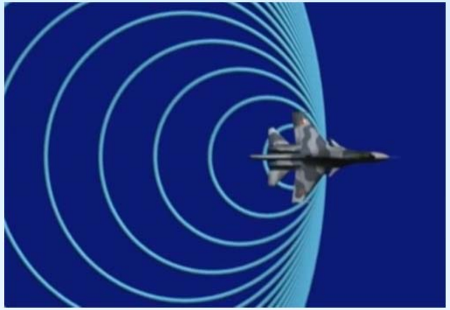
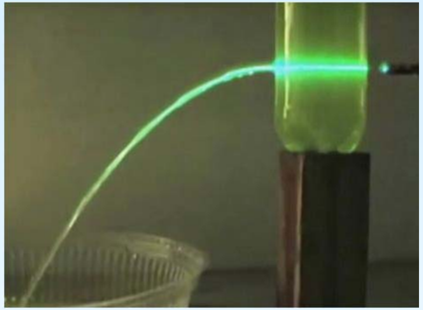
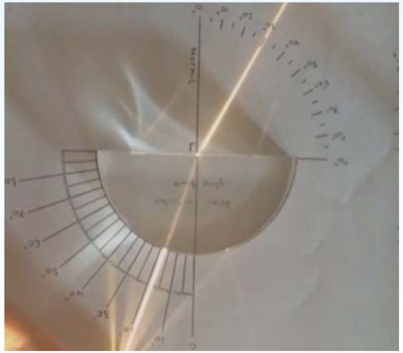
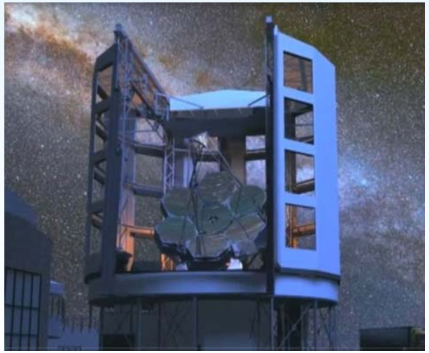
Wavemechanics and optics









Chapter 16 - Sound

Vincent Hedberg - Lunds Universitet

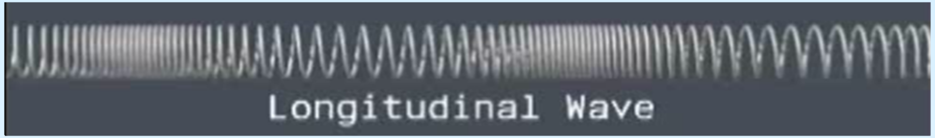
Sound = Pressure waves

Carreta Treme Treme

A Brazilian loud speaker truck

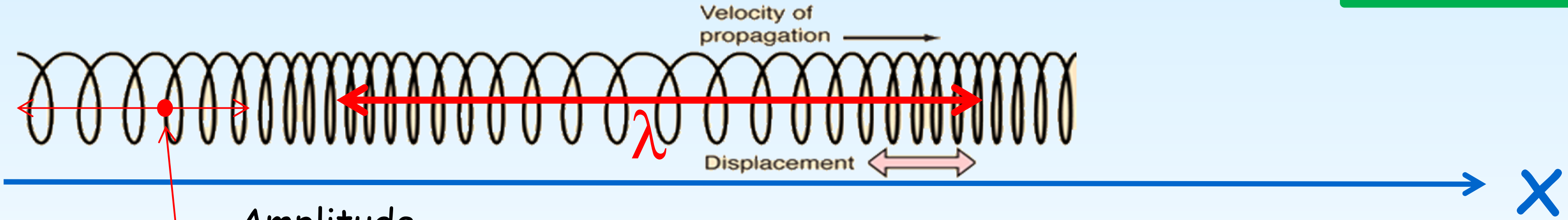
192 loudspeakers
33 amplifiers
240 batteries

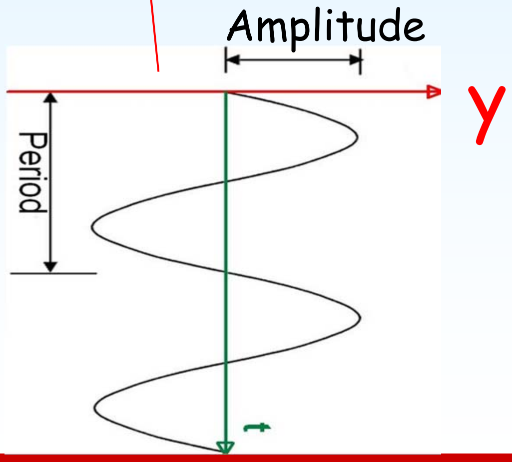
Sound: Longitudinal waves



Wave velocity

$$v = f \cdot \lambda = \frac{\omega}{k}$$





Mechanical longitudinal sinusoidal wave

$$y(x,t) = A\cos(kx - \omega t)$$

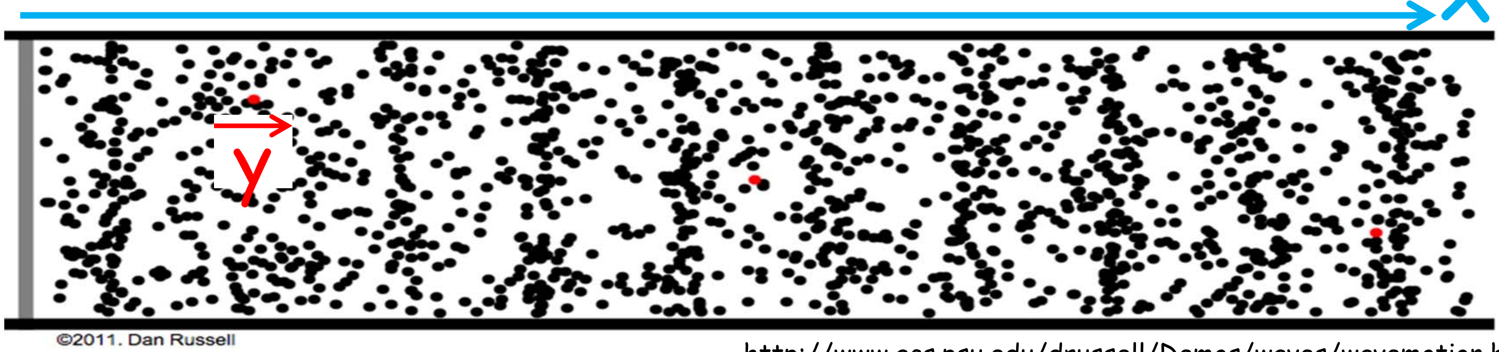
$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Sound & Pressure waves

Wave velocity v →

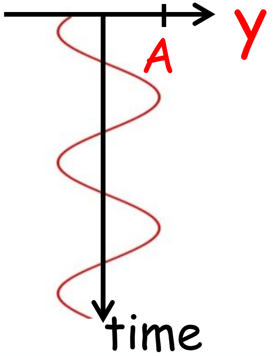
$$v = f \cdot \lambda = \frac{\omega}{k}$$

A piston moves in and out and create a longitudinal sine wave



http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

y: Movement of the air molecules



Wavefunction:

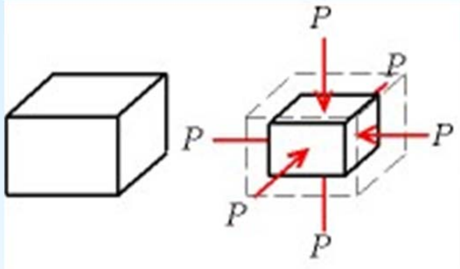
$$y(x,t) = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

How does the pressure vary with x and time?

Sound & Pressure waves

Bulk modulus



The bulk modulus measures a mediums resistance to uniform compression:

$$B = -V\frac{\Delta p}{\Delta V}$$

Δp → Pressure change

ΔV → Volyme change

Unit: N/m^2

The change in pressure after a change of volume:

$$\Delta p = -B\,\Delta V/V$$

Pressure increase: $\Delta p > 0$ and $\Delta V < 0$

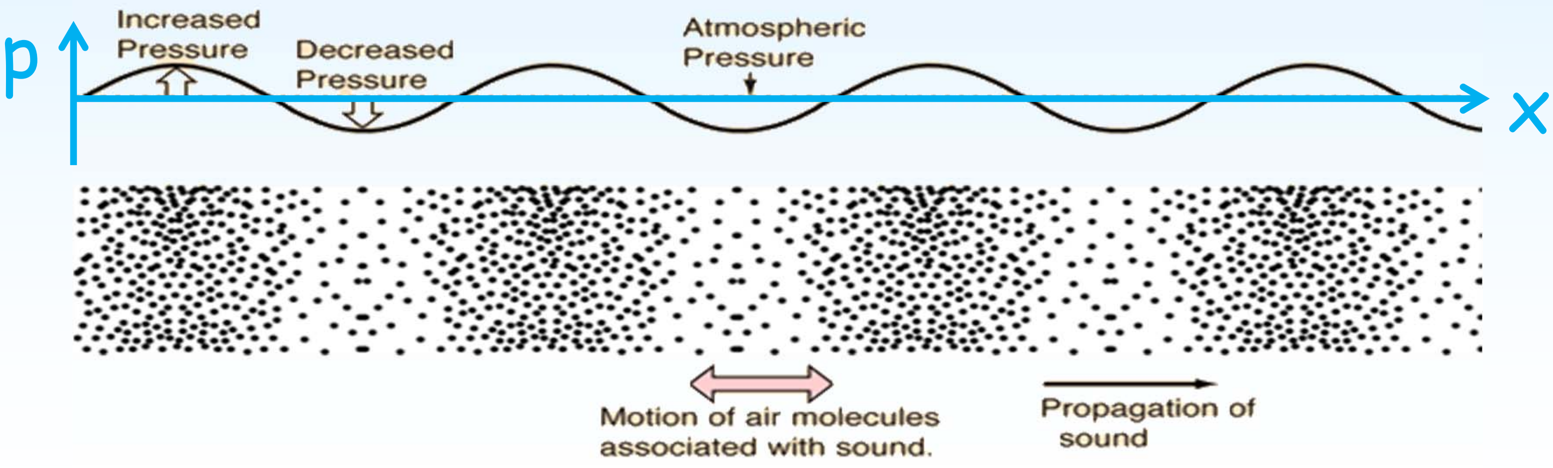
Sound & Pressure waves

Pressure function: $$p(x,t) = BkA\sin(kx - \omega t)$$

Pressure amplitude: $$p_{max} = BkA$$

Maximum pressure variation

At time = t

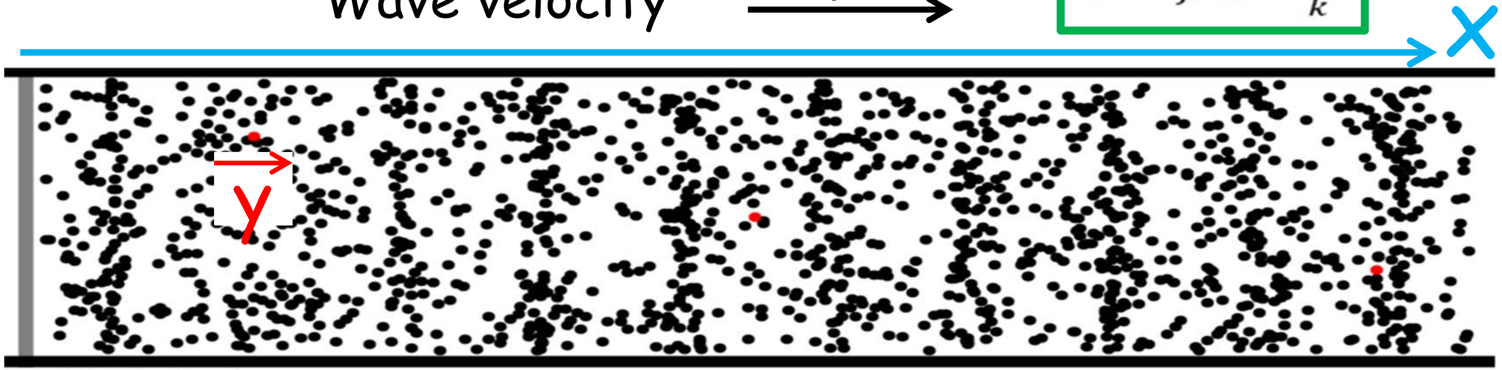


Sound & Pressure waves

Wave velocity $\overrightarrow{\quad V \quad}$

$$v = f \cdot \lambda = \frac{\omega}{k}$$

A piston moves in and out:



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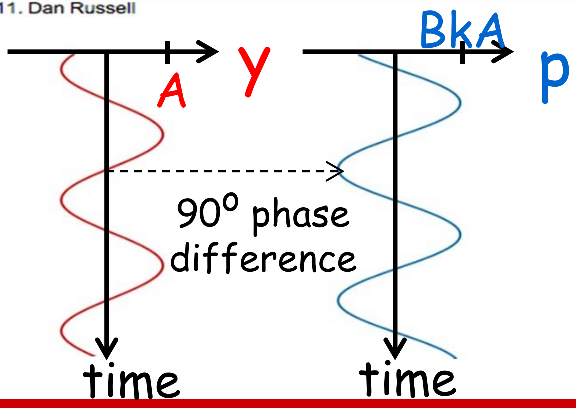
http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

y: Movement of air molecules

p: Pressure at position x



Wave function:

$$y(x,t) = A\cos(kx - \omega t)$$

Pressure function:

$$p(x,t) = BkA\sin(kx - \omega t)$$

The speed of sound in a liquid

SONAR

Sound Navigation And Ranging

www.youtube.com/watch?v=wTcaFYeUR10

The speed of sound

String: $v = \sqrt{\frac{F}{\mu}}$

F: String tension
μ: Mass per unit length

Liquids: $v = \sqrt{\frac{B}{\rho}}$

B: The Bulk modulus
ρ: The density

Solid material: $v = \sqrt{\frac{Y}{\rho}}$

Y: The Young's module
ρ: The density

Gas: $v = \sqrt{\frac{B}{\rho}}$

B: The Bulk modulus
ρ: The density

The power of sound

Sound: Power & Intensity

General for mechanical waves

Wave power (P): The instantaneous rate at which energy is transfered along the wave. (P = energy per unit of time)
Unit: W or J/s

Wave intensity (I): Average power per unit area through a surface perpendicular to the wave direction. (I =power per unit of area).
Unit: W/m^2

$$I = P_{av} / Area$$

The power in general:

$$P = \vec{F} \cdot \vec{v}$$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):

$$P(x, t) = F_y(x, t) v_y(x, t)$$

Sound: Power & Intensity

Wave power (P):

$$P(x,t) = F_y(x,t)v_y(x,t)$$

Pressure function (p):

$$p(x,t) = BkA\sin(kx - \omega t)$$

Pressure = Force per unit area

Wavefunction (y):

$$y(x,t) = A\cos(kx - \omega t)$$

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

Wavepower per unit of area:

$$\mathrm{P(x,t)} = p(x,t)v_y(x,t) = [BkA\sin(kx - \omega t)][\omega A\sin(kx - \omega t)]$$

$$= B\omega kA^2\sin^2(kx - \omega t)$$

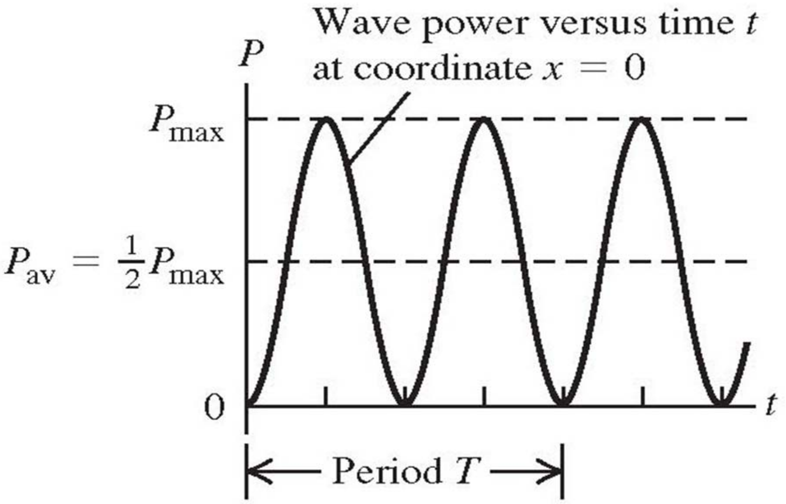
Power per m²

Pressure

Sound: Power & Intensity

Wavepower per unit of area:

$$P(x,t)/Area = B\omega kA^2 \sin^2(kx - \omega t)$$



Maximum power: $\frac{P_{max}}{Area} = B\omega kA^2$

Average power: $\frac{P_{av}}{Area} = \frac{1}{2}B\omega kA^2$

Exercise in algebra:

$$v = \frac{\omega}{k}$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$k = \frac{\omega}{\sqrt{\frac{B}{\rho}}}$$

$$v = \sqrt{\frac{B}{\rho}} \Rightarrow \sqrt{B} = v\sqrt{\rho}$$

$$\frac{P_{av}}{Area} = \frac{1}{2}B\omega k A^2 = \frac{1}{2}B\omega \frac{\omega}{\sqrt{\frac{B}{\rho}}}A^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

$$\frac{P_{av}}{Area} = \frac{1}{2}B\omega k A^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

Sound: Power & Intensity

Power general: $P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power - string:

$$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$$

$$P_{max} = Fk\omega A^2 = \sqrt{\mu F}\,\omega^2 A^2$$

$$P_{av} = \frac{1}{2} Fk\omega A^2 = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

Wave power - sound:

$$P(x,t)/Area = B\omega k A^2 sin^2(kx - \omega t)$$

$$P_{max}/Area = B\omega k A^2 = \sqrt{\rho B}\,\omega^2 A^2$$

$$P_{av}/Area = \frac{1}{2} B\omega k A^2 = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$$

$$P_{av} = \frac{1}{2}\mu(\omega A)^2 v = \frac{1}{2}\sqrt{\mu F}(\omega A)^2$$

$$P_{av}/Area = \frac{1}{2}\rho(\omega A)^2 v = \frac{1}{2}\sqrt{\rho B}(\omega A)^2$$

Sound: Intensity

Intensity = average power per unit area

When the Gulf Corvina fish spawn, it sends out audio signals that can reach an intensity level of 177 dB (202 dB = 10^8 W/m^2 for an entire shoal).

This is one of the loudest sounds in the animal world and can cause hearing damage to dolphins, seals and sea lions.

Sound: Intensity

Average power of a soundwave (P_{av}):
Unit: W or J/s

$$\frac{P_{av}}{Area} = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

Wave intensity (I): Average power per unit area through a surface perpendicular to the wave direction.
Unit: W/m^2

$$\mathrm{I} = \frac{Power}{Area}$$

$$I = \frac{P_{av}}{Area} = \frac{1}{2}B\omega kA^2 = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{1}{2}\rho(\omega A)^2 v$$

Sound: Power & Intensity

$$I = \frac{P_{av}}{Area} = \frac{1}{2} B\omega k A^2$$

Pressure function: $p(x,t) = BkA\sin(kx - \omega t)$

Pressure amplitude: $p_{max} = BkA \Rightarrow A^2 = \frac{p_{max}^2}{B^2k^2}$

$$I = \frac{1}{2} B\omega k A^2 = \frac{1}{2} B\omega k \frac{p_{max}^2}{B^2k^2} = \frac{1}{2B}\frac{\omega}{k} p_{max}^2 = \frac{1}{2B}\sqrt{\frac{B}{\rho}}\, p_{max}^2 = \frac{p_{max}^2}{2\sqrt{\rho B}}$$

$v = \frac{\omega}{k}$

$v = \sqrt{\frac{B}{\rho}}$

$$\frac{\omega}{k} = \sqrt{\frac{B}{\rho}}$$

The intensity is proportional to the square of the pressure amplitude.

$$I = \frac{p_{max}^2}{2\sqrt{\rho B}}$$

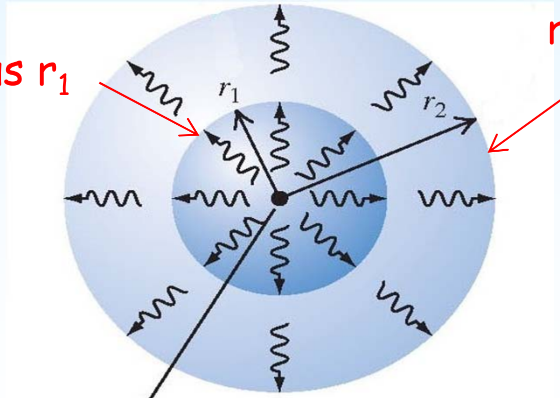
Sound: Power & Intensity

Wave intensity (I): The speed at which the wave transports energy through a surface perpendicular to the direction of the wave (I = Average power per area unit = energy per time and area unit). Units: $W/m^2 = J/s/m^2$

$$I = P_{av} / A_{rea}$$

Sphere with radius r_1

Sphere with radius r_2



Source with the average power P_{av}

The intensity through a sphere with radius r_1

$$I_1 = \frac{P_{av}}{4\pi r_1^2}$$

Ignoring power losses: $4\pi r_1^2 I_1 = 4\pi r_2^2 I_2$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

The decibel scale

Pain threshold:
120 dB = 1 W/m^2

Gulf Corvina:
200 dB = 10^8 W/m^2

Saturn V rocket:
220 dB = 10^{10} W/m^2

Krakatoa:
310 dB = 10^{19} W/m^2

Sound: Decibel

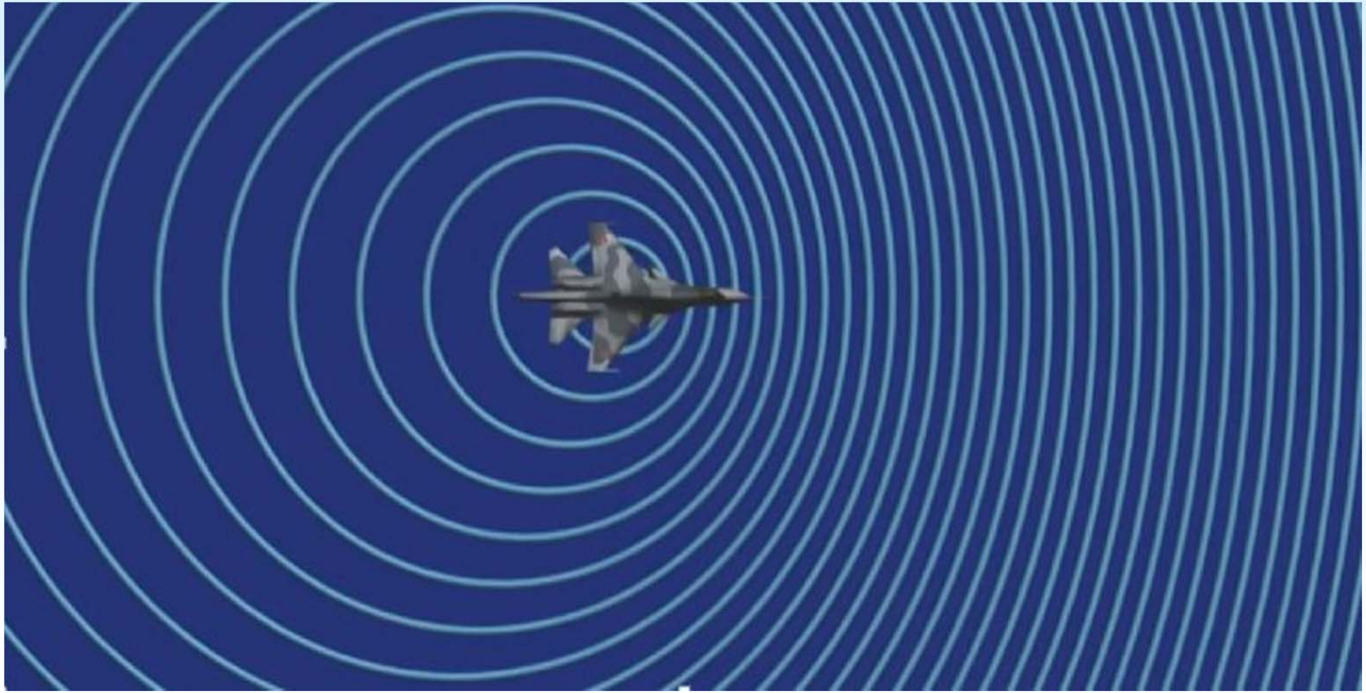
Intensity level (β) with decibel (dB) as the unit:

$$\beta = 10\,log\,\frac{I}{I_0} \longleftrightarrow I = I_0 \cdot 10^{\beta/10}$$

$I_0 = 10^{-12}$ W/m^2 is a reference level.
I_0 = approximately the limit of human hearing.

β = 0 dB when $I = I_0$
β = 120 dB when I = 1 W/m^2

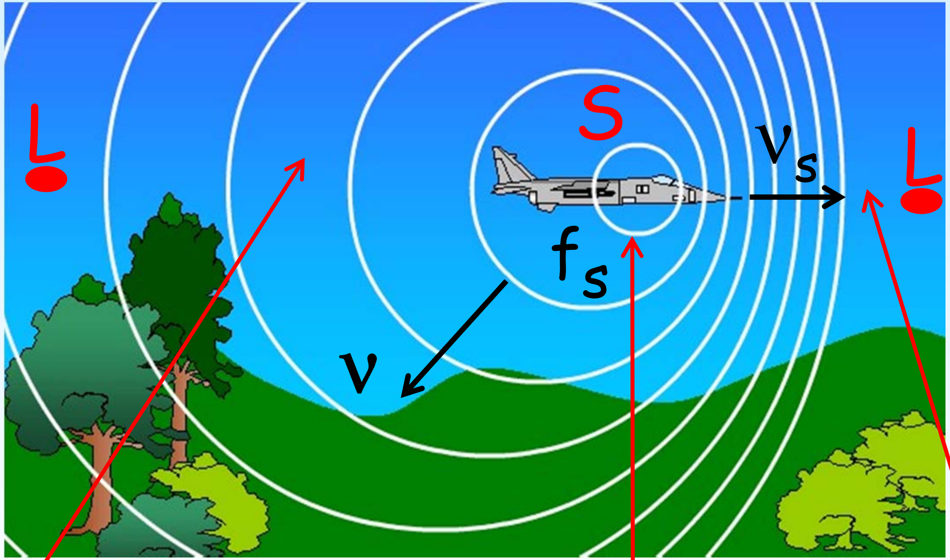
Doppler effect



https://www.youtube.com/watch?v=-Zu5SGIlmwc

Sound: The Doppler effect

The time for a sound wave to reach a listener (L) gets longer if the source (S) is moving away.



The time for a sound wave to reach a listener (L) gets shorter if the source is moving closer.

λ_{behind} longer

$\lambda_{in\ front}$ shorter

$$\lambda_{\text{behind}} = \frac{v + v_S}{f_S}$$

$$\lambda = \frac{v}{f_s}$$

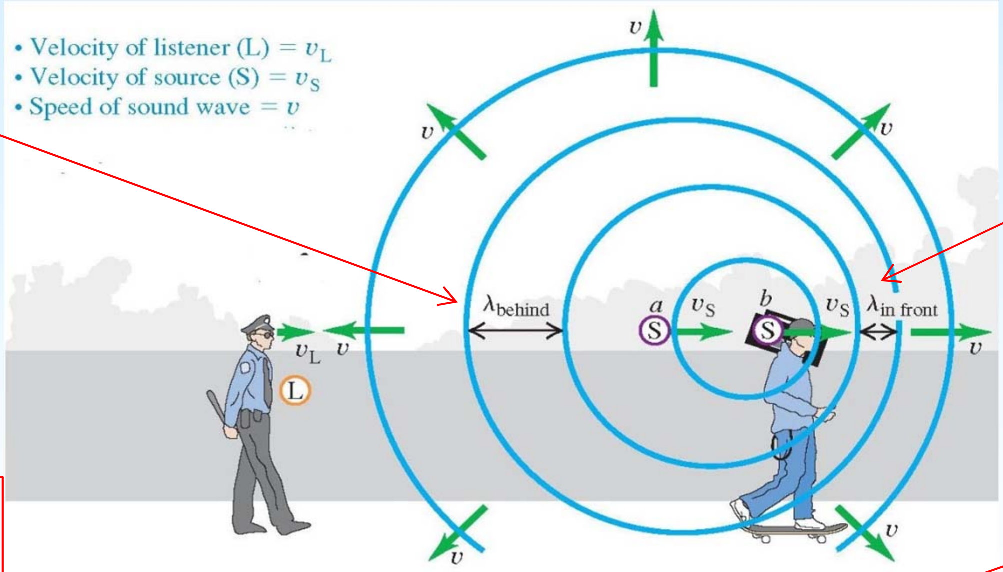
$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

More complicated: The listener moves too

$$\lambda_{behind} = \frac{v + v_S}{f_S}$$

General rule:

$$f = \frac{v}{\lambda}$$



$$\lambda_{in\ front} = \frac{v - v_S}{f_S}$$

The wave is approaching L by V + V_L

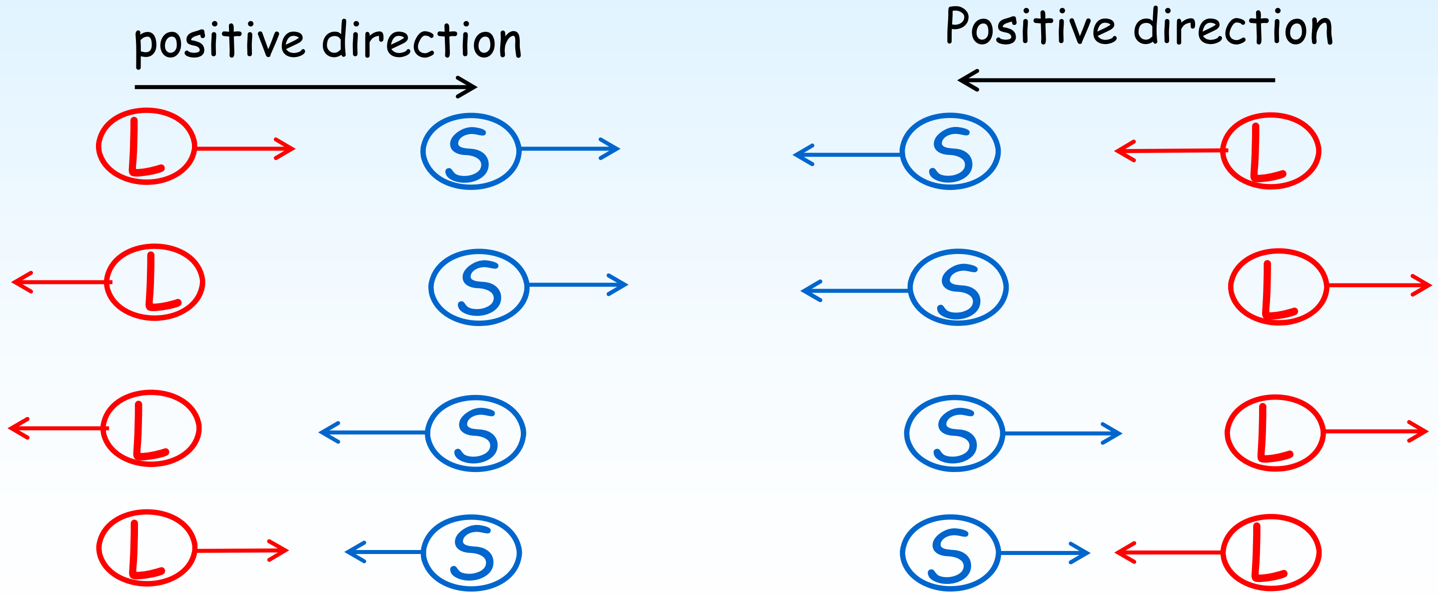
$$f_L = \frac{v + v_L}{\lambda_{behind}} = \frac{v + v_L}{(v + v_S)/f_S} = \frac{v + v_L}{v + v_S} f_S$$

Change of frequency

Sound: The Doppler effect

$$f_L = \frac{v+v_L}{v+v_S} f_S$$

This formula always works if positive velocity direction is defined from the listener towards the source !



$$f_L = \frac{v+v_L}{v+v_S} f_S$$

$$f_L = \frac{v-v_L}{v+v_S} f_S$$

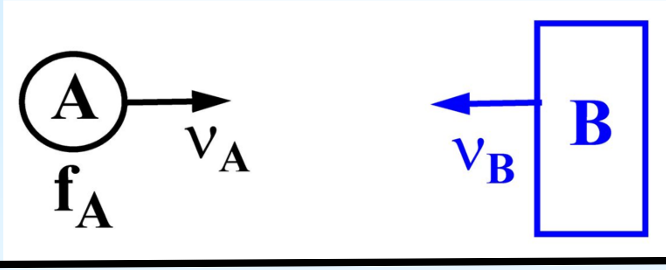
$$f_L = \frac{v-v_L}{v-v_S} f_S$$

$$f_L = \frac{v+v_L}{v-v_S} f_S$$

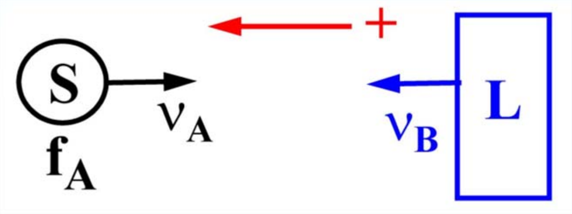
Sound: The Doppler effect

If the sound is reflected:

Sound is emitted by A and reflected against B.
What frequency does a listener in A hear?

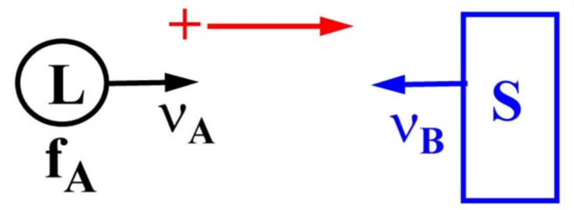


Step 1.
Consider B to be a listener.



$$f_L = \frac{v+v_L}{v+v_S} f_S = \frac{v+v_B}{v-v_A} f_A$$

Step 2.
Consider B to be a source of sound.



$$f_L = \frac{v + v_L}{v + v_S} f_S = \frac{v + v_A}{v - v_B} \frac{v + v_B}{v - v_A} f_A$$

SUMMARY

Sound

Sound: Summary

Wavefunction: $y(x,t) = A\cos(kx - \omega t)$

Pressure function: $p(x,t) = BkA\sin(kx - \omega t)$ → $p_{max} = BkA$

Speed of sound: $v = f\cdot\lambda = \dfrac{\omega}{k} = \sqrt{\dfrac{B}{\rho}}$

Power per unit area: $P(x,t) = B\omega kA^2\sin^2(kx - \omega t)$

Sound: Summary

Intensity
(average power per unit area)

$$I = P_{av} / A_{rea} = \frac{1}{2} B\omega k A^2 = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

The inverse-square law:

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

Intensity level (decibel):

$$\beta = 10 \log \frac{I}{I_0}$$

Doppler effect:

$$f_L = \frac{v + v_L}{v + v_S} f_S$$