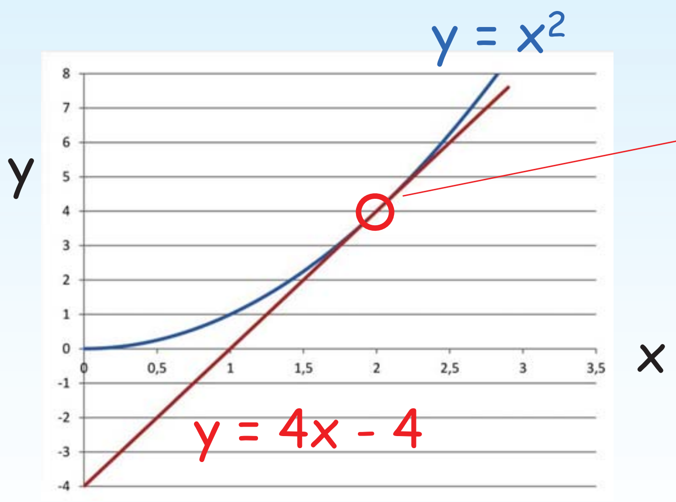
$$y(0,t) = 0.075\cos(-4\pi t) = 0.075\cos(4\pi t)$$

$$y(3,t) = 0.075\cos(\pi - 4\pi t) = -0.075\cos(4\pi t)$$

$\cos(-x) = \cos(x)$

Wave speed and the string characteristics

Mathematics: derivation



$$\frac{dy}{dx} = 2x = 4$$

The derivation gives the slope of the tangent.

Mechanical waves: Wave speed

Goal:

Figure out how the wave speed depends on the characteristics of the string.

Basic idea:

Look at the forces on a small string segment and apply Newtons law: F = m a

Mechanical waves: Wave speed

The wavespeed (v) in a string depend on the string tension given by the force on the string (F) and the mass per unith length of the string (μ).

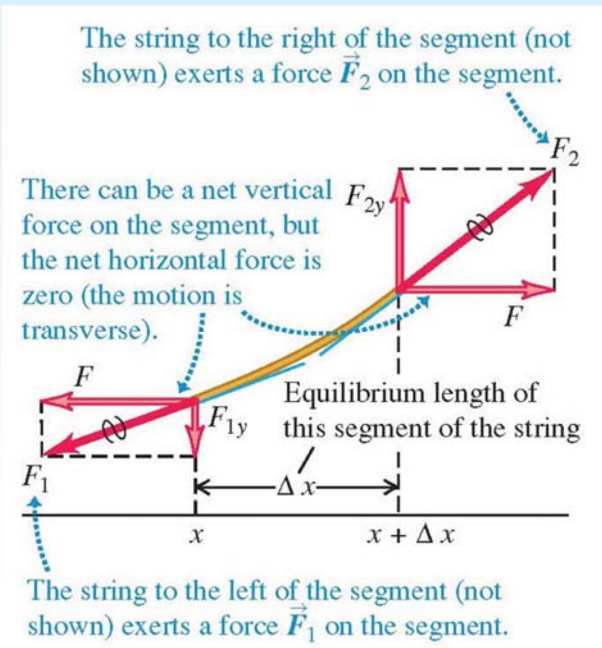
For a small string segment (Δx) the mass is $m = \mu \Delta x$

For a transverse wave the horisonthal force is zero.

The ratio of the force in the y-direction to the total force is the slope of the string. We can also get the slope by taking the derivative of the wavefunction:

$$\frac{F_{1y}}{F} = -\left(\frac{\partial y}{\partial x}\right)_x \qquad \frac{F_{2y}}{F} = \left(\frac{\partial y}{\partial x}\right)_{x+\Delta x}$$

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$



Mechanical waves: Wave speed

Newtons second law: F = m a and a = the second derivate on time.

$$F_y = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right] = \mu\,\Delta x \frac{\partial^2 y}{\partial t^2}$$

dividing by $F\Delta x$

$$\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\Delta x} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

When Δx goes to zero this is equivalent to the second derivative on x:

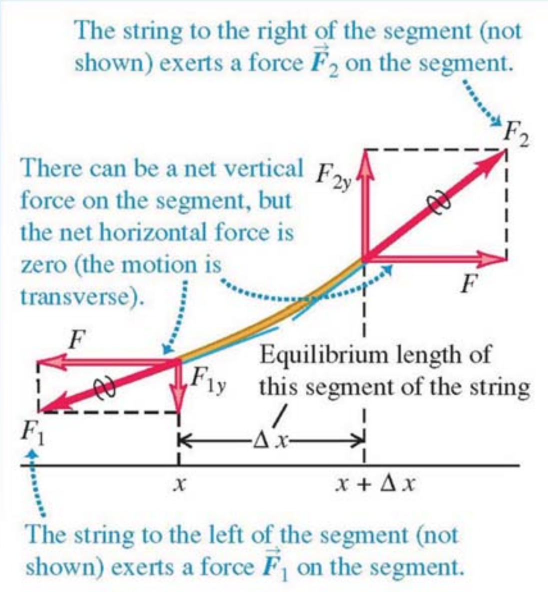
$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

The wave equation is:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

The wavespeed is then:

$$v = \sqrt{\frac{F}{\mu}}$$



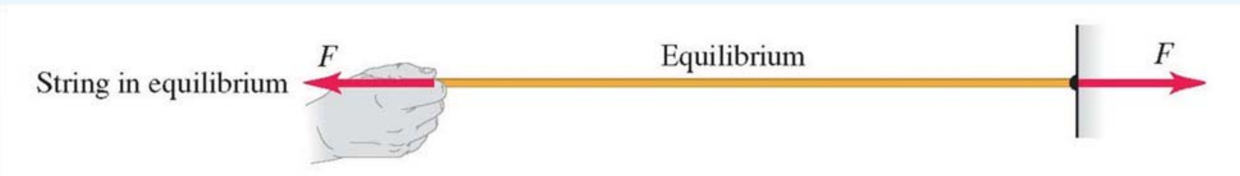
Mechanical waves: Wave speed

The wave speed in a string depends on two things:

$$v = \sqrt{\frac{F}{\mu}}$$

Force (or string trension)

String mass per unit length



More generally:

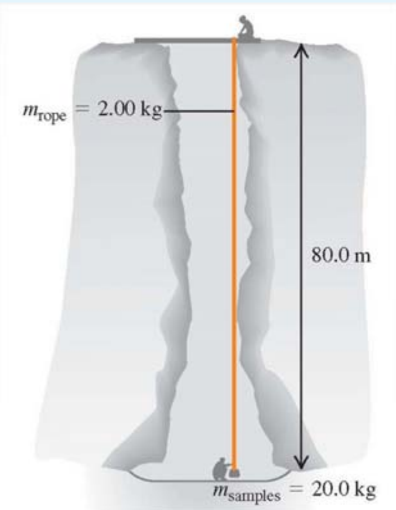
$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

Mechanical waves: Problem

Problem solving

Mechanical waves: Problem

One end of a 2.00-kg rope is tied to a support at the top of a mine shaft 80.0 m deep (Fig. 15.14). The rope is stretched taut by a 20.0-kg box of rocks attached at the bottom. (a) The geologist at the bottom of the shaft signals to a colleague at the top by jerking the rope sideways. What is the speed of a transverse wave on the rope? (b) If a point on the rope is in transverse SHM with $f = 2.00$ Hz, how many cycles of the wave are there in the rope's length?



The tension in the rope due to the box is

$$F = m_{box}g = (20.0\ \text{kg})(9.80\ \text{m/s}^2) = 196\ \text{N}$$

and the rope's linear mass density is

$$\mu = \frac{m_{rope}}{L} = \frac{2.00\ \text{kg}}{80.0\ \text{m}} = 0.0250\ \text{kg/m}$$

the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{196\ \text{N}}{0.0250\ \text{kg/m}}} = 88.5\ \text{m/s}$$

the wavelength is

$$\lambda = \frac{v}{f} = \frac{88.5\ \text{m/s}}{2.00\ \text{s}^{-1}} = 44.3\ \text{m}$$

There are $(80.0\ \text{m})/(44.3\ \text{m}) = 1.81$ wavelengths (that is, cycles of the wave) in the rope.

Mechanical waves: Power & Intensity

Power

Mechanical waves: Power & Intensity

The power in general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P): $P(x,t) = F_y(x,t)v_y(x,t)$

The instantaneous rate at which energy is transfered along the wave.

Unit: W or J/s

Wave intensity (I):

Average power per unit area through a surface perpendicular to the wave direction.

Unit: W/m^2

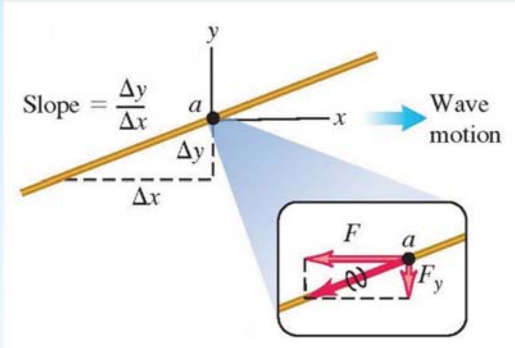
Mechanical waves: Power & Intensity

$$P = \vec{F} \cdot \vec{v}$$

$$P(x,t) = F_y(x,t)v_y(x,t)$$

The ratio of the force in the y-direction to the force in the x-direction is the slope of the string:

$$F_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}$$



$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

$$y(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

The wave power:

$$P(x,t) = Fk\omega A^2\sin^2(kx - \omega t)$$

Mechanical waves: Power & Intensity

The wave power:

$$P(x,t) = Fk\omega A^2\sin^2(kx - \omega t)$$

$$v = \sqrt{\frac{F}{\mu}}$$

$$v = \omega/k$$

$$k = \omega / \sqrt{\frac{F}{\mu}}$$

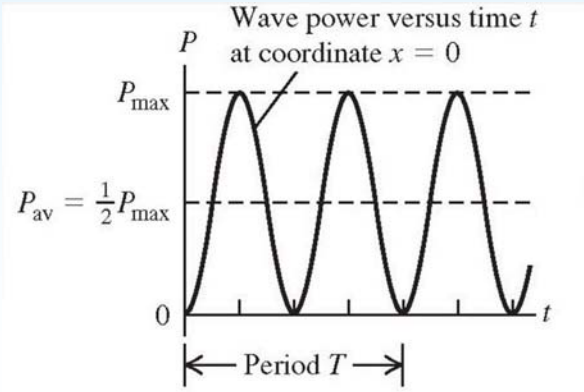
$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2\sin^2(kx - \omega t)$$

The maximum wave power:

$$P_{max} = \sqrt{\mu F}\,\omega^2 A^2$$

The average wave power:

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$



Problem solving

at what maximum rate does Throcky put energy into the clothesline? That is, what is his maximum instantaneous power? The linear mass density of the clothesline is $\mu = 0.250$ kg/m, and Throcky applies tension $F = 36.0$ N. (b) What is his average power?

A: Amplitude = 0.075 m

f: Frequency = 1 / T = 2.00 Hz

v: Wave speed = λ / T = 12.0 m/s

T: Period = 1 / f = 0.5 s

λ: Wavelength = v T = 6.00 m

ω: Angular frequency = 2 π f = 4π

k: Wave number = 2 π / λ = ⅓π

μ: Linear mass density = 0.250 kg/m

F: Tension = 36.0 N

Solution:

$$P_{max} = \sqrt{\mu F}\omega^2 A^2 = \sqrt{(0.250\text{ kg/m})(36.0\text{ N})}(4.00\pi\text{ rad/s})^2(0.075\text{ m})^2 = 2.66\text{ W}$$

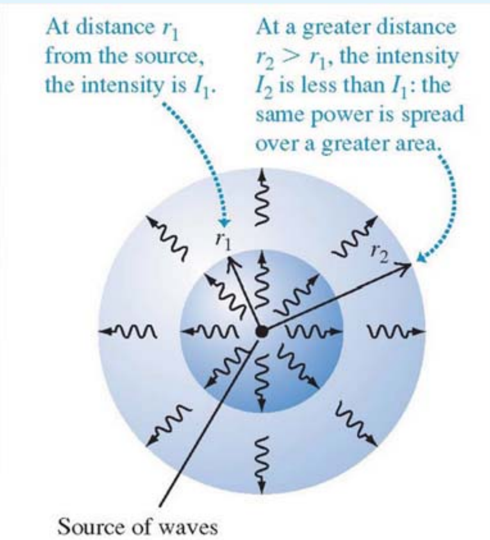
$$P_{av} = \tfrac{1}{2}P_{max} = \tfrac{1}{2}(2.66\text{ W}) = 1.33\text{ W}$$

Mechanical waves: Power & Intensity

Intensity

Mechanical waves: Power & Intensity

Wave intensity (I): The rate at which energy is transported by a wave through a surface perpendicular to the wave direction per unit surface area (average power per unit area). Unit: W/m^2



The intensity through a sphere with radius r_1

$$I_1 = \frac{P}{4\pi r_1^2}$$

If there is no loss of power:

$$4\pi r_1^2 I_1 = 4\pi r_2^2 I_2$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

Mechanical waves: Problem

Problem solving

Mechanical waves: Problem

A siren on a tall pole radiates sound waves uniformly in all directions. At a distance of 15.0 m from the siren, the sound intensity is 0.250 W/m^2. At what distance is the intensity 0.010 W/m^2?

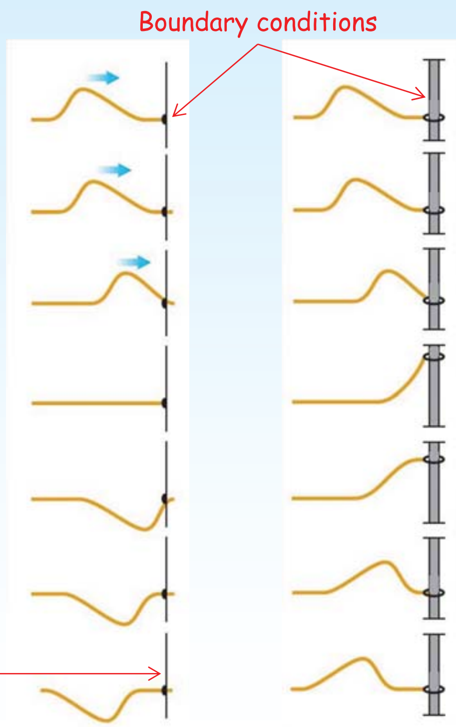
$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (15.0\ \text{m})\sqrt{\frac{0.250\ \text{W/m}^2}{0.010\ \text{W/m}^2}} = 75.0\ \text{m}$$

Reflections

Reflections of a wave

The support provides an opposite force which produces and inverted wave.



Mechanical waves: Reflections

The wavefunction of two waves is typically the sum of the individual wavefunctions.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

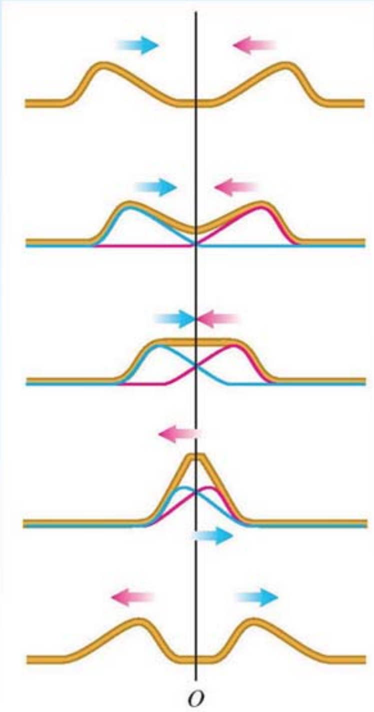
This is called the principle of superposition.

This is true if the wave equations for the waves are linear (they contain the function y(x,t) only to the first power).

For example can sinusoidal waves be superimposed like this because their wave equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

is linear.

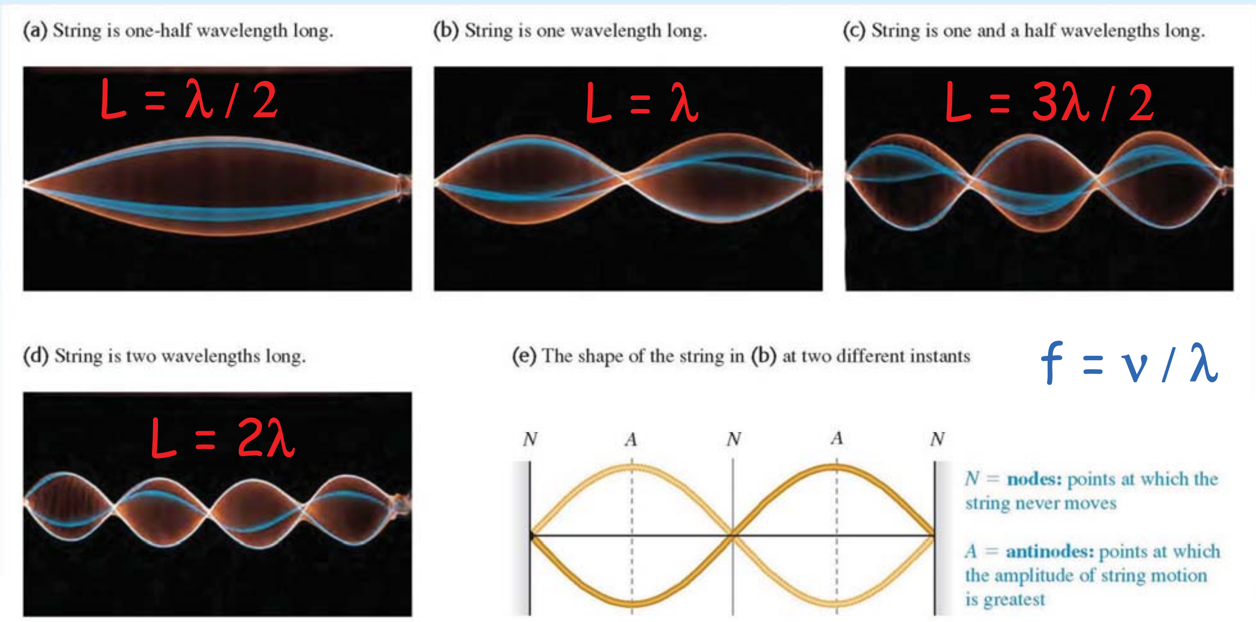


Mechanical waves: Standing waves

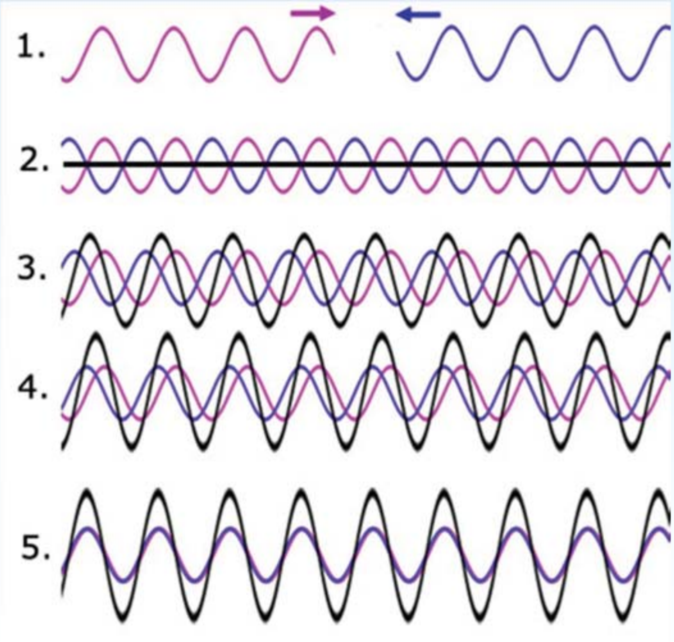
Standing waves

Mechanical waves: Standing waves

Mechanical waves: Standing waves



Mechanical waves: Standing waves



$$y_1(x,t) = -A\cos(kx + \omega t)$$

$$y_2(x,t) = A\cos(kx - \omega t)$$

Different times

$$y(x,t) = y_1(x,t) + y_2(x,t) = A[-\cos(kx + \omega t) + \cos(kx - \omega t)]$$

Mechanical waves: Standing waves

Wavefunction from superposition of two waves:

$$y(x,t) = y_1(x,t) + y_2(x,t) = A[-\cos(kx + \omega t) + \cos(kx - \omega t)]$$

Trigonometrical relationship: $\cos(a \mp b) = \cos a \cos b \pm \sin a \sin b$

⇓

Y(x,t)=A[-cos(kx)cos(ωt)+sin(kx)sin(ωt) +cos(kx)cos(ωt)+sin(kx)sin(ωt)]

$$y(x,t) = y_1(x,t) + y_2(x,t) = 2A\sin kx \ \sin\omega t$$

Nodes are given by sin(kx) = 0

$$kx = 0, \pi, 2\pi, 3\pi, \ldots \qquad k = 2\pi/\lambda$$

$$x = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots$$

$$= 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

λ = v / f

Mechanical waves: Standing waves

Wavefunction:

$$y(x,t) = 2A \sin(kx) \sin(\omega t)$$

Velocity:

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} \Rightarrow v_y(x,t) = 2A\omega \sin(kx) \cos(\omega t)$$

Acceleration:

$$a_y(x,t) = \frac{\partial v_y(x,t)}{\partial t} = \frac{\partial^2 y(x,t)}{\partial t^2} \Rightarrow a_y(x,t) = -2A\omega^2 \sin(kx) \sin(\omega t)$$

Mechanical waves: Problem

Problem solving

Mechanical waves: Problem

A guitar string lies along the x-axis when in equilibrium. The end of the string at $x = 0$ (the bridge of the guitar) is fixed. A sinusoidal wave with amplitude $A = 0.750$ mm $= 7.50 \times 10^{-4}$ m and frequency $f = 440$ Hz, travels along the string in the $-x$-direction at 143 m/s. It is reflected from the fixed end, and the superposition of the incident and reflected waves forms a standing wave. (a) Find the equation giving the displacement of a point on the string as a function of position and time.

$$y(x,t) = 2A \sin(kx) \sin(\omega t)$$

$$A = 0.750 \text{ mm} = 7.50 \times 10^{-4} \text{ m}$$

$$\omega = 2\pi f = (2\pi \text{ rad})(440 \text{ s}^{-1}) = 2760 \text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{2760 \text{ rad/s}}{143 \text{ m/s}} = 19.3 \text{ rad/m}$$

Mechanical waves: Problem

(b) Locate the nodes.

v = 143 m/s
f = 440 Hz
A = 0.075 m
ω = 2760 rad/s
k = 19.3 rad/m

The nodes are at $x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$

$f = v / \lambda$ → $\lambda = v/f = (143 \text{ m/s})/(440 \text{ Hz})$

the nodes are at $x = 0, \; 0.163 \text{ m}, \; 0.325 \text{ m},$

Mechanical waves: Problem

(c) Find the amplitude of the standing wave and the maximum transverse velocity and acceleration.

v = 143 m/s
f = 440 Hz
A = 0.075 m
ω = 2760 rad/s
k = 19.3 rad/m

$$y(x,t) = 2A \sin(kx) \sin(\omega t)$$

Amplitude = 2A = 0.15 m

$$v_y(x,t) = 2A\omega \sin(kx) \cos(\omega t)$$

$$v_y(x,t)_{max} = 2A\omega = 4.14 \text{ m/s}$$

$$a_y(x,t) = -2A\omega^2 \sin(kx) \sin(\omega t)$$

$$a_y(x,t)_{max} = 2A\omega^2 = 11426 \text{ m/s}^2$$

Mechanical waves: Stringed instrument

Stringed instrument

Mechanical waves: Stringed instrument

Instrument with strings of length L has nodes at both ends.

Nodes when $\sin(kx) = 0$ $\quad x = 0, \frac{\pi}{k}, \frac{2\pi}{k}, \frac{3\pi}{k}, \ldots = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$

$$L = n\frac{\lambda}{2} \quad (n = 1, 2, 3, \ldots)$$

$$\lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)$$

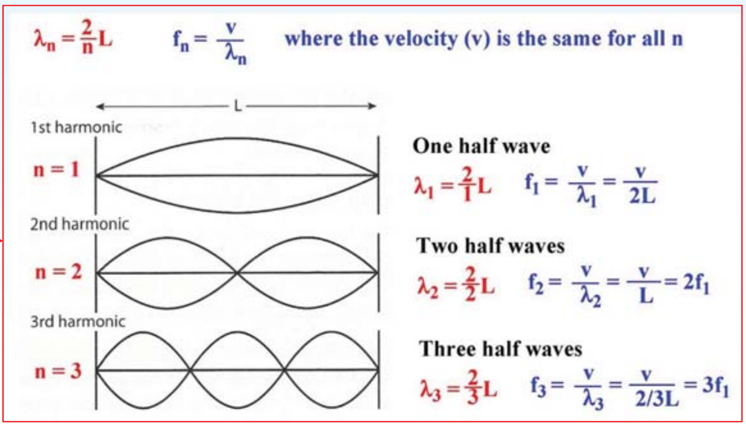
$v = \lambda / T = \lambda f$

$\lambda = v / f$

$f = v / \lambda$

$$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots)$$

f_1, f_2, f_3.... Harmonic frequencies
f_1: Fundamental frequency
f_2, f_3, f_4.... Overtones



Mechanical waves: Stringed instrument

$$f_1 = v/2L$$

$$v = \sqrt{F/\mu}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Long string: Low frequency
Thick string: Low frequency
Large tension: High frequency

A stringed instrument does not produce only harmonic frequencies but a superposition of many normal modes.

Mechanical waves: Problem

Problem solving

Mechanical waves: Problem

build a bass viol with strings of length 5.00 m between fixed points. One string, with linear mass density 40.0 g/m, is tuned to a 20.0-Hz fundamental frequency (the lowest frequency that the human ear can hear). Calculate (a) the tension of this string

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

→

$$F = 4\mu L^2 f_1{}^2 = 4(40.0 \times 10^{-3}\,\text{kg/m})(5.00\,\text{m})^2(20.0\,\text{s}^{-1})^2$$
$$= 1600\,\text{N}$$

Mechanical waves: Problem

(b) the frequency and wavelength on the string of the second harmonic

(c) the frequency and wavelength on the string of the second overtone.

f_1 = 20.0 Hz
L = 5.00 m
μ = 40.0g/m
F = 1600 N

$$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots)$$

$$f_2 = 2f_1 = 2(20.0\text{ Hz}) = 40.0\text{ Hz}$$

$$\lambda_2 = \frac{2L}{2} = \frac{2(5.00\text{ m})}{2} = 5.00\text{ m}$$

$$\lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)$$

(c) The second overtone is the "second tone over" (above) the fundamental—that is, $n = 3$. Its frequency and wavelength are

$$f_3 = 3f_1 = 3(20.0\text{ Hz}) = 60.0\text{ Hz}$$

$$\lambda_3 = \frac{2L}{3} = \frac{2(5.00\text{ m})}{3} = 3.33\text{ m}$$

Mechanical waves: Problem

What are the frequency and wavelength of the sound waves produced in the air when the string in Example 15.7 is vibrating at its fundamental frequency? The speed of sound in air at 20°C is 344 m/s.

f_1 = 20.0 Hz
L = 5.00 m
μ = 40.0g/m
F = 1600 N

$$v = \lambda / T = \lambda f$$

$$\lambda = v / f$$

$$f = f_1 = 20.0\text{ Hz}$$

$$\lambda_{1(\text{sound})} = \frac{v_{\text{sound}}}{f_1} = \frac{344\text{ m/s}}{20.0\text{ Hz}} = 17.2\text{ m}$$