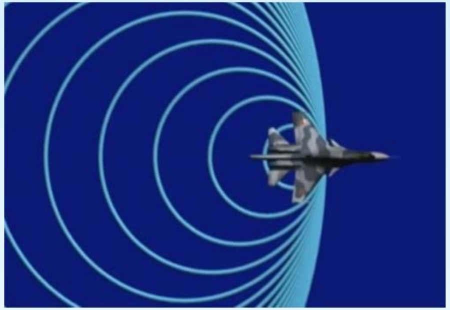
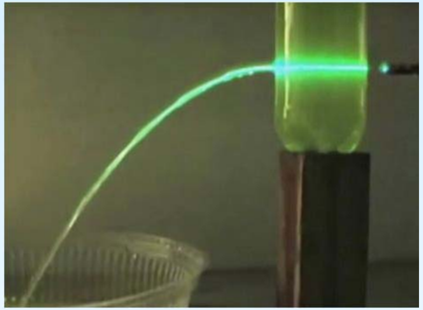
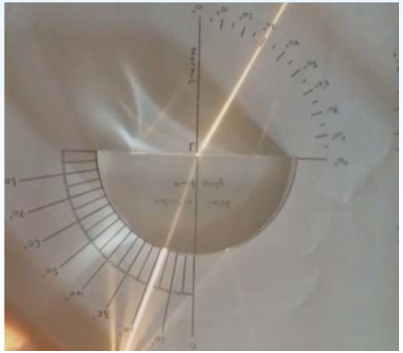
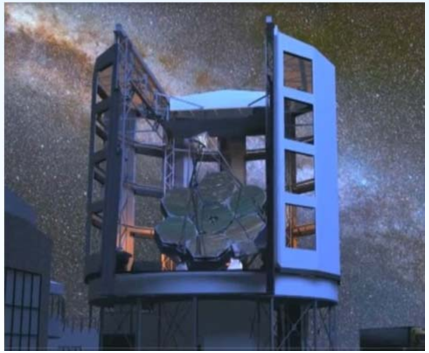
# Wavemechanics and optics





## Chapter 16 - Sound

Vincent Hedberg - Lunds Universitet

# Content

# Part 1. Sound = Pressure waves


Carreta Treme Treme

A Brazilian loud speaker truck

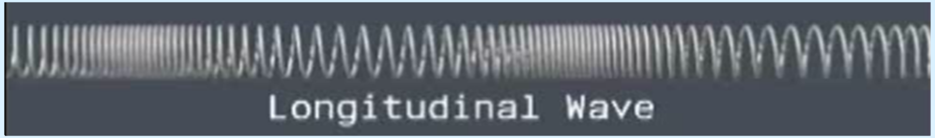
192 loudspeakers
33 amplifiers
240 batteries

# Sound: Longitudinal waves

## Mechanical longitudinal wave:

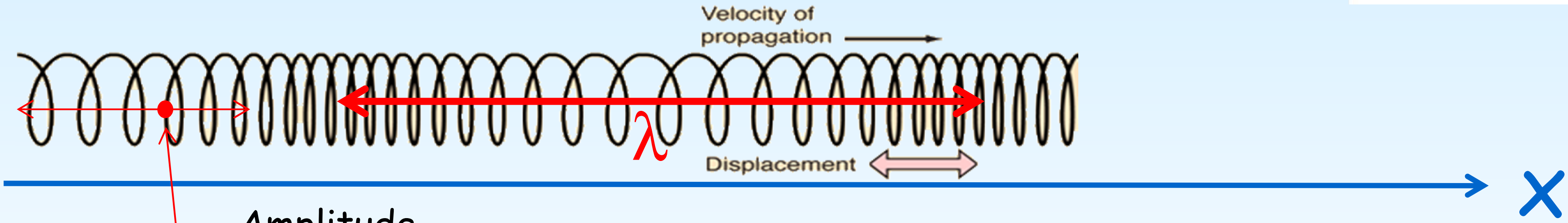
**The medium is moving in the same direction as the wave**

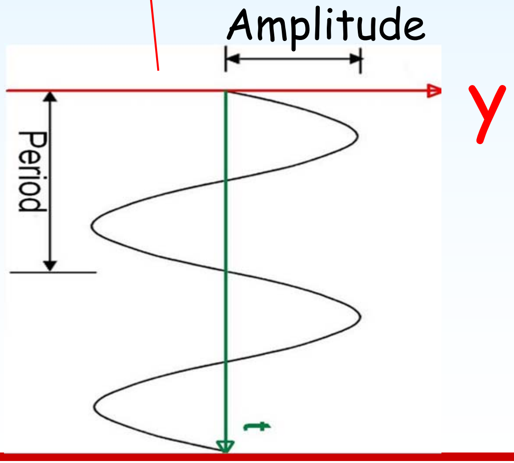
# Sound: Longitudinal waves




Wave velocity

$$v = f \cdot \lambda = \frac{\omega}{k}$$




Mechanical longitudinal sinusoidal wave

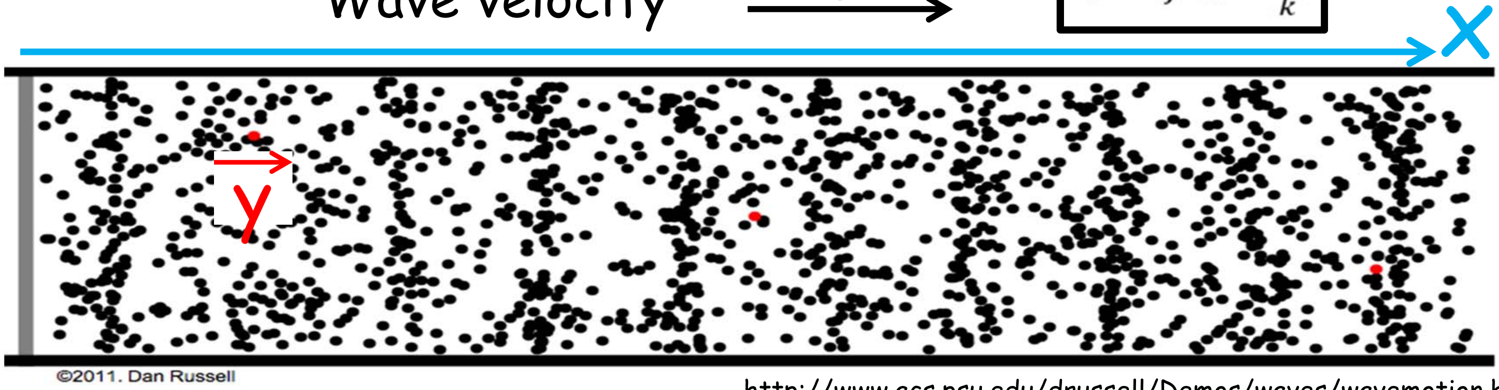
$$y(x, t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$$

# Sound & Pressure waves

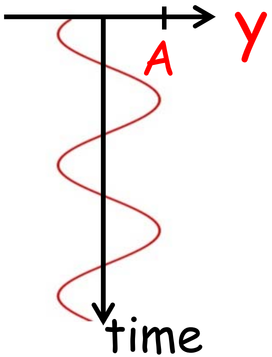
Wave velocity $\xrightarrow{\;v\;}$ $v = f \cdot \lambda = \frac{\omega}{k}$

A piston moves in and out and create a longitudinal sine wave




http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

**y:**
Movement of the air molecules




**Wavefunction:**

$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

How does the pressure vary with x and time?

# Sound & Pressure waves

Given

The wavefunction: $y(x, t) = A\cos(kx - \omega t)$

Goal

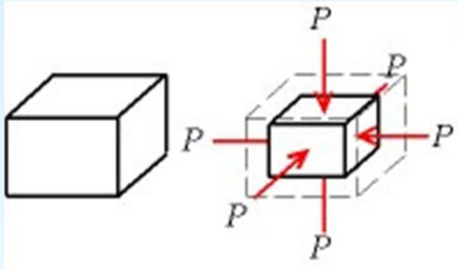
Derive a function for the pressure !

How

See how a pressure change causes a volume change in a small cylindrical volume element.

# Sound & Pressure waves


## Bulk modulus




**The bulk modulus measures a mediums resistance to uniform compression:**

$$B = -V\frac{\Delta p}{\Delta V}$$

**Δp → Pressure change**

**ΔV → Volyme change**

Unit: $N/m^2$

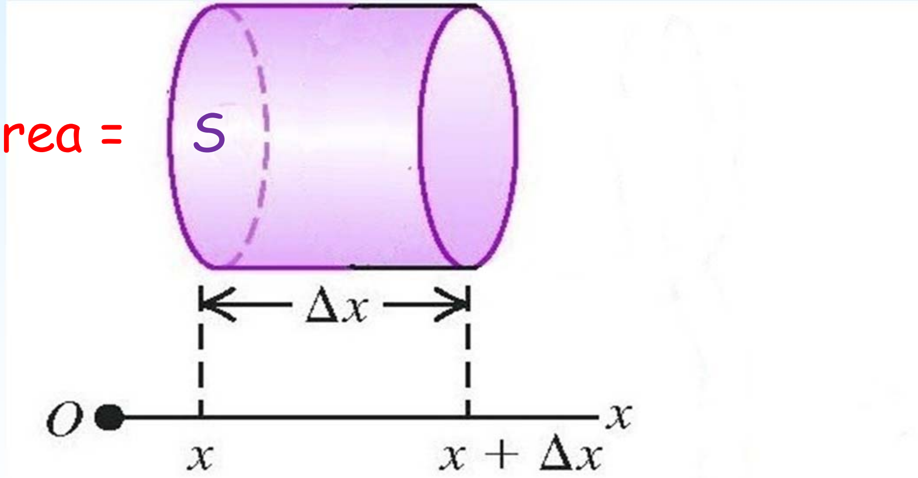
The change in pressure after a change of volume:

$$\Delta p = -B\ \Delta V/V$$

**Pressure increase: $\Delta p > 0$ and $\Delta V < 0$**

A sound wave passes a cylinder shaped volume element:

Area = 



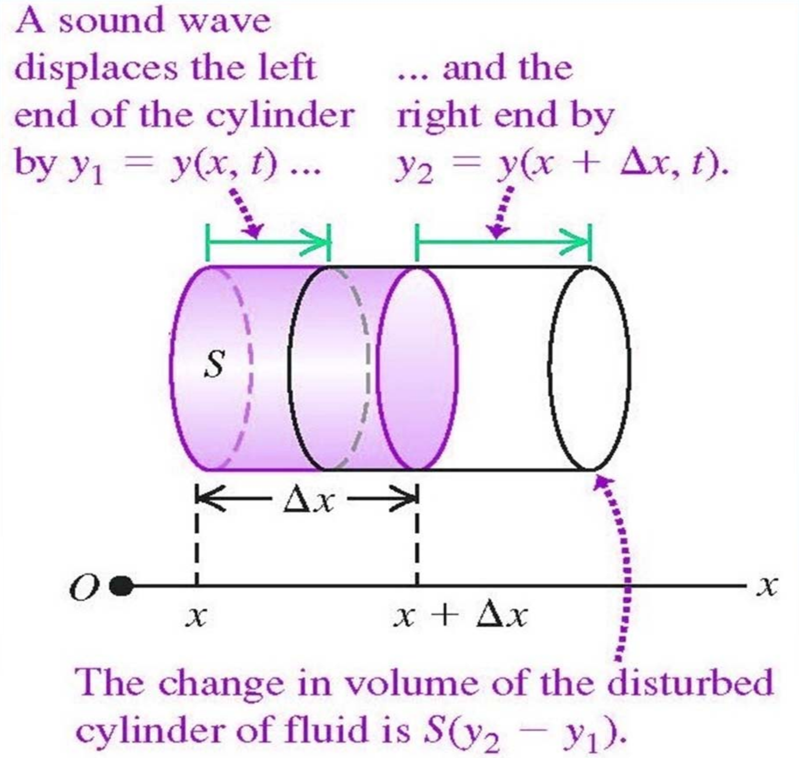

Volume:

$V = S\,\Delta x$

How is this volume changed by a sound wave?

How does the pressure change?

# Sound & Pressure waves




$V_1 = S\Delta x$ $\quad V_2 = S(x+\Delta x+y_2-(x+y_1))$

**Pressure change:** $\Delta p = -B\dfrac{\Delta V}{V}$

Volume: $V_1 = S\Delta x$

Volume change: $\Delta V = V_2 - V_1 = S(y_2 - y_1)$

$$\Delta V = S[y(x+\Delta x,t) - y(x,t)]$$

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[y(x+\Delta x, t) - y(x, t)]}{S\,\Delta x} = \frac{\partial y(x, t)}{\partial x}$$

**Time-dependent pressure variations**

$$p(x, t) = -B\frac{\partial y(x, t)}{\partial x}$$

# Sound & Pressure waves

Pressure variations:

$$p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$$

+

Wave function:

$$y(x,t) = A\cos(kx - \omega t)$$

=

Pressure function:

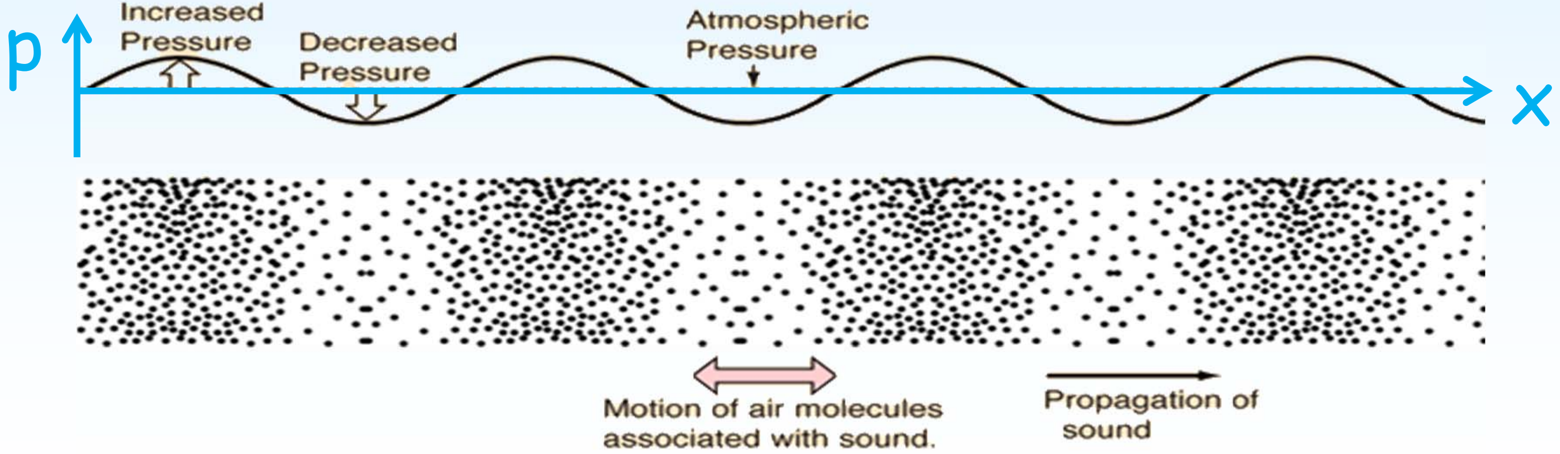
$$p(x,t) = BkA\sin(kx - \omega t)$$

# Sound & Pressure waves

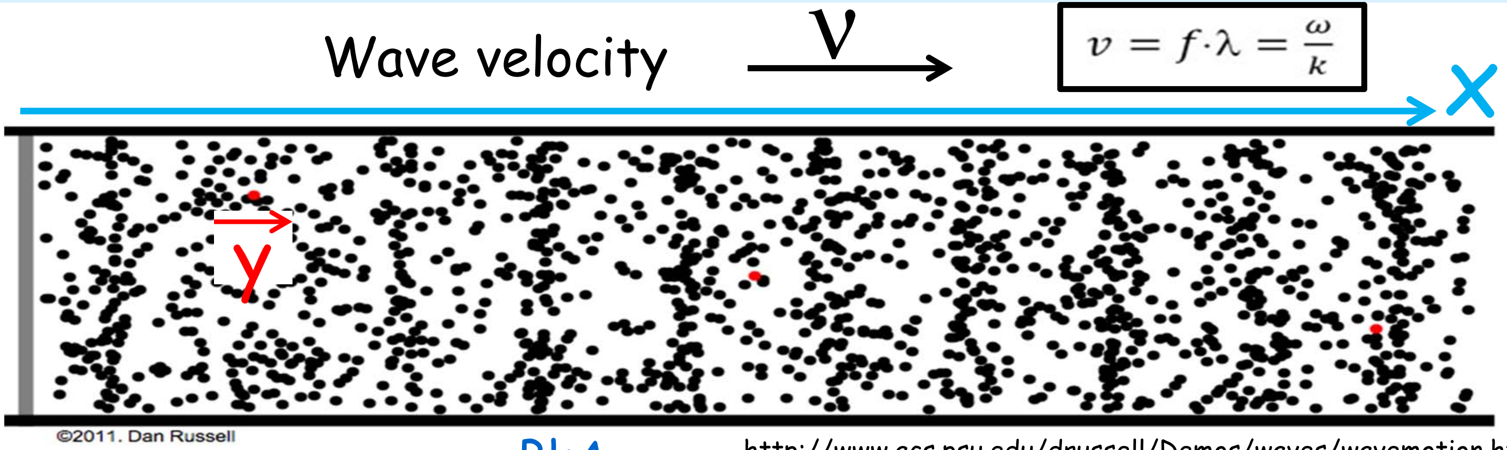
Pressure function: $$p(x,t) = BkA\sin(kx - \omega t)$$

Pressure amplitude: $$p_{\max} = BkA$$ Maximum pressure variation

At time = t



# Sound & Pressure waves



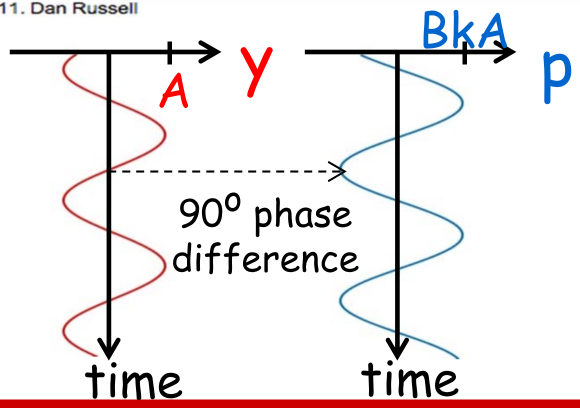

$$v = f \cdot \lambda = \frac{\omega}{k}$$

A piston moves in and out:

http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

**y**: Movement of air molecules

**p**: Pressure at position x




**Wave function:**

$$y(x,t) = A\cos(kx - \omega t)$$

**Pressure function:**

$$p(x,t) = BkA\sin(kx - \omega t)$$

## Human hearing

Audible frequency range: 20-20 kHz

Loudness: Higher pressure amplitud → Larger loudness (at the same frequency)

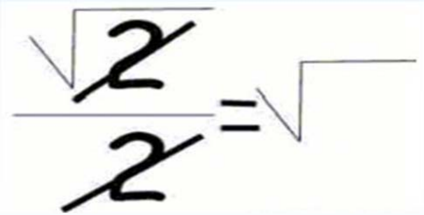
Changed frequency → Changed loudness (at the same amplitude)

Pitch: Higher frequency → Higher pitch

Higher pressure amplitude → Normally higher pitch

Timbre: Instruments with the same fundamental frequency may have different content of overtones e.g. different timbre

# Part 2. Problems



# Sound: Problems

A sinusoidal sound wave has a frequency of 1000 Hz and a pressure amplitude of $3.0 \times 10^{-2}$ Pa.

Air: $v = 344$ m/s, $B = 1.42 \times 10^5$ Pa

What will be the maximum movement of the air due to this sound wave ?

$$p_{max} = BkA$$

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}$$

$$A = \frac{p_{max}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}$$

# Part 3. The speed of sound in a liquid

SONAR

Sound Navigation
And Ranging

www.youtube.com/watch?v=wTcaFYeUR10

# The speed of sound

Given

Pressure change from a volume change: $$\Delta p = -B \frac{\Delta V}{V}$$

Goal

Derive a formula for the speed of sound in a liquid !

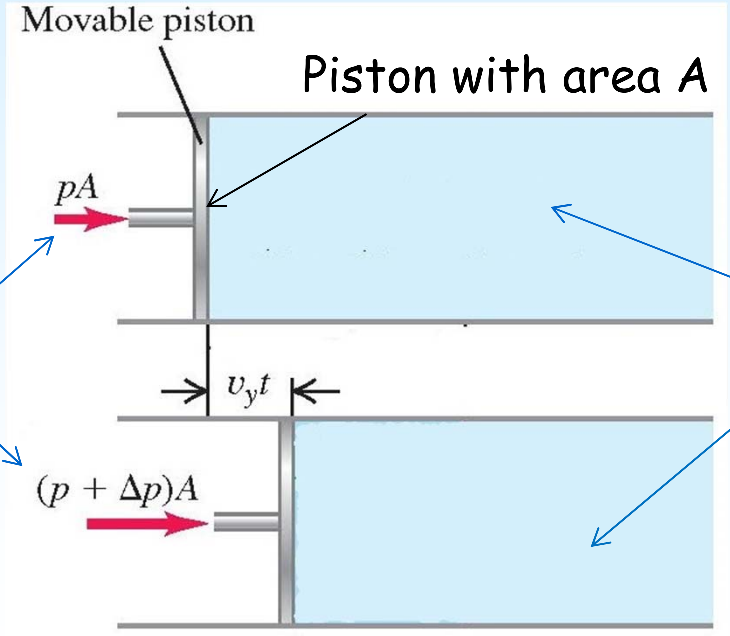
How

See how a pressure change causes a volume change in a small cylindrical volume element.

# The speed of sound

## Derivation of the formula for the sound velocity in a liquid

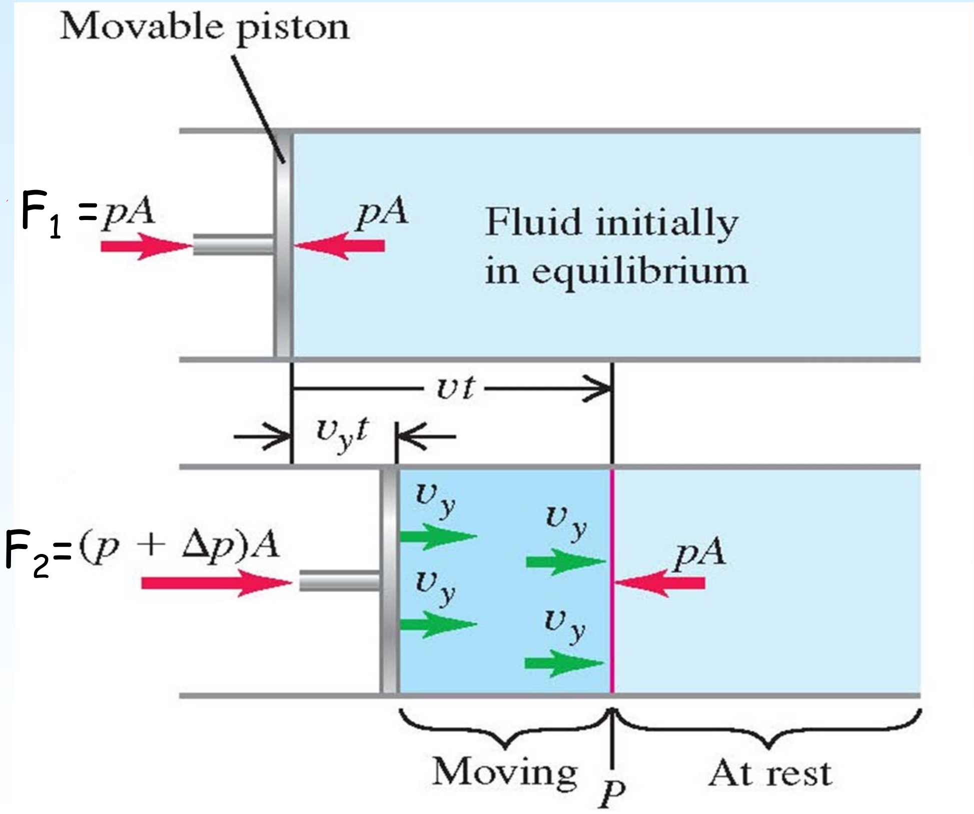
Assume: A piston is pushed into a cylinder with velocity $v_y$ and creates a pressure wave.




Force on piston
=
Pressure x Area

Cylinder
filled with liquid
with the bulk modulus B
and the density ρ

# The speed of sound




## Variables

Time = 0:

$p$ = Pressure in the liquid

$A$ = Area of the piston

$F_1$ = Force on the piston

$\rho$ = Density of the liquid

Time = t:

$v_y$ = Velocity of piston

$v$ = Velocity of wave

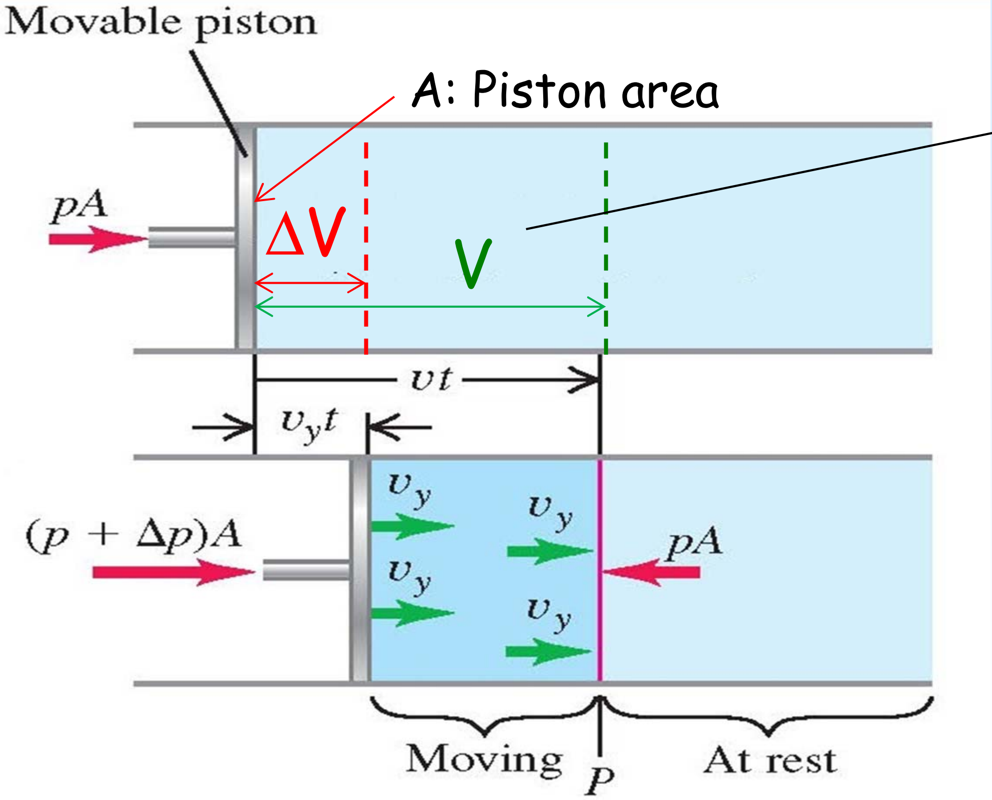
$v_y t$ = Distance the piston has moved

$vt$ = Distance the wave has moved

$\Delta p$ = Pressure change

$F_2$ = Force on the piston

# The speed of sound




$V = Avt$ Volume

$\Delta V = -Av_y t$ Change of volume

Decreasing volume

Pressure change: $\Delta p = -B \frac{\Delta V}{V}$

$$\Delta p = B \frac{v_y}{v}$$

$v_y$ ← Piston velocity

$v$ ← Wave velocity

# Kinematics

Momentum: $\vec{p} = m\vec{v}$

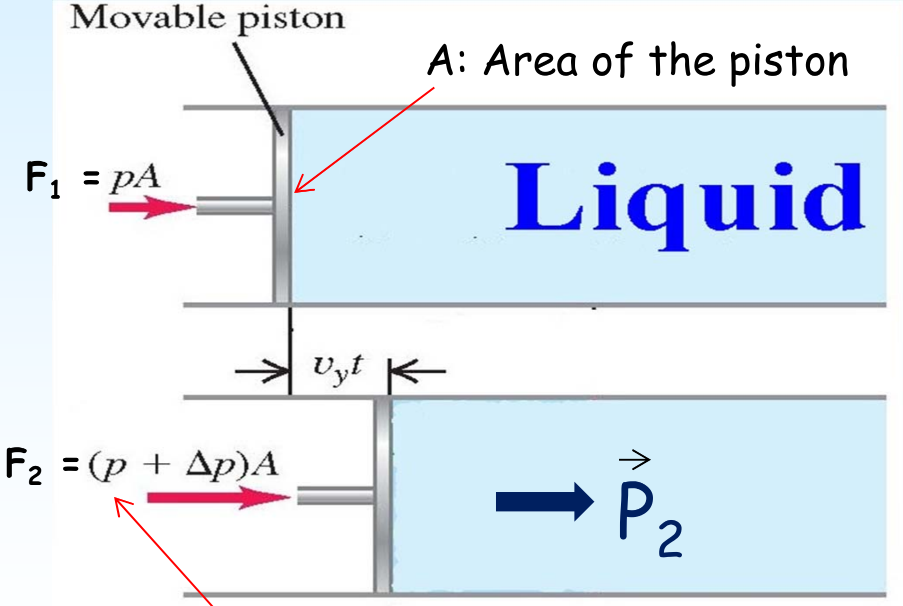
Impulse: $\vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F}\, dt = (\vec{F}_2 - \vec{F}_1)\, t$

The Momentum-Impulse Theorem: $\vec{J} = \vec{p}_2 - \vec{p}_1$

The impulse is equal to the change of momentum !

# The speed of sound

The impulse if a piston is pushed into a cylinder with the velocity $v_y$ and sets the volume element V in motion:




Pressure

**Method 1:**

$$\vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F}\, dt = (\vec{F}_2 - \vec{F}_1)\, t$$

$$\vec{J} = (\vec{F}_2 - \vec{F}_1)\, t = \Delta p\, A\, t$$

**Method 2:**

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \quad \text{where} \quad \vec{p} = m\vec{v}$$

$$\vec{J} = \vec{P}_2 - \vec{P}_1 = \vec{P}_2 - 0 = m\, v_y$$

The momentum of the water

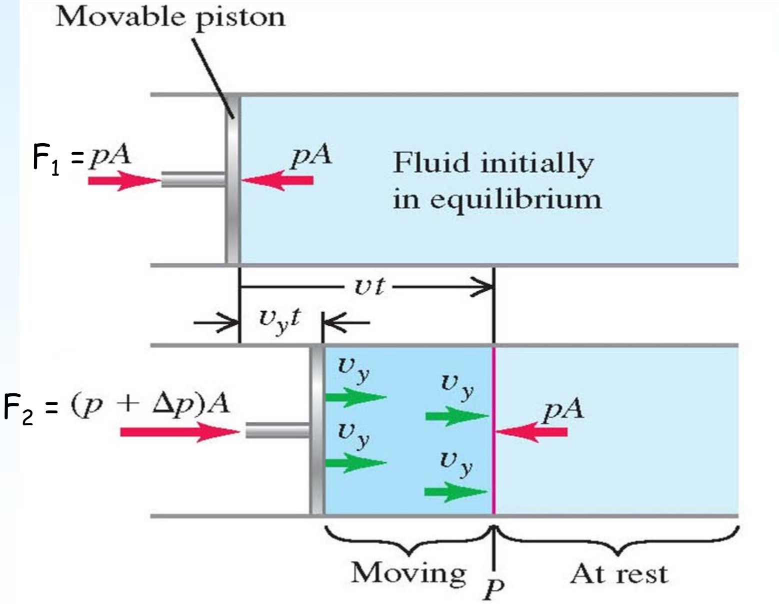
# The speed of sound

**Method 1:**

$$\vec{J} = \int_{t_1}^{t_2} \Sigma \vec{F}\, dt$$

$$\vec{J} = (\vec{F}_2 - \vec{F}_1)\, t = \Delta p\, A\, t = B\, A\, t\, v_y / v$$

$$\Delta p = B\frac{v_y}{v}$$




**Method 2:**

$$\vec{J} = \vec{p}_2 - \vec{p}_1 = \vec{p}_2$$

$$\vec{J} = \vec{P}_2 = m\, v_y = \rho\, V\, v_y = \rho\, A\, v\, t\, v_y$$

**Method 1 = Method 2**

$$\vec{J} = \vec{J} \qquad B\frac{v_y}{v}At = \rho v t A v_y$$

$$v = \sqrt{\frac{B}{\rho}} \quad \text{(speed of a longitudinal wave in a fluid)}$$

# The speed of sound

String: $v = \sqrt{\frac{F}{\mu}}$

F: String tension
μ: Mass per unit length

Liquids: $v = \sqrt{\frac{B}{\rho}}$

B: The Bulk modulus
ρ: The density

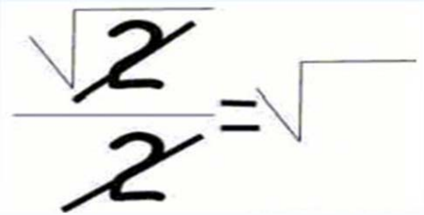
Solid material: $v = \sqrt{\frac{Y}{\rho}}$

Y: The Young's module
ρ: The density

Gas: $v = \sqrt{\frac{B}{\rho}}$

B: The Bulk modulus
ρ: The density

# Part 4. Problems



# Sound: Problems

A human can hear frequencies between 20 and 20000 Hz. What wavelengths does this correspond to ?

Assume that v = 344 m/s

$$v = f \cdot \lambda = \frac{\omega}{k}$$

→

$\lambda = v / f$

$\lambda = 344 / 20 = 17$ m for f = 20 Hz

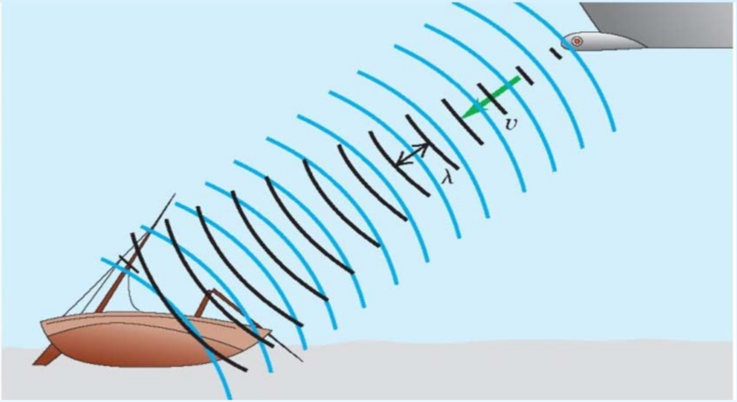
$\lambda = 344 / 20000 = 1.7$ cm for f = 20 kHz

# Sound: Problems

A sonar system sends out sound waves at a frequency of 262 Hz.

What will be the speed and wavelength of this sound wave if B = $2.18 \times 10^9$ Pa ?

What will be the velocity and wavelength of the wave in air if B = $1.42 \times 10^5$ Pa and the density 1.225 kg/m$^3$




$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \times 10^9 \text{ Pa}}{1.00 \times 10^3 \text{ kg/m}^3}} = 1480 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{1480 \text{ m/s}}{262 \text{ s}^{-1}} = 5.65 \text{ m}$$

V = 340 m/s in air

λ = 1.3 m in air

# Part 5. The power of sound

The highest sound ever measured:

When the Krakatoa volcano exploded in 1883, the sound was heard in Perth at a distance of 3100 km.

The explosion was equivalent to 10000 atom bombs.

# Sound: Power & Intensity

## General for mechanical waves

Wave power (P): The instantaneous rate at which energy is transfered along the wave. (P = energy per unit of time)
Unit: W or J/s

Wave intensity (I): Average power per unit area through a surface perpendicular to the wave direction. (I =power per unit of area).
Unit: $W/m^2$

$$I = P_{av} / A_{rea}$$

The power in general:

$$P = \vec{F} \cdot \vec{v}$$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):

$$P(x,t) = F_y(x,t)\,v_y(x,t)$$

# Sound: Power & Intensity

**Wave power (P):**

$$P(x,t) = F_y(x,t)v_y(x,t)$$

**Pressure function (p):**

$$p(x,t) = BkA\sin(kx - \omega t)$$

Pressure = Force per unit area

**Wavefunction (y):**

$$y(x,t) = A\cos(kx - \omega t)$$

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

Wavepower per unit of area:

$$\mathrm{P(x,t)} = p(x,t)v_y(x,t) = [BkA\sin(kx - \omega t)][\omega A\sin(kx - \omega t)]$$

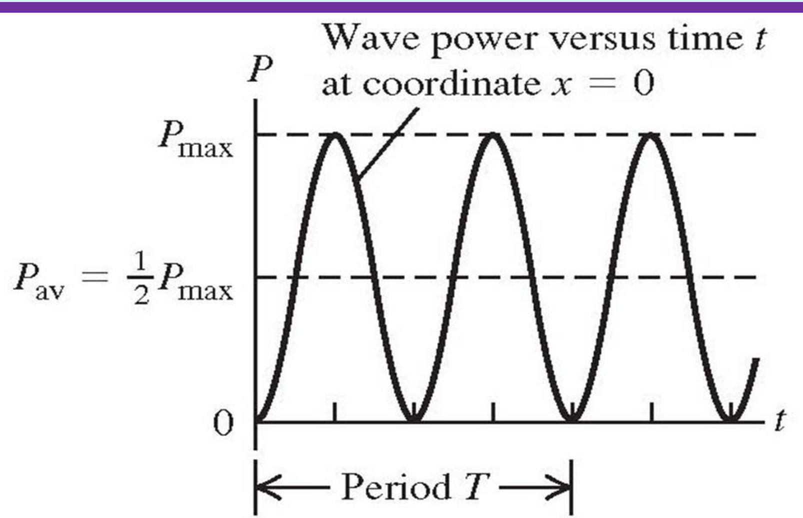
$$= B\omega kA^2\sin^2(kx - \omega t)$$

Power per m$^2$    Pressure

# Sound: Power & Intensity

Wavepower per unit of area:

$$P(x,t)/Area = B\omega k A^2 \sin^2(kx - \omega t)$$




Maximum power: $$\frac{P_{max}}{Area} = B\omega k A^2$$

Average power: $$\frac{P_{av}}{Area} = \frac{1}{2} B\omega k A^2$$

## Exercise in algebra:

$$v = \frac{\omega}{k}$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$k = \frac{\omega}{\sqrt{\frac{B}{\rho}}}$$

$$v = \sqrt{\frac{B}{\rho}} \Rightarrow \sqrt{B} = v\sqrt{\rho}$$

$$\frac{P_{av}}{Area} = \frac{1}{2}B\omega k A^2 = \frac{1}{2}B\omega \frac{\omega}{\sqrt{\frac{B}{\rho}}}A^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

$$\frac{P_{av}}{Area} = \frac{1}{2}B\omega k A^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

# Sound: Power & Intensity

## Compare power for string and sound:

Power general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

**Wave power - string:**

$$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$$

$$P_{max} = Fk\omega A^2 = \sqrt{\mu F}\omega^2 A^2$$

$$P_{av} = \frac{1}{2}Fk\omega A^2 = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$$

Unit: N

**Wave power - sound:**

$$P(x,t)/Area = B\omega k A^2 sin^2(kx - \omega t)$$

$$P_{max}/Area = B\omega k A^2 = \sqrt{\rho B}\omega^2 A^2$$

$$P_{av}/Area = \frac{1}{2}B\omega k A^2 = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$$

Unit: N/m$^2$

# Part 6. Intensity = average power per unit area

When the Gulf Corvina fish spawn, it sends out audio signals that can reach an intensity level of 177 dB (202 dB = $10^8$ W/m$^2$ for an entire shoal).

This is one of the loudest sounds in the animal world and can cause hearing damage to dolphins, seals and sea lions.

# Sound: Intensity

Average power of a soundwave ($P_{av}$):
Unit: W or J/s

$$\frac{P_{av}}{Area} = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

Wave intensity (I): Average power per unit area through a surface perpendicular to the wave direction.
Unit: $W/m^2$

$$\mathrm{I} = \frac{Power}{Area}$$

$$I = \frac{P_{av}}{Area} = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

# Sound: Power & Intensity

$$I = \frac{P_{av}}{Area} = \frac{1}{2} B\omega k A^2$$

Pressure function: $p(x,t) = BkA\sin(kx - \omega t)$

Pressure amplitude: $p_{max} = BkA \Rightarrow A^2 = \frac{{p_{max}}^2}{B^2k^2}$

$$I = \frac{1}{2} B\omega k A^2 = \frac{1}{2} B\omega k \frac{{p_{max}}^2}{B^2k^2} = \frac{1}{2B}\frac{\omega}{k}{p_{max}}^2 = \frac{1}{2B}\sqrt{\frac{B}{\rho}}{p_{max}}^2 = \frac{{p_{max}}^2}{2\sqrt{\rho B}}$$

$$v = \frac{\omega}{k}$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$\frac{\omega}{k} = \sqrt{\frac{B}{\rho}}$$

The intensity is proportional to the square of the pressure amplitude.

$$I = \frac{{p_{max}}^2}{2\sqrt{\rho B}}$$

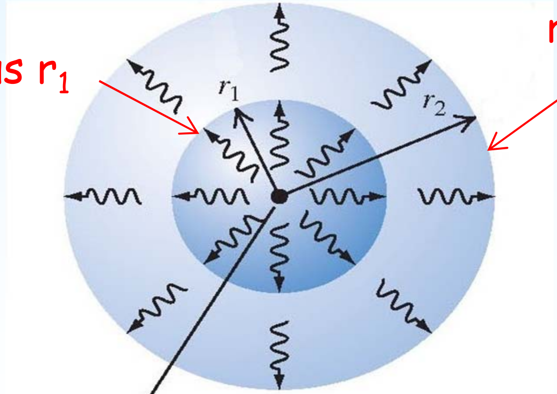
# Sound: Power & Intensity

**Wave intensity (I):** The speed at which the wave transports energy through a surface perpendicular to the direction of the wave (I = Average power per area unit = energy per time and area unit). Units: $W/m^2 = J/s/m^2$

$$I = P_{av} / A_{rea}$$

Sphere with radius $r_1$

Sphere with radius $r_2$




Source with the average power $P_{av}$

The intensity through a sphere with radius $r_1$

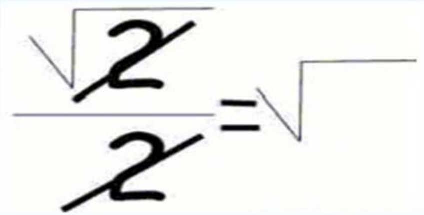
$$I_1 = \frac{P_{av}}{4\pi r_1^2}$$

Ignoring power losses:

$$4\pi r_1^2 I_1 = 4\pi r_2^2 I_2$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

# Part 7. Problems



# Sound: Problems

A siren sends out sound waves uniformly in all directions. The sound intensity is 0.250 W/m$^2$ at a distance of 15.0 m.

At what distance is the intensity 0.010 W/m$^2$ ?

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \qquad \text{(inverse-square law for intensity)}$$

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (15.0\text{ m})\sqrt{\frac{0.250\text{ W/m}^2}{0.010\text{ W/m}^2}} = 75.0\text{ m}$$

# Sound: Problems

Calculate the sound intensity if the pressure amplitude is $3.0 \times 10^{-2}$ Pa, the air density is 1.20 kg/m$^3$ and the speed of sound is 344 m/s !

$$I = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$\sqrt{\rho B} = v\,\rho$$

$$I = \frac{p_{\max}{}^2}{2\rho v} = \frac{(3.0 \times 10^{-2}\ \mathrm{Pa})^2}{2(1.20\ \mathrm{kg/m^3})(344\ \mathrm{m/s})}$$

$$= 1.1 \times 10^{-6}\ \mathrm{J/(s \cdot m^2)} = 1.1 \times 10^{-6}\ \mathrm{W/m^2}$$

# Sound: Problems

What is the pressure amplitude of a sound wave with f = 20 Hz if it has the same intensity as a sound wave with f = 1000 Hz, $I = 1.1 \times 10^{-6}$ W/m$^2$ and $p_{max} = 3.0 \times 10^{-2}$ Pa. Assume that $\rho = 1.20$ kg/m$^3$ and $v = 344$ m/s

**Wave 1:** f = 1000 Hz, $p_{max} = 3.0 \times 10^{-2}$ Pa, $\rho = 1.20$ kg/m$^3$, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m$^2$

**Wave 2:** f = 20 Hz, $p_{max}$ = ???????????, $\rho = 1.20$ kg/m$^3$, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m$^2$

$$I = \frac{{p_{max}}^2}{2\sqrt{\rho B}}$$

Since $\rho B$ = constant and $I_1 = I_2$ then follows that $p_{max2} = p_{max1} = 3.0\times10^{-2}$ Pa

**Wave 2:** f = 20 Hz, $p_{max} = 3.0 \times 10^{-2}$ Pa, $\rho = 1.20$ kg/m$^3$, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m$^2$

# Sound: Problems

What is the displacement amplitude of Wave 2 in the previous problem ?

Wave 2: $f = 20$ Hz, $p_{max} = 3.0 \times 10^{-2}$ Pa, $\rho = 1.20$ kg/m$^3$, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m$^2$

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$$

$$I = \frac{p_{max}{}^2}{2\sqrt{\rho B}} \Rightarrow \sqrt{\rho B} = p_{max}{}^2/2I$$

$\Rightarrow I = (p_{max}{}^2/2I)\,\omega^2 A^2/2$

$I = (p_{max}{}^2/2I)\,\omega^2 A^2/2 \Rightarrow I^2 = p_{max}{}^2\omega^2 A^2/4 \Rightarrow I = p_{max}\omega A/2$

$A = 2I / p_{max}\omega = 2 \times 1.1 \times 10^{-6} / (3.0 \times 10^{-2} \times 2\pi \times 20) = 0.58\ \mu$m

# Sound: Problems

At a concert you want a sound intensity that is 1 W/m$^2$ at a distance of 20 m from the speakers.
What output power do the speakers need?

Intensity is the average power per unit area: $I = P_{av} / A_{rea}$

The intensity through a sphere with radius r: $I = \frac{P}{4\pi r^2}$

The intensity through a hemisphere with radius r: $I = \frac{P}{2\pi r^2}$

$$P = 2\,\pi\, r^2\, I = 2.5\ \text{kW}$$

# Part 8. The decibel scale

Pain threshold:
120 dB = 1 $W/m^2$

Gulf Corvina:
200 dB = $10^8$ $W/m^2$

Saturn V rocket:
220 dB = $10^{10}$ $W/m^2$

Krakatoa:
310 dB = $10^{19}$ $W/m^2$

# Sound: Decibel

Intensity level (β) with decibel (dB) as the unit:

$$\beta = 10\,log\frac{I}{I_0} \quad \longleftrightarrow \quad I = I_0 \cdot 10^{\beta/10}$$

$I_0 = 10^{-12}$ W/m$^2$ is a reference level.
$I_0$ = approximately the limit of human hearing.

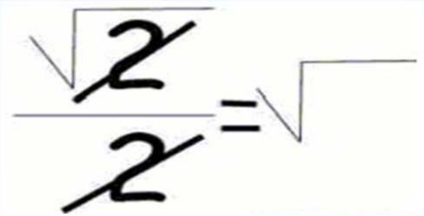
$\beta$ = 0 dB when $I = I_0$
$\beta$ = 120 dB when $I = 1$ W/m$^2$

# Sound: Decibel

| Source or Description of Sound | Sound Intensity Level, $\beta$ (dB) | Intensity, $I$ (W/m$^2$) |
|---|---|---|
| Military jet aircraft 30 m away | 140 | $10^2$ |
| Threshold of pain | 120 | 1 |
| Riveter | 95 | $3.2 \times 10^{-3}$ |
| Elevated train | 90 | $10^{-3}$ |
| Busy street traffic | 70 | $10^{-5}$ |
| Ordinary conversation | 65 | $3.2 \times 10^{-6}$ |
| Quiet automobile | 50 | $10^{-7}$ |
| Quiet radio in home | 40 | $10^{-8}$ |
| Average whisper | 20 | $10^{-10}$ |
| Rustle of leaves | 10 | $10^{-11}$ |
| Threshold of hearing at 1000 Hz | 0 | $10^{-12}$ |
| Saturn V rocket: | 220 | $10^{10}$ |

A Saturn V rocket produces a 100 million times higher intensity than a jet aircraft !

# Part 9. Problems



# Sound: Problems

After 10 minutes at 120 dB, the human hearing threshold is temporarily changed from 0 dB to 28 dB if f = 1000 Hz.

After 10 years of 92 dB, the limit for human hearing is permanently changed from 0 dB to 28 dB if f = 1000 Hz.

What sound intensity corresponds to 28 dB and 92 dB ?

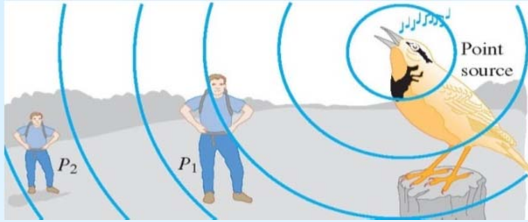
$$\beta = 10 \, log \frac{I}{I_0}$$

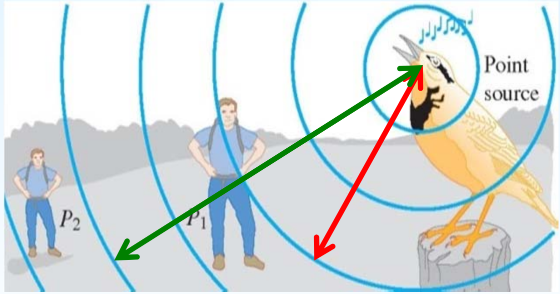
$$I = I_0 \cdot 10^{\beta/10}$$ with $I_0 = 10^{-12}$ W/m$^2$

$$I_{28\ \mathrm{dB}} = (10^{-12}\ \mathrm{W/m^2})10^{2.8} = 6.3 \times 10^{-10}\ \mathrm{W/m^2}$$

$$I_{92\ \mathrm{dB}} = (10^{-12}\ \mathrm{W/m^2})10^{9.2} = 1.6 \times 10^{-3}\ \mathrm{W/m^2}$$

A bird sings with constant power.
How many decibels does the intensity level go down if the listener doubles the distance to the bird?




$r_2 = 2r_1$ $\beta_2$ $I_2$

$r_1$ $\beta_1$ $I_1$

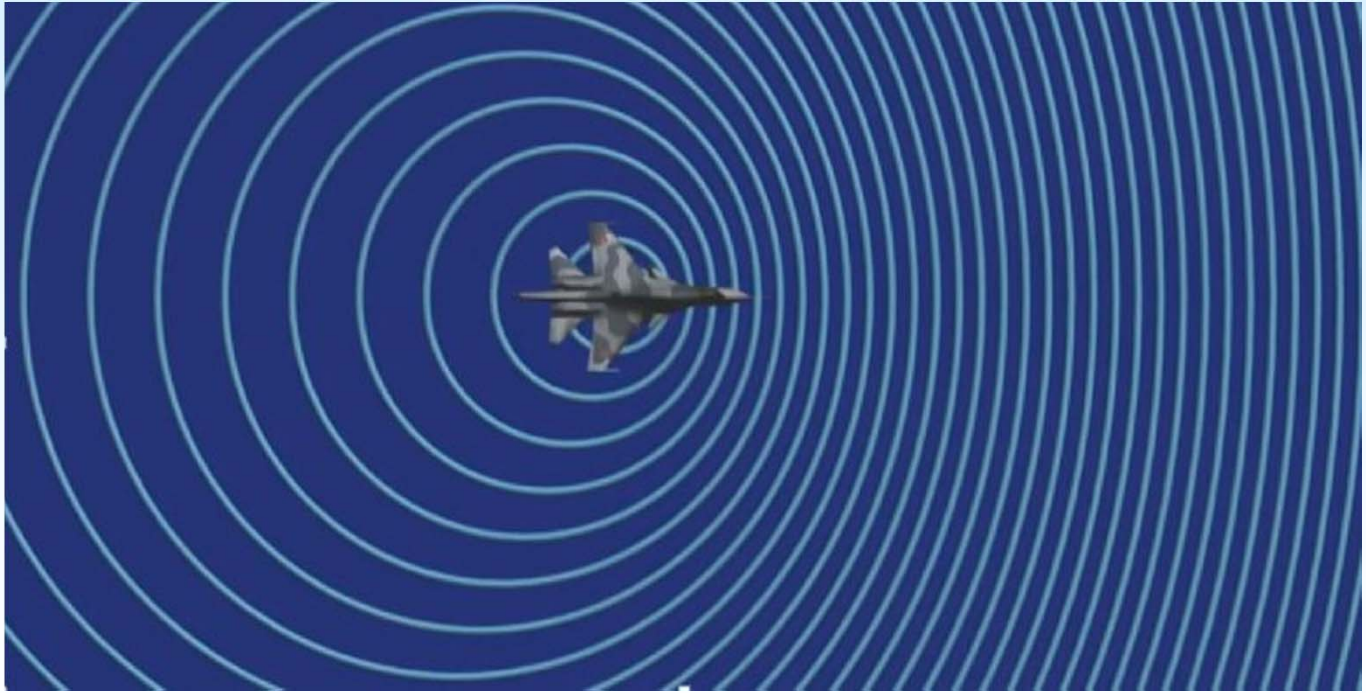
$$\frac{I_1}{I_2} = \frac{r_2{}^2}{r_1{}^2} = \frac{4r_1{}^2}{r_1{}^2} = 4$$

$$\beta = (10\text{ dB}) \log \frac{I}{I_0}$$

$$\begin{aligned} \beta_2 - \beta_1 &= (10\text{ dB})\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right) \\ &= (10\text{ dB})[(\log I_2 - \log I_0) - (\log I_1 - \log I_0)] \\ &= (10\text{ dB})\log\frac{I_2}{I_1} \end{aligned}$$

$$\beta_2 - \beta_1 = (10\text{ dB})\log\frac{I_2}{I_1} = (10\text{ dB})\log\tfrac{1}{4} = -6.0\text{ dB}$$

# Part 10. Doppler effect



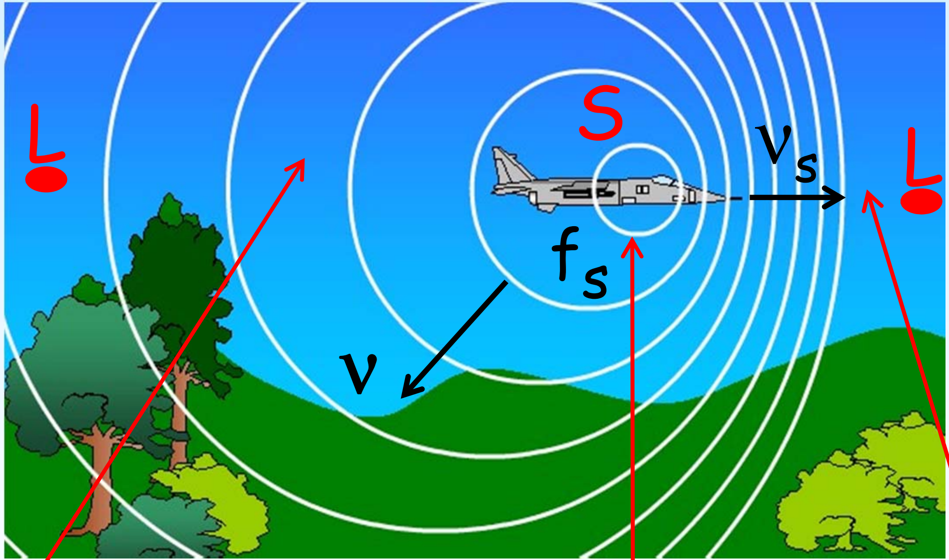
https://www.youtube.com/watch?v=-Zu5SGllmwc

# Sound: The Doppler effect

The time for a sound wave to reach a listener (L) gets longer if the source (S) is moving away.

$\lambda_{behind}$ longer

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S}$$



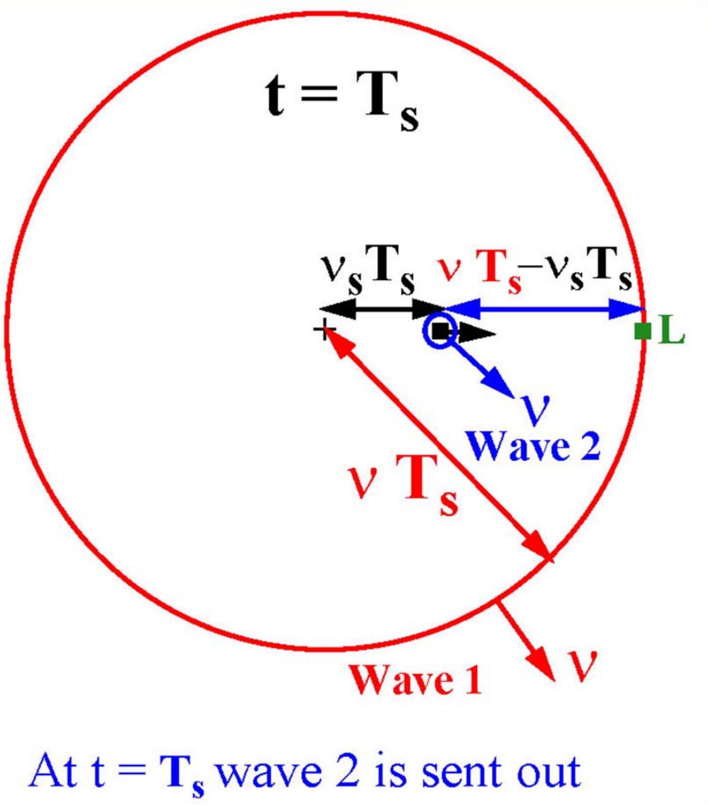

$$\lambda = \frac{v}{f_s}$$

The time for a sound wave to reach a listener (L) gets shorter if the source is moving closer.

$\lambda_{in\ front}$ shorter

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

# Sound: The Doppler effect




- $v$ does not change because of $v_s$ since it only depends on the medium.

The time it takes for the listener to detect wave 2 is given by $T_L$ = distance/speed:

$$T_L = \frac{\vartheta T_s - \vartheta_s T_s}{\vartheta}$$

$T_L$ is also the time between the two waves (the period).

$$f_L = \frac{1}{T_L} = \frac{\vartheta}{\vartheta - \vartheta_s} f_S$$

$$\lambda_L = \frac{\upsilon}{f_L} = \frac{\vartheta - \vartheta_s}{f_s}$$

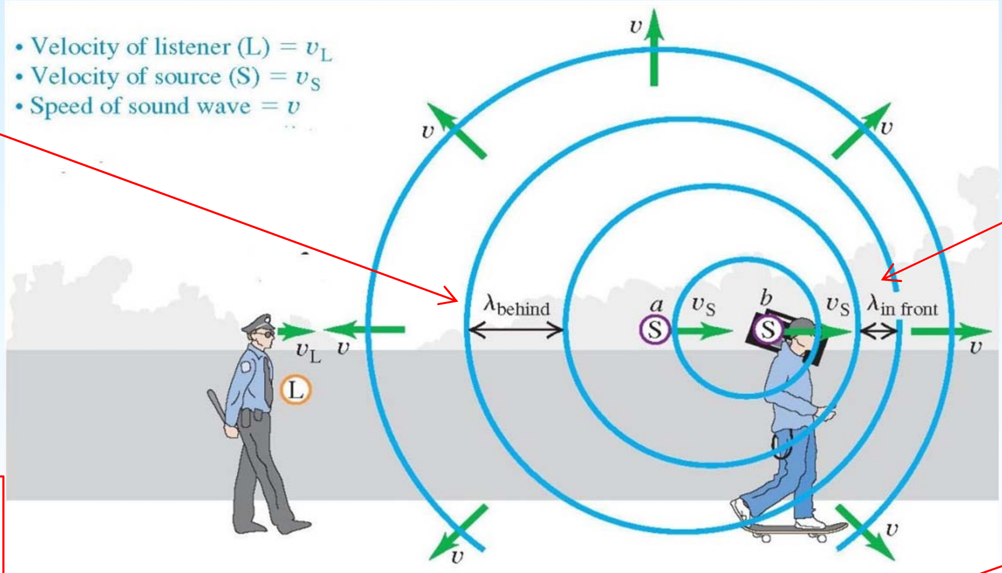
# Sound: The Doppler effect

## More complicated: The listener moves too

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S}$$

General rule:

$$f = \frac{v}{\lambda}$$




$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

The wave is approaching L by V + $V_L$

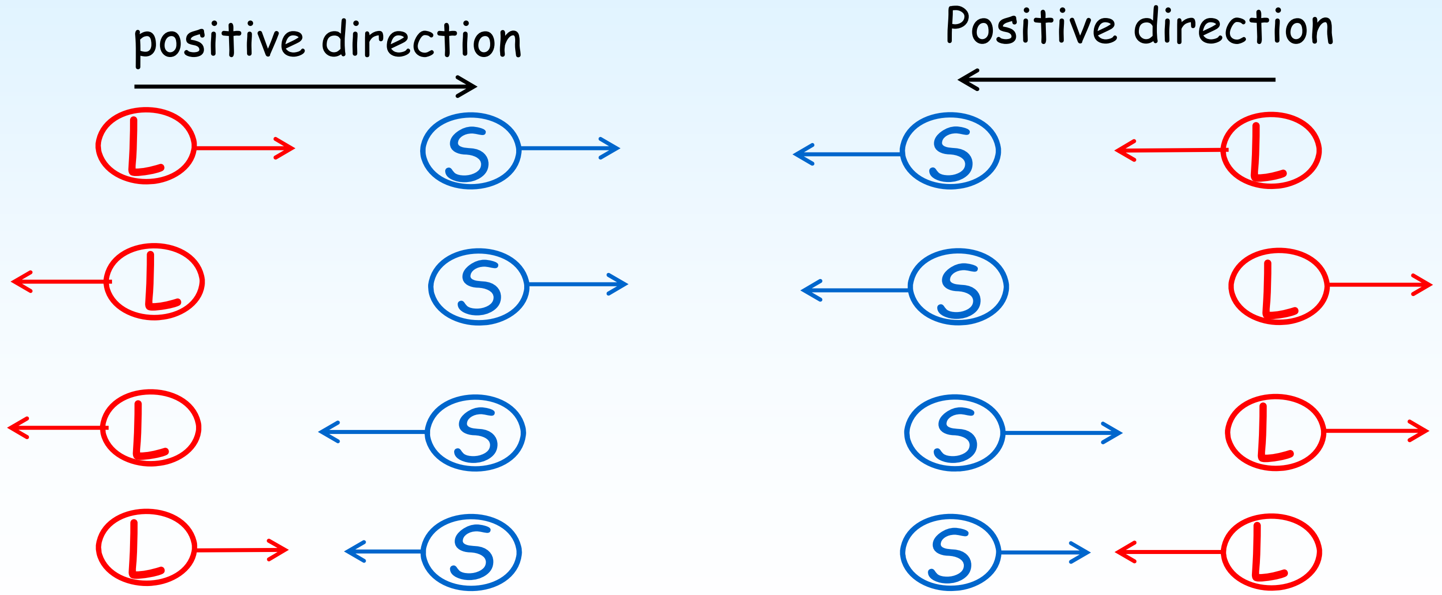
$$f_L = \frac{v + v_L}{\lambda_{\text{behind}}} = \frac{v + v_L}{(v + v_S)/f_S} = \frac{v + v_L}{v + v_S} f_S$$

Change of frequency

# Sound: The Doppler effect

$$f_L = \frac{v+v_L}{v+v_S} f_S$$

This formula always works if positive velocity direction is defined from the listener towards the source !




$$f_L = \frac{v+v_L}{v+v_S} f_S$$

$$f_L = \frac{v-v_L}{v+v_S} f_S$$

$$f_L = \frac{v-v_L}{v-v_S} f_S$$

$$f_L = \frac{v+v_L}{v-v_S} f_S$$

# Sound: The Doppler effect

Electromagnetic waves such as light also have a Doppler effect.

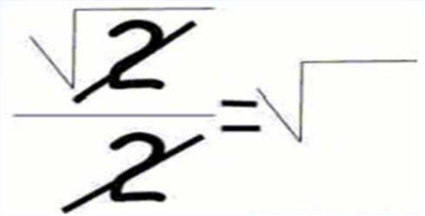
It can be calculated with the theory of relativity:

$$f_O = \sqrt{\frac{c - v}{c + v}} f_S$$

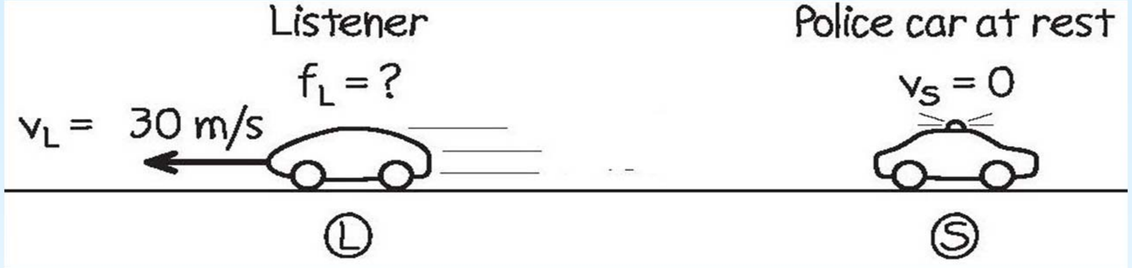
$f_s$ = frequency of the light source
$f_o$ = frequency of the light detected
c = the speed of light
v = The relative speed of the light source with respect to the observer

v is positive if the observer moves away from the source.
v is negative if the observer moves towards the light source.

# Part 11. Problems



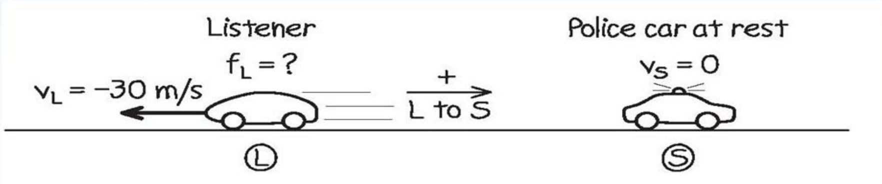
# Sound: Problems




f = 300 Hz

Speed of sound = 340 m/s

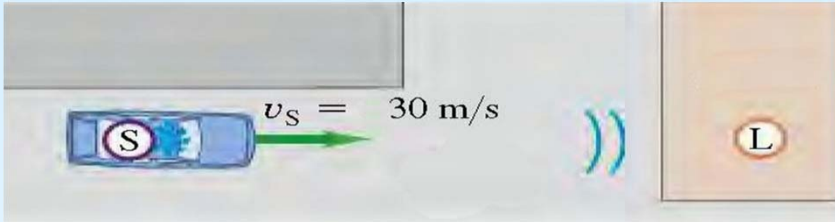
What frequency does the listener hear ?

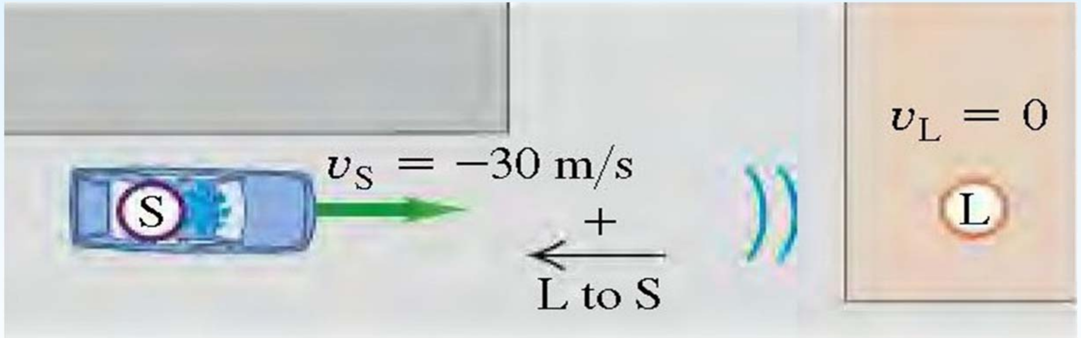



$$f_L = \frac{v + v_L}{v + v_S} f = \frac{340 \text{ m/s} + (-30 \text{ m/s})}{340 \text{ m/s}} (300 \text{ Hz}) = 274 \text{ Hz}$$

# Sound: Problems

A police car with a siren of f = 300 Hz drives towards a house at the speed of 30 m/s. What frequency does a listener hear in the house?



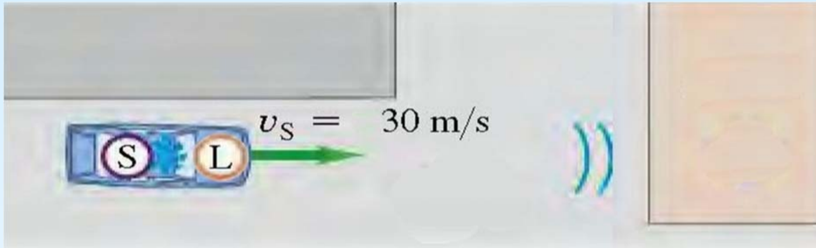




$$f_L = \frac{v + v_L}{v + v_S} f_S$$

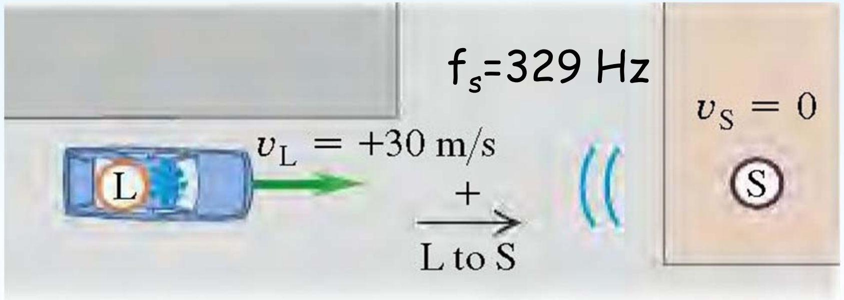
$$f_W = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + (-30 \text{ m/s})} (300 \text{ Hz}) = 329 \text{ Hz}$$

# Sound: Problems

A police car with a siren of f = 300 Hz drives towards a house at the speed of 30 m/s. What frequency does a listener hear in the police car if the sound is reflected back to it?




The house becomes a sound source with the frequency 329 Hz as calculated earlier:




$$f_L = \frac{v+v_L}{v+v_S} f_s$$

$$f_L = \frac{v + v_L}{v} f_s = \frac{340\ \text{m/s} + 30\ \text{m/s}}{340\ \text{m/s}} (329\ \text{Hz}) = 358\ \text{Hz}$$

# Part 12. Summary

# Sound: Summary

Wavefunction:

$$y(x,t) = A\cos(kx - \omega t)$$

Pressure function:

$$p(x,t) = BkA\sin(kx - \omega t)$$

$$p_{\max} = BkA$$

Speed of sound:

$$v = f\cdot\lambda = \frac{\omega}{k} = \sqrt{\frac{B}{\rho}}$$

Power per unit area:

$$P(x,t) = B\omega kA^2\sin^2(kx - \omega t)$$

# Sound: Summary

Intensity
(average power per unit area)

$$I = P_{av} / A_{rea} = \frac{1}{2} B \omega k A^2 = \frac{1}{2} \sqrt{\rho B} \omega^2 A^2 = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

The inverse-square law:

$$\frac{I_1}{I_2} = \frac{r_2^{\,2}}{r_1^{\,2}} \quad \text{(inverse-square law for intensity)}$$

Intensity level (decibel):

$$\beta = 10 \, log \frac{I}{I_0}$$

Doppler effect:

$$f_L = \frac{v + v_L}{v + v_S} f_S$$