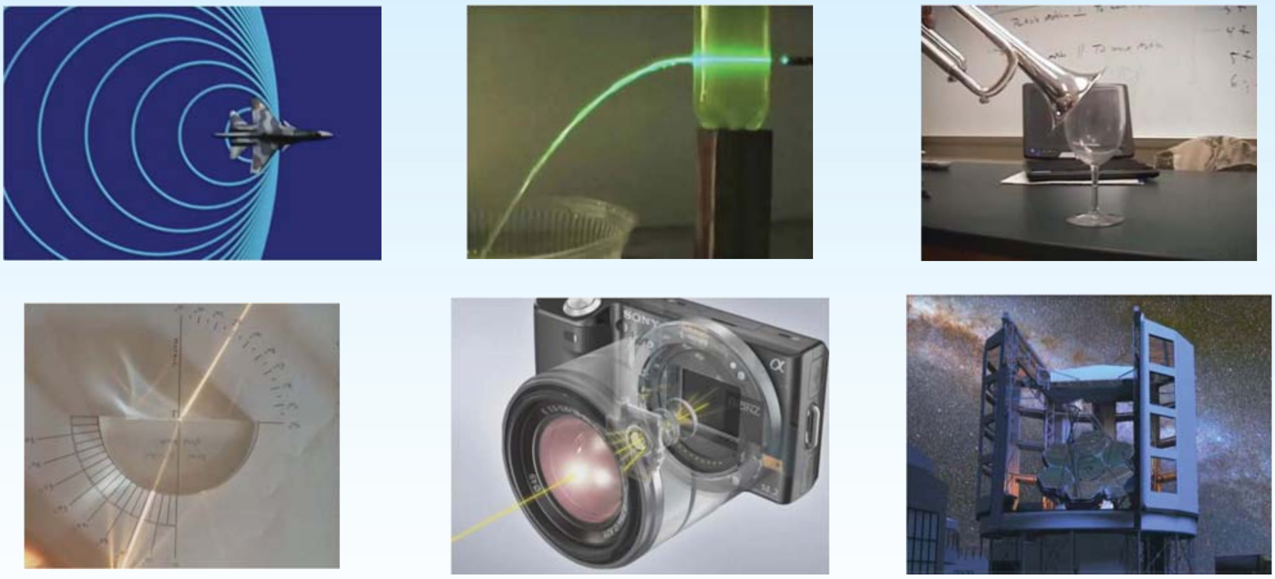
Vågrörelselära och optik



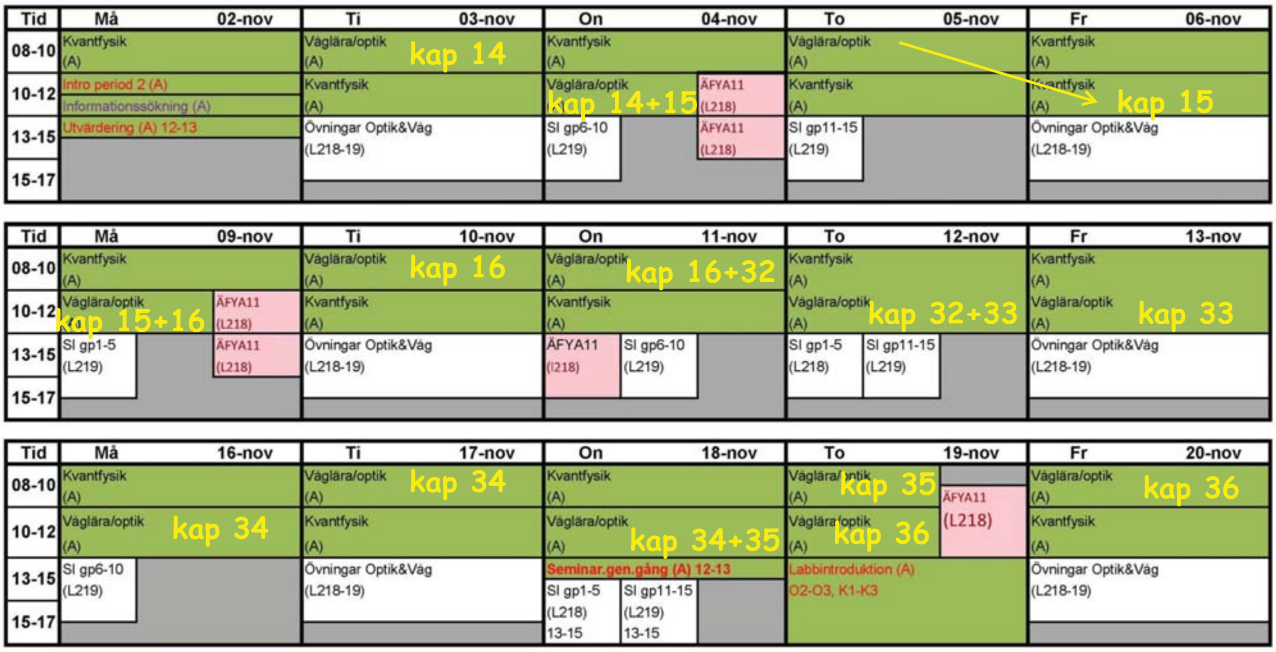
Kapitel 32 – Elektromagnetiska vågor

Vågrörelselära och optik

Kurslitteratur: University Physics by Young & Friedman

Harmonisk oscillator:	Kapitel 14.1 – 14.4
Mekaniska vågor:	Kapitel 15.1 – 15.8
Ljud och hörande:	Kapitel 16.1 – 16.9
Elektromagnetiska vågor:	Kapitel 32.1 & 32.3 & 32.4
Ljusets natur:	Kapitel 33.1 – 33.4 & 33.7
Stråloptik:	Kapitel 34.1 – 34.8
Interferens:	Kapitel 35.1 – 35.5
Diffraktion:	Kapitel 36.1 - 36.5 & 36.7

Vågrörelselära och optik



Tid	Må 02-nov	Ti 03-nov	On 04-nov	To 05-nov	Fr 06-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 14	Kvantfysik (A)	Våglära/optik (A)	Kvantfysik (A)
10-12	Intro period 2 (A) Informationssökning (A)	Kvantfysik (A)	Våglära/optik kap 14+15 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A) kap 15
13-15	Utvärdering (A) 12-13	Övningar Optik&Våg (L218-19)	SI gp6-10 (L219) ÄFYA11 (L218)	SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 09-nov	Ti 10-nov	On 11-nov	To 12-nov	Fr 13-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 16	Våglära/optik (A) kap 16+32	Kvantfysik (A)	Kvantfysik (A)
10-12	Våglära/optik kap 15+16 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A)	Våglära/optik (A) kap 32+33	Våglära/optik (A) kap 33
13-15	SI gp1-5 (L219) ÄFYA11 (L218)	Övningar Optik&Våg (L218-19)	ÄFYA11 (l218) SI gp6-10 (L219)	SI gp1-5 (L218) SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 16-nov	Ti 17-nov	On 18-nov	To 19-nov	Fr 20-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 35 ÄFYA11 (L218)	Våglära/optik (A) kap 36
10-12	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 34+35	Våglära/optik (A) kap 36	Kvantfysik (A)
13-15	SI gp6-10 (L219)	Övningar Optik&Våg (L218-19)	Seminar.gen.gång (A) 12-13 SI gp1-5 (L218) 13-15 SI gp11-15 (L219) 13-15	Labbintroduktion (A) 02-03, K1-K3	Övningar Optik&Våg (L218-19)
15-17					

Electromagnetic waves
Maxwell's equations

Maxwell's equations

Electromagnetic waves
Maxwell's equations

Maxwells equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for magnetism)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad \text{(Ampere's law)}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

$\bar{\mathrm{E}}$ - the electric field intensity (N/C)

ϕ_{E} - electric flux (Nm^2/C)

$\bar{\mathrm{B}}$ - magnetic field strength (A/m)

ϕ_{B} - magnetic flux (T/m^2)

Electromagnetic waves
Maxwell's equations

The implications of Maxwell's Equations for magnetic and electric fields:

1. A static electric field can exist in the absence of a magnetic field e.g. a capacitor with a static charge has an electric field without a magnetic field.

2. A constant magnetic field can exist without an electric field e.g. a conductor with constant current has a magnetic field without an electric field.

3. Where electric fields are time-variable, a non-zero magnetic field must exist.

4. Where magnetic fields are time-variable, a non-zero electric field must exist

5. Magnetic fields can be generated by permanent magnets, by an electric current or by a changing electric field.

6. Magnetic monopoles cannot exist. All lines of magnetic flux are closed loops.

Electromagnetic waves
Maxwell's equations

The speed of light from Maxwell's equations

$E = c\,B$ from Faraday´s law

$E = B\,/\,(\varepsilon_0\,\mu_0\,c)$ from Ampere´s law

ε_0 is the permittivity in vacuum $= 8.85 \times 10^{-12}$ F/m

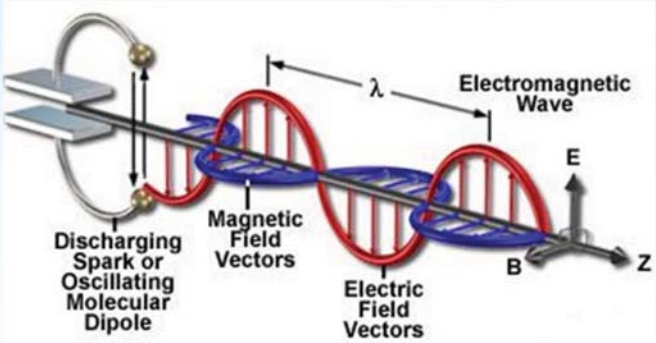
μ_0 is the permeability in vacuum $= 1.26 \times 10^{-6}$ N/A^2

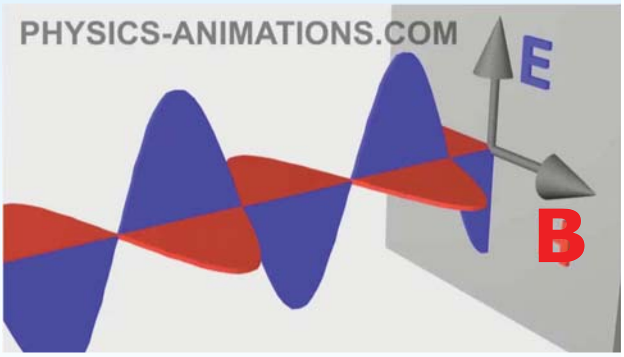
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

Permittivity: A mediums ability to form an electric field in itself.
Permeability: A mediums ability to form a magnetic field in itself.

Electromagnetic waves
Maxwell's equations

The electromagnetic wave



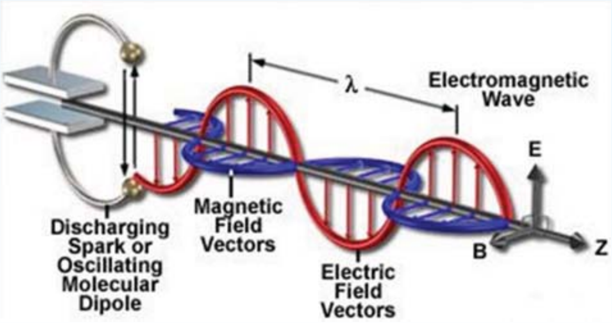


Electromagnetic waves Maxwell’s equations

Electromagnetic waves are produced by the vibration of charged particles.

An **electromagnetic wave** is a wave that is capable of transmitting its energy through **a vacuum**.

The propagation of an electromagnetic wave, which has been generated by a discharging capacitor or an oscillating molecular dipole.



The field is strongest at 90 degrees to the moving charge and zero in the direction of the moving charge.

As the current oscillates up and down in the spark gap a magnetic field is created that oscillates in a horizontal plane.

The changing magnetic field, in turn, induces an electric field so that a series of electrical and magnetic oscillations combine to produce a formation that propagates as an electromagnetic wave.

Electromagnetic waves Maxwell’s equations

Experiment that demonstrates how moving charges creates an electromagnetic field

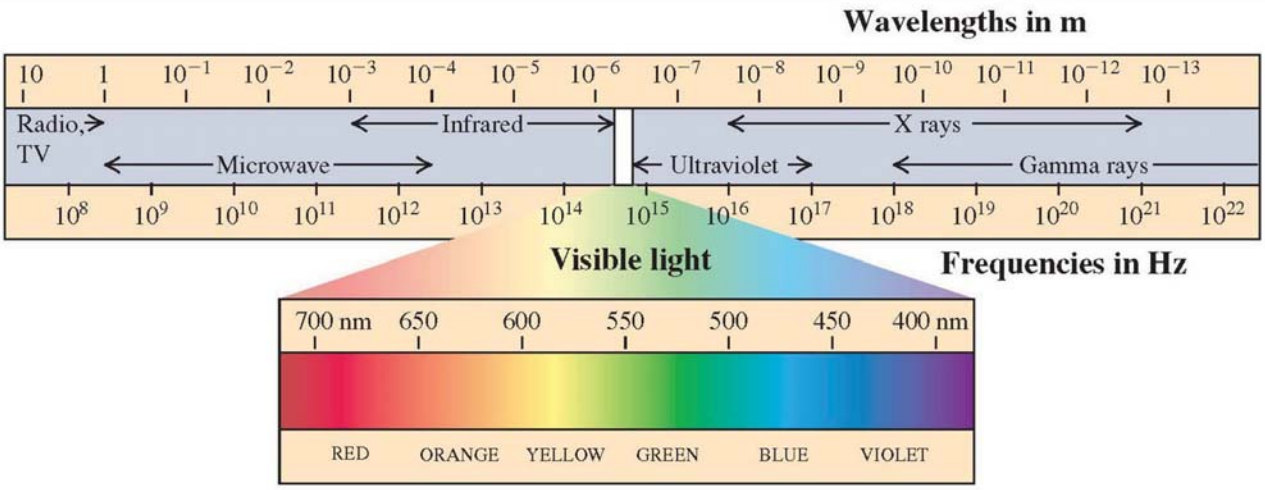
Electromagnetic waves

Electromagnetic waves

Electromagnetic waves

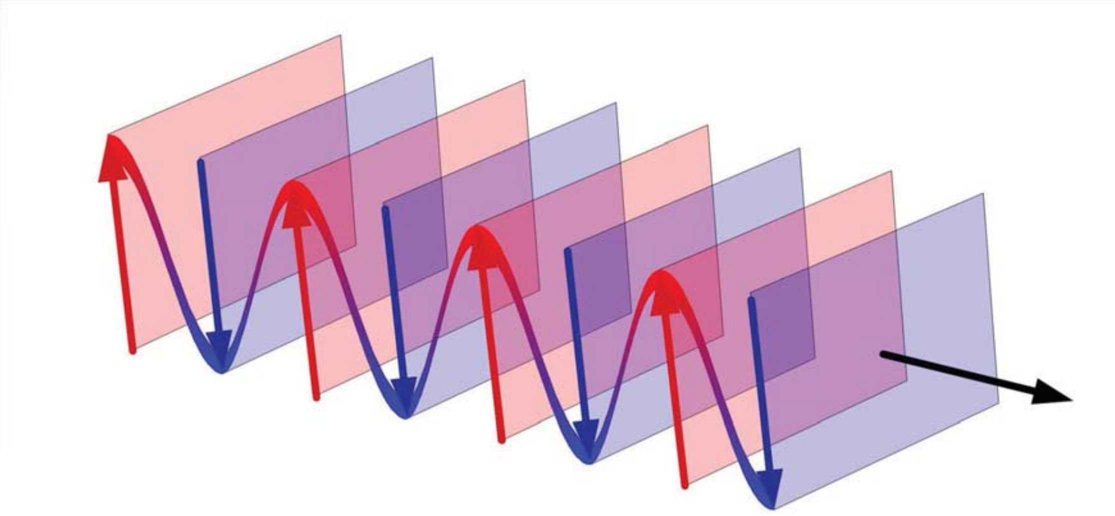
The electromagnetic spectrum

$\lambda = c / f$



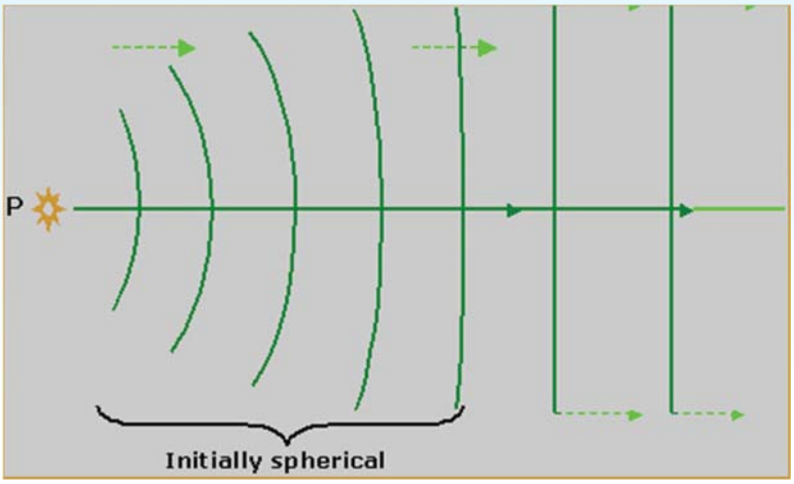
Electromagnetic waves

Wavefronts: surfaces with constant phase



Electromagnetic waves

Wavefronts depends on the distance to the source



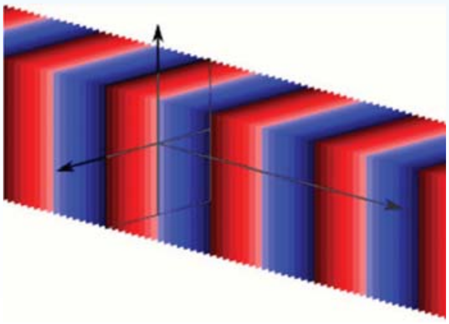
Electromagnetic waves

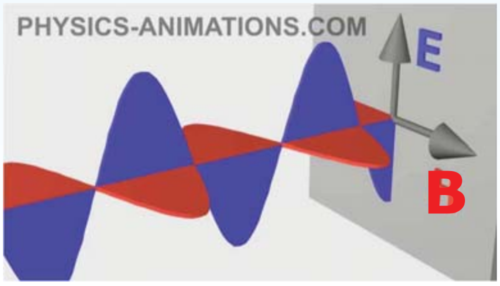
A **plane wave** is a constant-frequency wave whose wavefronts are infinite parallel planes of constant peak-to-peak amplitude normal to the phase velocity vector.

At a particular point and time all E and B vectors in the plane have the same magnitude.

No true plane waves since only a plane wave of infinite extent will propagate as a plane wave. However, many waves are approximately plane waves in a localized region of space.

In a plane electromagnetic wave the E and B fields are perpendicular to the direction of propagation so it is a transverse wave.





Electromagnetic waves
The wave function

The wavefunction

Mechanical waves:
The wavefunction

$y(x, t) = A\cos(kx - \omega t)$ (sinusoidal wave moving in $+x$-direction)

$v = \lambda / T$
$f = 1 / T$

Amplitude: A

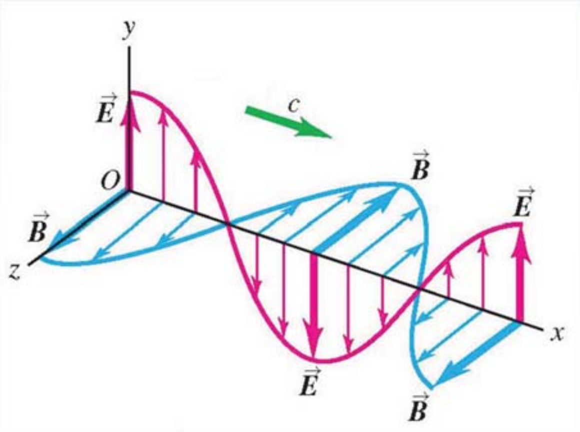
Wavenumber: $k = 2\pi/\lambda$

Angular frequency: $\omega = 2\pi/T$

$$\boxed{v = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k}$$

Electromagnetic waves
The wave function

The electromagnetic wavefunction



$$\vec{E}(x, t) = \hat{j}E_{\max}\cos(kx - \omega t)$$
$$\vec{B}(x, t) = \hat{k}B_{\max}\cos(kx - \omega t)$$

not the same

Electromagnetic waves
The wave function

$$\vec{E}(x,t) = \hat{j}E_{\max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{\max}\cos(kx - \omega t)$$

$c = \lambda / T$

$f = 1 / T$

Amplitude: $E_{max} = c\,B_{max}$

Wavenumber: $k = 2\pi/\lambda$

Angular frequency: $\omega = 2\pi/T$

$$\boxed{c = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k}$$

Electromagnetic waves
The wave function

In a dielectric medium the speed of light is smaller than c !

Electromagnetic waves in matter:

$c \rightarrow v$

$\mu_0 \rightarrow \mu$

$\varepsilon_0 \rightarrow \varepsilon$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$$

Dielectric constant

$K = \varepsilon / \varepsilon_0$

Relative permeability

$K_m = \mu / \mu_0$

Electromagnetic waves
The wave function

Electromagnetic wave in vacuum

$E = cB$ from Faraday´s law

$E = B / (\varepsilon_0 \mu_0 c)$ from Ampere´s law

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Electromagnetic wave in matter

$E = vB$ from Faraday´s law

$E = B / (\varepsilon \mu v)$ from Ampere´s law

$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

Permettivity Permability

$$\frac{c}{v} = n = \frac{\frac{1}{\sqrt{\epsilon_0 \mu_0}}}{\frac{1}{\sqrt{\epsilon \mu}}} = \sqrt{KK_m} \cong \sqrt{K}$$

Refractive index Dielectric constant Relative permeability

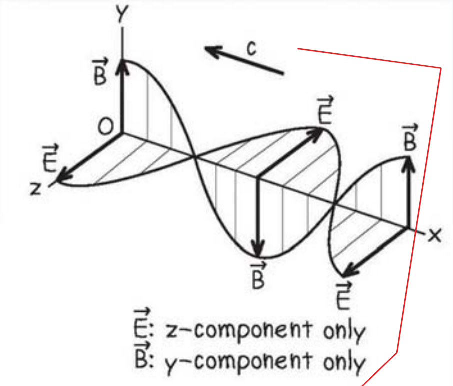
$K = \varepsilon/\varepsilon_0$ $K_m = \mu/\mu_0$

Electromagnetic waves
problems

Problem solving

Electromagnetic waves problems

A carbon dioxide laser emits a sinusoidal electromagnetic wave that travels in vacuum in the negative x-direction. The wavelength is 10.6 μm the $\vec{E}$ field is parallel to the z-axis, with $E_{\max} = 1.5$ MV/m. Write vector equations for $\vec{E}$ and $\vec{B}$ as functions of time and position.



$$\vec{E}(x, t) = \hat{k} E_{\max} \cos(kx + \omega t)$$
$$\vec{B}(x, t) = \hat{j} B_{\max} \cos(kx + \omega t)$$

$E_{max} = c\ B_{max}$

$k = 2\pi/\lambda$

$c = \omega/k$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{1.5 \times 10^6 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 5.0 \times 10^{-3} \text{ T}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{10.6 \times 10^{-6} \text{ m}} = 5.93 \times 10^5 \text{ rad/m}$$

$$\omega = ck = (3.00 \times 10^8 \text{ m/s})(5.93 \times 10^5 \text{ rad/m})$$
$$= 1.78 \times 10^{14} \text{ rad/s}$$

$$\vec{E}(x, t) = \hat{k}(1.5 \times 10^6 \text{ V/m}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

$$\vec{B}(x, t) = \hat{j}(5.0 \times 10^{-3} \text{ T}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

Electromagnetic waves problems

Visiting a jewelry store one evening, you hold a diamond up to the light of a sodium-vapor street lamp. The heated sodium vapor emits yellow light with a frequency of 5.09×10^{14} Hz. Find the wavelength in vacuum and the wave speed and wavelength in diamond, for which $K = 5.84$ and $K_m = 1.00$ at this frequency.

The wavelength in vacuum of the sodium light is

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 5.89 \times 10^{-7} \text{ m} = 589 \text{ nm}$$

The wave speed and wavelength in diamond are

$$v_{\text{diamond}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(5.84)(1.00)}} = 1.24 \times 10^8 \text{ m/s}$$

$$\lambda_{\text{diamond}} = \frac{v_{\text{diamond}}}{f} = \frac{1.24 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}}$$
$$= 2.44 \times 10^{-7} \text{ m} = 244 \text{ nm}$$

Electromagnetic waves problems

90.0-MHz radio wave (in the FM radio band) passes from vacuum into an insulating ferrite (a ferromagnetic material used in computer cables to suppress radio interference). Find the wavelength in vacuum and the wave speed and wavelength in the ferrite, for which $K = 10.0$ and $K_m = 1000$ at this frequency.

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}} = 3.33 \text{ m}$$

$$v_{\text{ferrite}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(10.0)(1000)}} = 3.00 \times 10^6 \text{ m/s}$$

$$\lambda_{\text{ferrite}} = \frac{v_{\text{ferrite}}}{f} = \frac{3.00 \times 10^6 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}}$$

$$= 3.33 \times 10^{-2} \text{ m} = 3.33 \text{ cm}$$

Electromagnetic waves
Power & Intensity

Power & Intensity

Mechanical waves:
Power & Intensity

The power in general:

$P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):

The instantaneous rate at which energy is transfered along the wave.

Unit: W or J/s

Wave intensity (I):

Average power per unit area through a surface perpendicular to the wave direction.

Unit: W/m^2

Sound – power & intensity

The power in general:

$P = \vec{F} \cdot \vec{v}$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):
The instantaneous rate at which energy is transfered along the wave.

Unit: W or J/s

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

Wave intensity (I):
Average power per unit area through a surface perpendicular to the wave direction.

Unit: W/m^2

$$\mathrm{I} = \mathrm{P_{av}} / \mathrm{A_{rea}}$$

Electromagnetic waves
Power & Intensity

Total energy density (u):

Energy per unit volume due to an electric and magnetic field.

Unit: J/m^3

Power (P):

The instantaneous rate at which energy is transfered along a wave.

Unit: W or J/s

The Poynting vector ($\bar{S}$):

Energy transferred per unit time per unit area = Power per unit area.

Unit: W/m^2

Intensity (I):

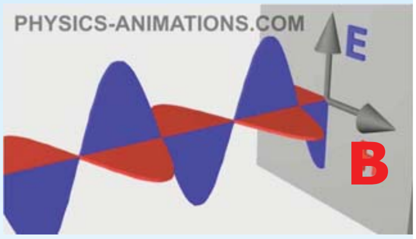
Average power per unit area through a surface perpendicular to the wave direction = the average value of S.

Unit: W/m^2

Electromagnetic waves
Power & Intensity

The total energy density (energy per unit volume) due to an electric and magnetic field is

$$u = \tfrac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$



E = c B from Faraday's law

E = B / ($\varepsilon_0\,\mu_0$ c) from Ampere's law

$$+ \quad \boxed{c = \frac{1}{\sqrt{\epsilon_0\mu_0}}} \quad \Longrightarrow \quad B^2 = \varepsilon_0\,\mu_0\,E^2$$

$$u = \tfrac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}E)^2 = \epsilon_0 E^2$$

where $E\ (x, t) = E_{\max}\cos(kx - \omega t)$

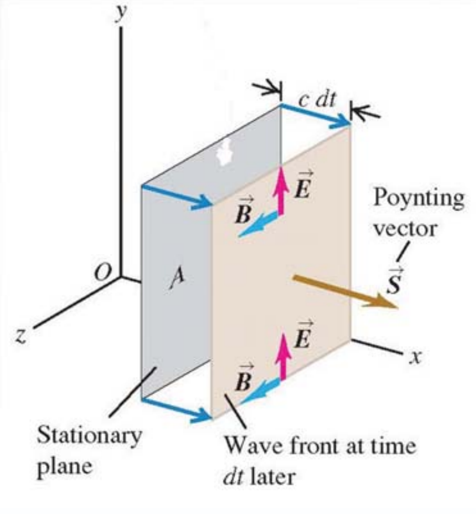
Energy E-field Energy B-field

Conclusions: The electric and magnetic fields carry the same amount of energy.
The energy density varies with position and time.

Electromagnetic waves
Power & Intensity

Energy transfer = energy transferred per unit time per unit area.

S = Power per unit area = Energy transfer = Energy flow



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \text{(Poynting vector in vacuum)}$$

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t)$$
$$= \frac{1}{\mu_0}[\hat{j}E_{\max}\cos(kx-\omega t)] \times [\hat{k}B_{\max}\cos(kx-\omega t)]$$

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx-\omega t)$$

Amplitude = maximum energy transfer

Electromagnetic waves
Power & Intensity

Intensity = the average value of S

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx-\omega t)$$

The average of $\cos^2(x) = 1/2$

$$I = S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c}$$

E = c B

Electromagnetic waves in matter:

$\mu_0 \rightarrow \mu$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$$

Electromagnetic waves problems

Problem solving

Electromagnetic waves problems

$E = 100\ \mathrm{V/m} = 100\ \mathrm{N/C}$. Find the value of B, the energy density u, and the rate of energy flow per unit area S.

maximum

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

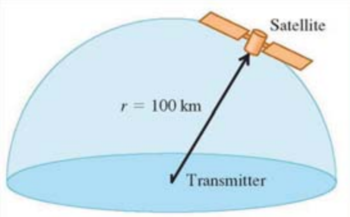
$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

$$B = \frac{E}{c} = \frac{100\ \mathrm{V/m}}{3.00 \times 10^{8}\ \mathrm{m/s}} = 3.33 \times 10^{-7}\ \mathrm{T}$$

$$u = \epsilon_0 E^2 = (8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(100\ \mathrm{N/C})^2$$
$$= 8.85 \times 10^{-8}\ \mathrm{N/m^2} = 8.85 \times 10^{-8}\ \mathrm{J/m^3}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \longrightarrow S = \frac{EB}{\mu_0} = \frac{(100\ \mathrm{V/m})(3.33 \times 10^{-7}\ \mathrm{T})}{4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}}$$
$$= 26.5\ \mathrm{V \cdot A/m^2} = 26.5\ \mathrm{W/m^2}$$

Electromagnetic waves problems



A radio station on the earth's surface emits a sinusoidal wave with average total power 50 kW . Assuming that the transmitter radiates equally in all directions above the ground (which is unlikely in real situations), find the electric-field and magnetic-field amplitudes $E_{\max}$ and $B_{\max}$ detected by a satellite 100 km from the antenna. $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

The surface area of a hemisphere of radius $r = 100\ \mathrm{km} = 1.00 \times 10^5\ \mathrm{m}$ is

$$A = 2\pi R^2 = 2\pi(1.00 \times 10^5\ \mathrm{m})^2 = 6.28 \times 10^{10}\ \mathrm{m}^2$$

$$I = S_{\mathrm{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c}$$

$$I = \frac{P}{A} = \frac{P}{2\pi R^2} = \frac{5.00 \times 10^4\ \mathrm{W}}{6.28 \times 10^{10}\ \mathrm{m}^2} = 7.96 \times 10^{-7}\ \mathrm{W/m^2}$$

$I = S_{\mathrm{av}} = E_{\max}{}^2/2\mu_0 c$, so

$$\begin{aligned} E_{\max} &= \sqrt{2\mu_0 c S_{\mathrm{av}}} \\ &= \sqrt{2(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(3.00 \times 10^8\ \mathrm{m/s})(7.96 \times 10^{-7}\ \mathrm{W/m^2})} \\ &= 2.45 \times 10^{-2}\ \mathrm{V/m} \end{aligned}$$

$$B_{\max} = \frac{E_{\max}}{c} = 8.17 \times 10^{-11}\ \mathrm{T}$$

Electromagnetic waves Momentum & forces

Momentum & forces

Electromagnetic waves
Momentum & forces

Kinematics

Impuls: $\vec{J} = \int_{t_1}^{t_2} \Sigma\vec{F}\, dt$

The Momentum-Impuls theorem: $\vec{J} = \vec{p}_2 - \vec{p}_1$

A change of momentum is equal to the impulse.

Electromagnetic waves
Momentum & forces

Electromagnetic waves carry momentum (p = E/c).

If the wave is absorbed or reflected this momentum is transferred to the surface.

The momentum transfer creates a force (F) on the surface.

Radiation pressure (p_{rad}) = force per unit area ($p_{rad} = F/A$).

$$p_{\mathrm{rad}} = \frac{S_{\mathrm{av}}}{c} = \frac{I}{c} \quad \text{(radiation pressure, wave totally absorbed)}$$

$$p_{\mathrm{rad}} = \frac{2S_{\mathrm{av}}}{c} = \frac{2I}{c} \quad \text{(radiation pressure, wave totally reflected)}$$

Electromagnetic waves Momentum & forces

Crooke's radiometer

Radiation pressure or thermal effect ?

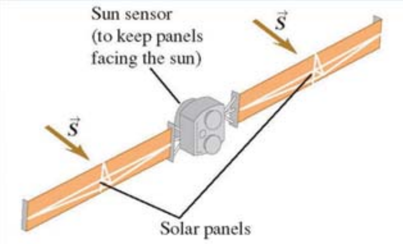
Electromagnetic waves problems

Problem solving

Electromagnetic waves

An earth-orbiting satellite has solar energy–collecting panels with a total area of 4.0 m^2 If the sun's radiation is perpendicular to the panels and is completely absorbed, find the average solar power absorbed and the average radiation-pressure force.

The intensity I (power per unit area) is $1.4 \times 10^3\ \text{W/m}^2$.



Intensity = power per unit area:

$$P = IA = (1.4 \times 10^3\ \text{W/m}^2)(4.0\ \text{m}^2) = 5.6 \times 10^3\ \text{W} = 5.6\ \text{kW}$$

$$p_{\text{rad}} = \frac{S_{\text{av}}}{c} = \frac{I}{c} \quad \text{(radiation pressure, wave totally absorbed)}$$

$p_{\text{rad}} = 1.4 \times 10^3 / 3.0 \times 10^8 = 4.7 \times 10^{-6}\ \text{N/m}^2$

$$F = p_{\text{rad}}A = (4.7 \times 10^{-6}\ \text{N/m}^2)(4.0\ \text{m}^2) = 1.9 \times 10^{-5}\ \text{N}$$