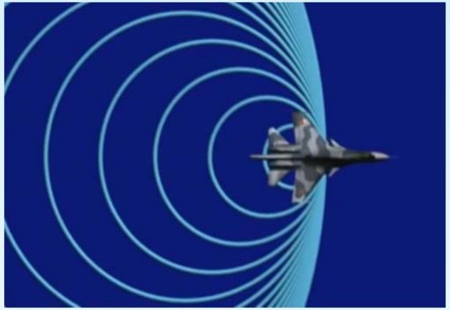
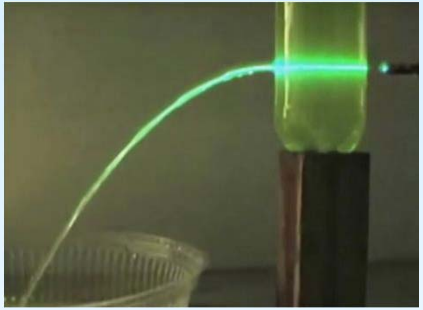
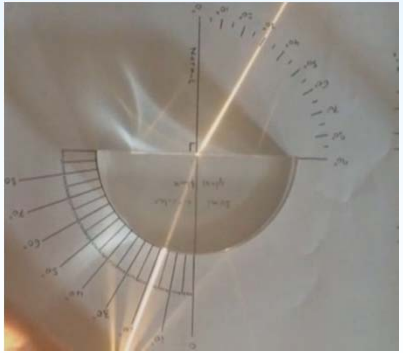
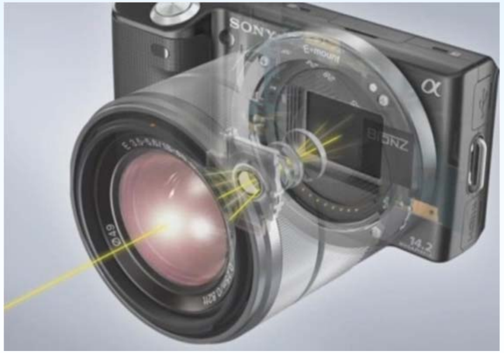
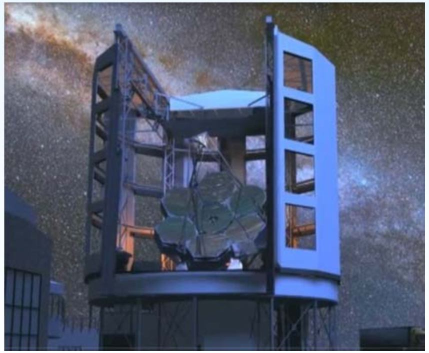
Wavemechanics and optics











Chapter 32 – Electromagnetic waves

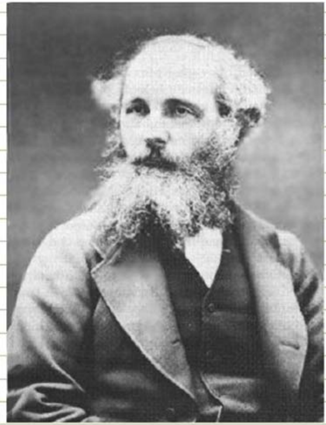
Vincent Hedberg - Lunds Universitet

Content

Part 1. Maxwell's equations

James Clerk Maxwell

- **Unified** Electric and Magnetic Theory.
- Predicted Electromagnetic **Wave Propagation**
- Theorized that **light** was an electromagnetic wave.
- Could "**low-frequency**" waves be generated ?



James Clerk Maxwell (1831-1879)

Maxwell's equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for magnetism)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad \text{(Ampere's law)}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

$\bar{E}$ - the electric field intensity (N/C)

ϕ_E - electric flux (Nm^2/C)

$\bar{B}$ - magnetic field strength (A/m)

ϕ_B - magnetic flux (T/m^2)

Electromagnetic waves: Maxwell's equations

The implications of Maxwell's Equations for magnetic and electric fields:

1. A static electric field can exist in the absence of a magnetic field e.g. a capacitor with a static charge has an electric field without a magnetic field.

2. A constant magnetic field can exist without an electric field e.g. a conductor with constant current has a magnetic field without an electric field.

3. Where electric fields are time-variable, a non-zero magnetic field must exist.

4. Where magnetic fields are time-variable, a non-zero electric field must exist

5. Magnetic fields can be generated by permanent magnets, by an electric current or by a changing electric field.

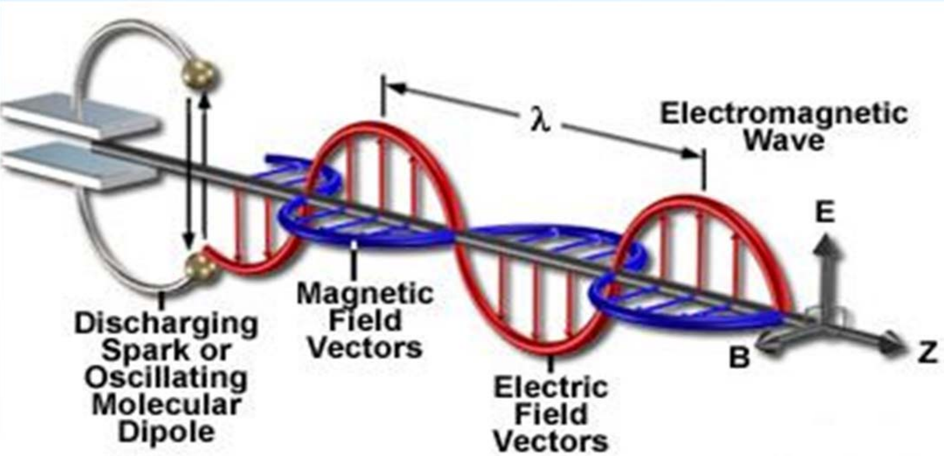
6. Magnetic monopoles do not exist. All lines of magnetic flux are closed loops.

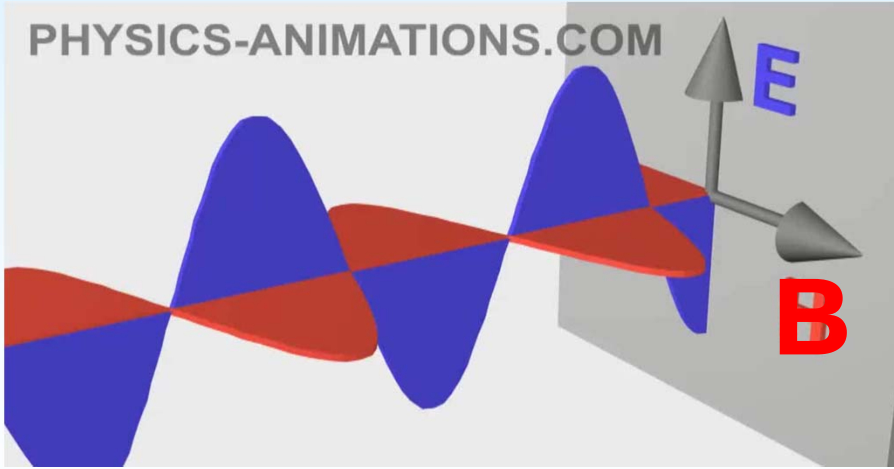
6. Magnetic monopoles do not exist according to the experiments conducted so far.

According to some theories, magnetic monopoles may exist and several experiments around the world are looking for them. The ATLAS experiment for example:

No experiment has found monopoles !

The electromagnetic wave consists of an electric and a magnetic field.

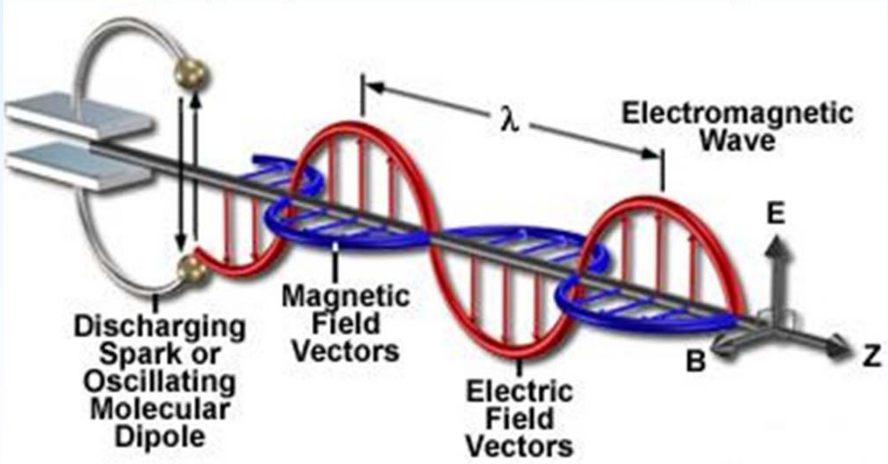




Electromagnetic waves: Maxwell's equations

- Electromagnetic waves are produced by the movement of charged particles.
- An **electromagnetic wave** is capable of transmitting energy through **a vacuum**.

An electromagnetic wave can be generated by a discharging capacitor or an oscillating molecular dipole.



The field is strongest at 90 degrees to the moving charge and zero in the direction of the moving charge.

As the current oscillates up and down in the spark gap a magnetic field is created that oscillates in a horizontal plane.

The changing magnetic field, in turn, induces an electric field so that a series of electrical and magnetic oscillations combine to produce a formation that propagates as an electromagnetic wave.

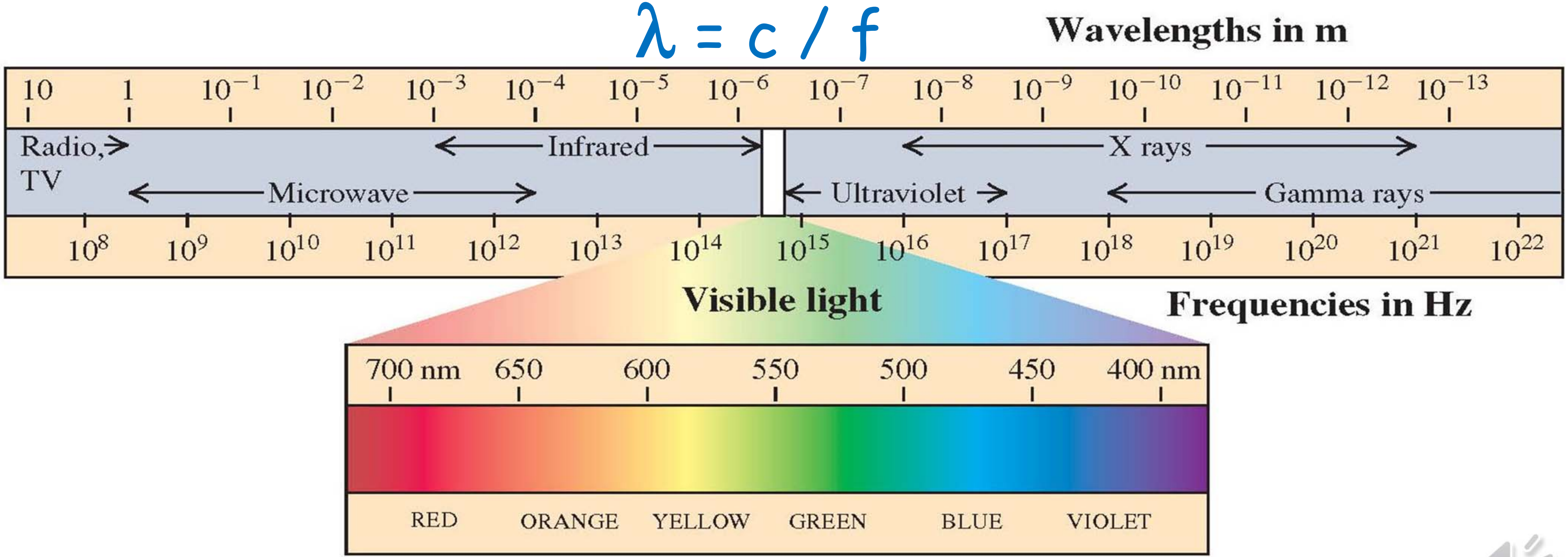
Hertz experiment demonstrated how moving charges creates an electromagnetic field

https://www.youtube.com/watch?v=9gDFll6Ge7g

Part 2. Electromagnetic waves

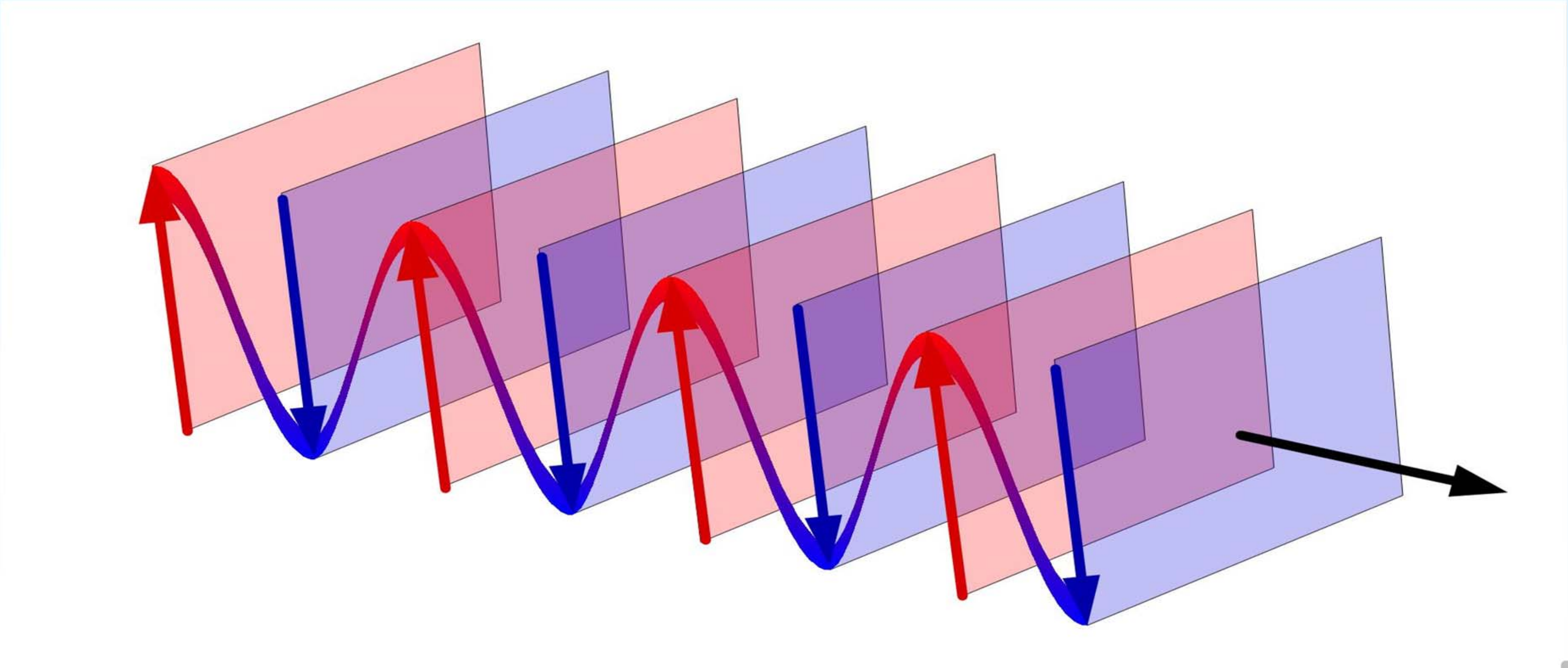
The electromagnetic spectrum

$$\lambda = c / f$$



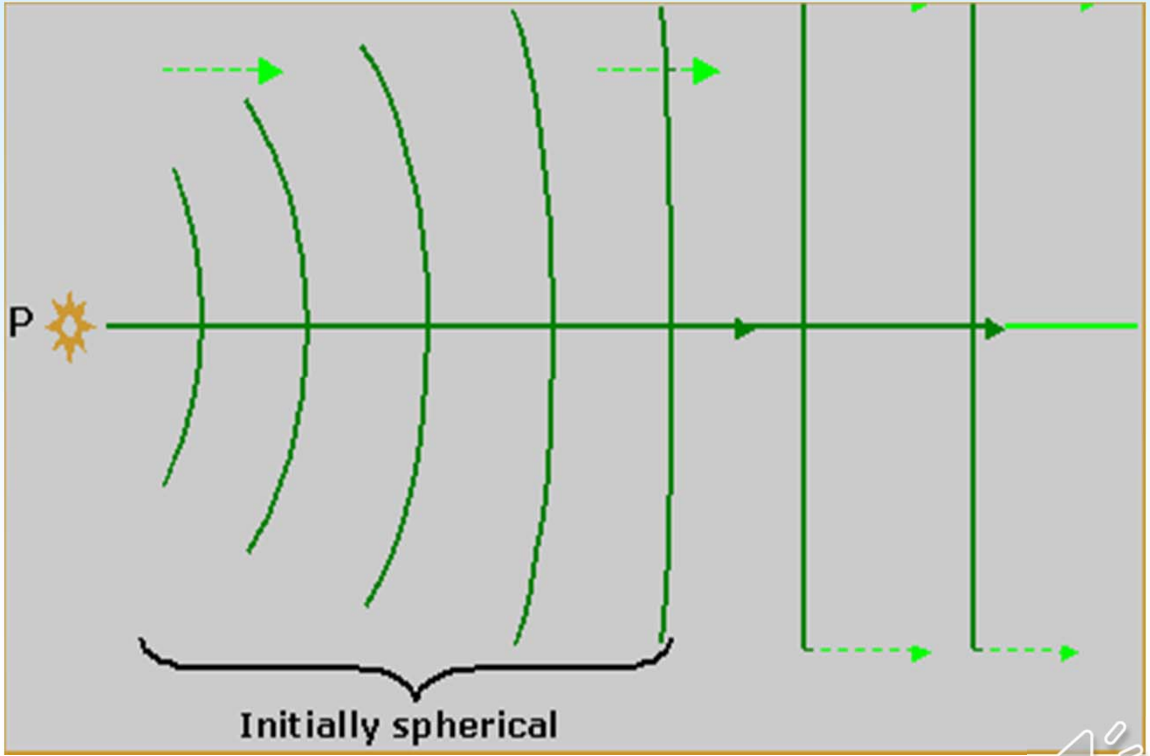
Electromagnetic waves

Wavefronts: surfaces with constant phase



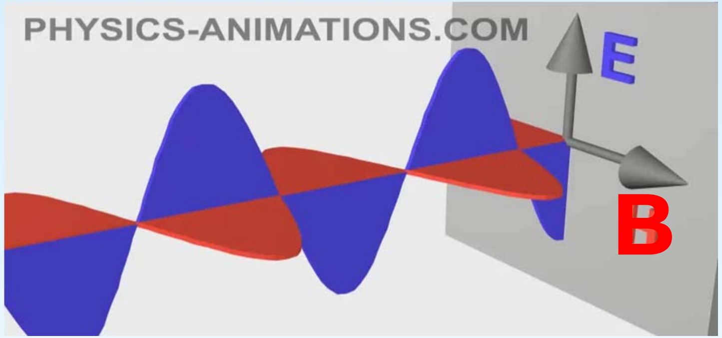
Electromagnetic waves

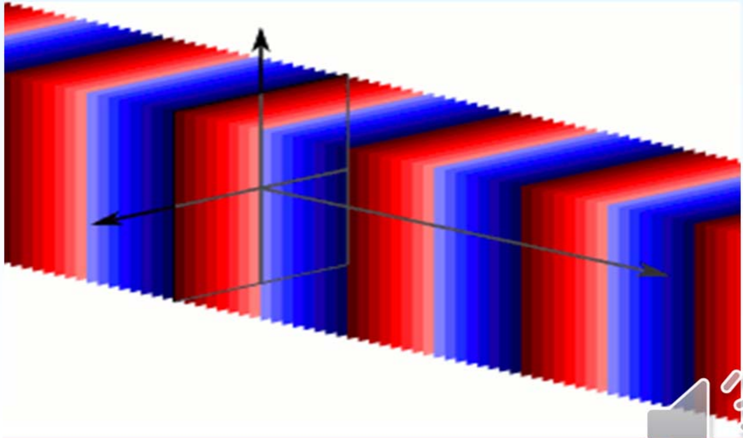
Wavefronts depends on the distance to the source



Electromagnetic waves

- Electromagnetic waves are transverse because the E and B fields are perpendicular to the direction of propagation.
- A plane wave is a wave with constant frequency whose wave fronts are infinite parallel planes with constant peak-to-peak amplitude.
- At a certain point and time, all the E and B vectors in the plane are of the same size.
- Completely flat waves do not exist because only an infinitely large wave can be flat. But many waves are approximately flat in a localized area of space.





Electromagnetic waves

For flat Electromagnetic waves, one can find relations between the magnitude of the magnetic and electric fields from two of Maxwell's equations:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}$$

Flat wave → $E = cB$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \quad \text{(Ampere's law)}$$

Flat wave → $E = \dfrac{B}{\varepsilon_0 \mu_0 c}$

c = The speed of light.
ε = Permittivity: A mediums ability to form an electric field in itself.
μ = Permeability: A mediums ability to form a magnetic field in itself.

Electromagnetic waves

The speed of light from Maxwell's equations:

$E = cB$ from Faraday´s law

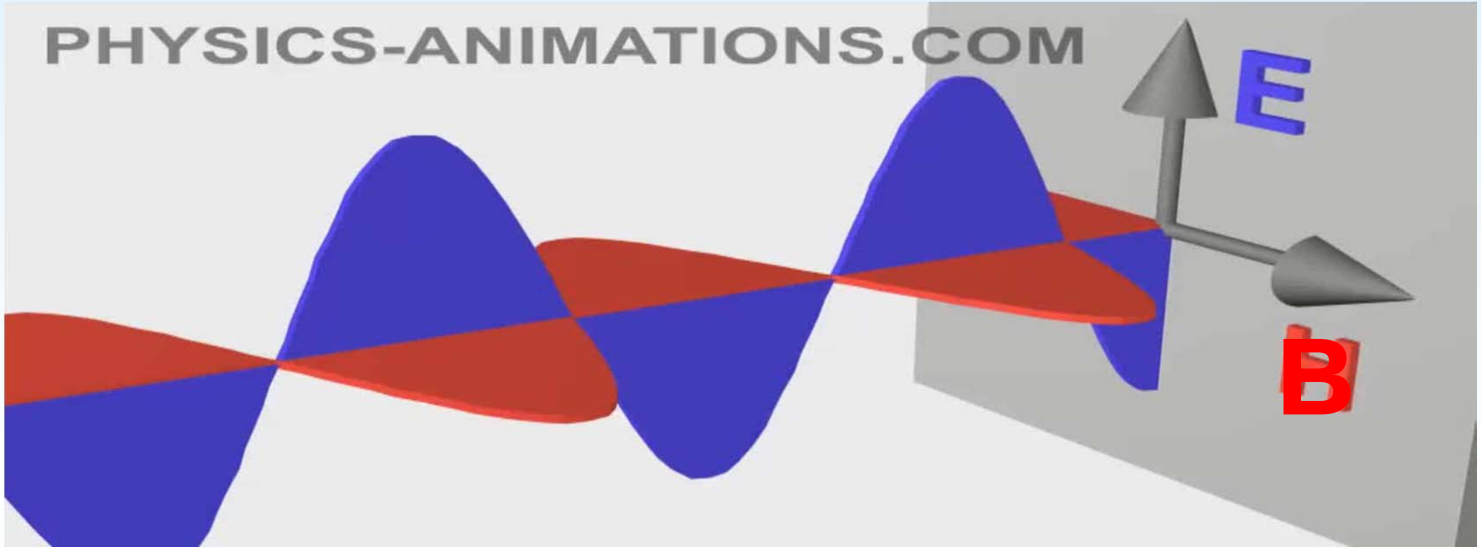
$E = B / (\varepsilon_0 \mu_0 c)$ from Ampere´s law

ε_0 is the permittivity in vacuum $= 8.85 \times 10^{-12}$ F/m

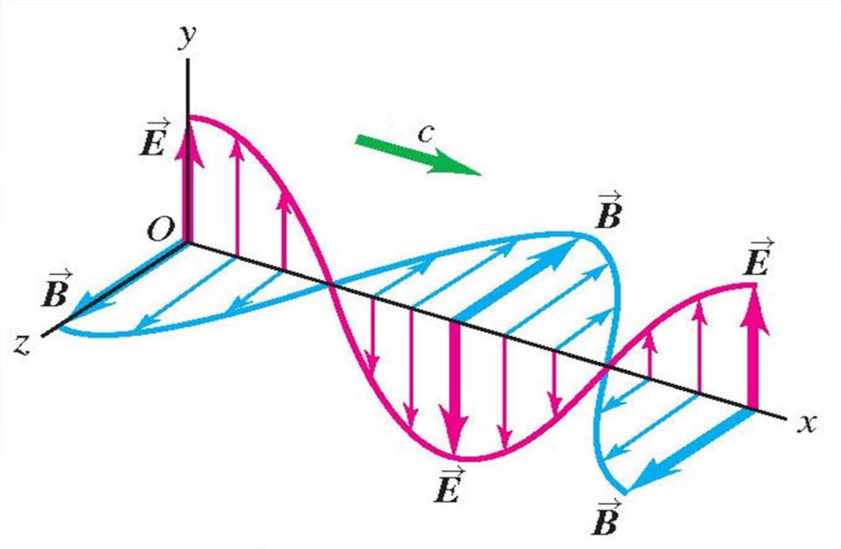
μ_0 is the permeability in vacuum $= 1.26 \times 10^{-6}$ N/A^2

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

Part 3. The wavefunction



The electromagnetic wavefunction for sinusoidal waves



$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Not the same k

(one is a direction vector and the other is the wave number)

Electromagnetic waves: The wavefunction

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Amplitude: $E_{max} = c\, B_{max}$

$c = \lambda / T$
$f = 1 / T$

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$c = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k$

Task: Show that $E_{max} = c\, B_{max}$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

Plane wave

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$

The wavefunction:

$$\vec{E}(x,t) = \hat{j} E_{max} \cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k} B_{max} \cos(kx - \omega t)$$

$$-E_{max} k \sin(kx - \omega t) = -B_{max}\, \omega \sin(kx - \omega t)$$

$$E_{max} = \frac{\omega}{k} B_{max} = c B_{max}$$

Compare wavefunctions

Mechanical waves

$$y(x, t) = A\cos(kx - \omega t)$$

Amplitude: A

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$v = \lambda / T = \omega / k$

Electromagnetic waves

$$\vec{E}(x, t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x, t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Amplitude: $E_{max} = c\,B_{max}$

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$c = \lambda / T = \omega / k$

Electromagnetic waves: Wavefunction

In a dielectric medium the speed of light is smaller than c !

Electromagnetic waves in matter:

$c \longrightarrow v$

$\mu_0 \longrightarrow \mu$

$\varepsilon_0 \longrightarrow \varepsilon$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon \mu}}$$

Dielectric constant

$K = \varepsilon / \varepsilon_0$

Relative permeability

$K_m = \mu / \mu_0$

Electromagnetic waves: Wavefunction

Electromagnetic waves in vacuum

$E = c B$ from Faraday´s law

$E = B / (\varepsilon_0 \mu_0 c)$ from Ampere´s law

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Electromagnetic waves in matter

$E = v B$ from Faraday´s law

$E = B / (\varepsilon \mu v)$ from Ampere´s law

$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

Permability (→ μ)

Permittivity (→ ϵ)

$$\frac{c}{v} = n = \frac{\frac{1}{\sqrt{\epsilon_0 \mu_0}}}{\frac{1}{\sqrt{\epsilon \mu}}} = \sqrt{K K_m} \cong \sqrt{K}$$

$$\Rightarrow \quad v = \frac{c}{\sqrt{K K_m}}$$

Refractive index (→ n)

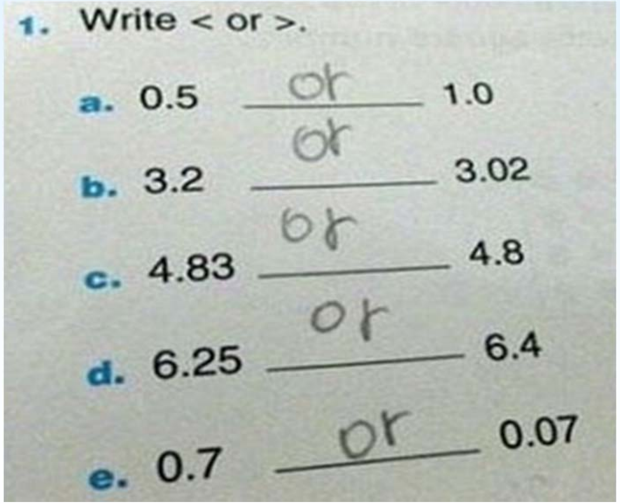
Dielectric constant (→ K)

Relative permeability (→ K_m)

$K = \varepsilon / \varepsilon_0$

$K_m = \mu / \mu_0$

Part 4. Problems


1. Write < or >.

a. 0.5 or 1.0

b. 3.2 or 3.02

c. 4.83 or 4.8

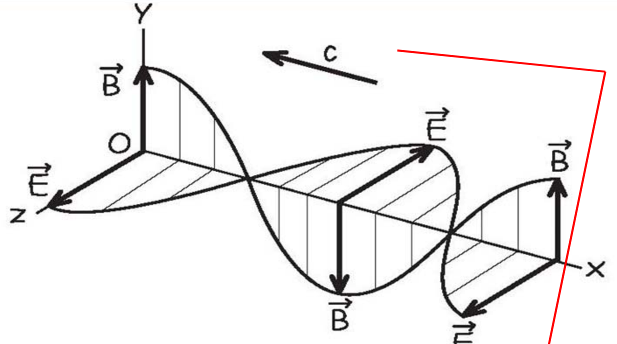
d. 6.25 or 6.4

e. 0.7 or 0.07

Electromagnetic waves: Problems

A laser sends out a sinusoidal electromagnetic wave in the negative x-direction with the wavelength 10.6 μm. The E-field is in the z-direction and E_{max} = 1.5 MV/m.

Give the wave function of the laser beam.



$\vec{E}$: z-component only
$\vec{B}$: y-component only

$$\vec{E}(x, t) = \hat{k}E_{max}\cos(kx + \omega t)$$
$$\vec{B}(x, t) = \hat{j}B_{max}\cos(kx + \omega t)$$

$E_{max} = c\ B_{max}$

$k = 2\pi/\lambda$

$c = \omega/k$

$$B_{max} = \frac{E_{max}}{c} = \frac{1.5 \times 10^6 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 5.0 \times 10^{-3} \text{ T}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{10.6 \times 10^{-6} \text{ m}} = 5.93 \times 10^5 \text{ rad/m}$$

$$\omega = ck = (3.00 \times 10^8 \text{ m/s})(5.93 \times 10^5 \text{ rad/m}) = 1.78 \times 10^{14} \text{ rad/s}$$

$$\vec{E}(x, t) = \hat{k}(1.5 \times 10^6 \text{ V/m}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

$$\vec{B}(x, t) = \hat{j}(5.0 \times 10^{-3} \text{ T}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

Electromagnetic waves: Problems

Yellow light with f = 5.09x10[14] Hz goes from vacuum into a diamond.

What is the wavelength in vacuum?
What is the wavelength and wave velocity in the diamond if K = 5.84 & K_m=1.00

$v=c=\lambda/T=\lambda f$

Vacuum:

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 5.89 \times 10^{-7} \text{ m} = 589 \text{ nm}$$

$$v = \frac{c}{\sqrt{KK_m}}$$

Diamond:

$$v_{\text{diamond}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(5.84)(1.00)}} = 1.24 \times 10^8 \text{ m/s}$$

$$\lambda_{\text{diamond}} = \frac{v_{\text{diamond}}}{f} = \frac{1.24 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 2.44 \times 10^{-7} \text{ m} = 244 \text{ nm}$$

Electromagnetic waves: Problems

Radio waves with 90.0 MHz go from vacuum into insulating ferrite.

What is the wavelength in vacuum?
What is the wavelength and wave velocity in the ferrite if K = 10.0 & K_m=1000 ?

$v=\lambda/T=\lambda f=c$

$$v = \frac{c}{\sqrt{KK_m}}$$

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}} = 3.33 \text{ m}$$

$$v_{\text{ferrite}} = \frac{c}{\sqrt{KK_\text{m}}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(10.0)(1000)}} = 3.00 \times 10^6 \text{ m/s}$$

$$\lambda_{\text{ferrite}} = \frac{v_{\text{ferrite}}}{f} = \frac{3.00 \times 10^6 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}}$$

$$= 3.33 \times 10^{-2} \text{ m} = 3.33 \text{ cm}$$

Part 5. Power and intensity

Blue Laser
Power = 1 W

Mechanical waves: Power & Intensity

Wave power (P): The instantaneous rate at which energy is transfered along the wave. (P = energy per unit of time)
Unit: W eller J/s

Wave intensity (I): Average power per unit area through a surface perpendicular to the wave direction. (I =power per unit of area).
Unit: W/m²

$$I = P_{av} / A_{rea}$$

The power in general:

$$P = \vec{F} \cdot \vec{v}$$ (instantaneous rate at which force $\vec{F}$ does work on a particle)

Wave power (P):

$$P(x,t) = F_y(x,t)v_y(x,t)$$

if y is the only direction where the velocity is not zero

Electromagnetic waves: Power & Intensity

Total energy density (u):

Energy per unit volume due to an electric and magnetic field. Unit: J/m^3

Power (P):

The instantaneous rate at which energy is transfered along a wave.

Unit: W or J/s

The Poynting vector ($\bar{S}$):

Energy transferred per unit time per unit area = Power per unit area.

Unit: W/m^2

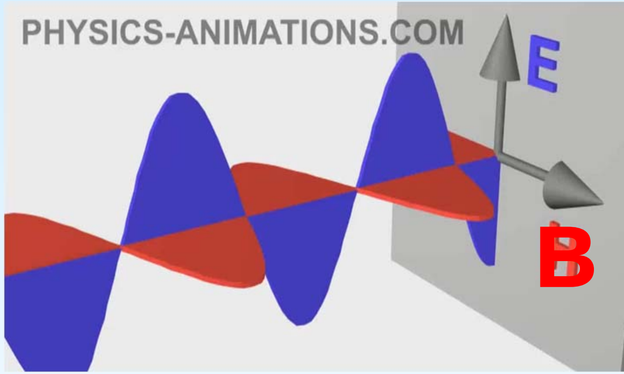
Intensity (I):

Average power per unit area through a surface perpendicular to the wave direction = the average value of S.

Unit: W/m^2

Energydensity (energy per unit volume) due to an electromagnetic field.

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$



Faraday's law: $E = cB$ ⟹ $B = \sqrt{\varepsilon_0 \mu_0}\, E$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\left(\sqrt{\epsilon_0 \mu_0}\, E\right)^2 = \epsilon_0 E^2$$

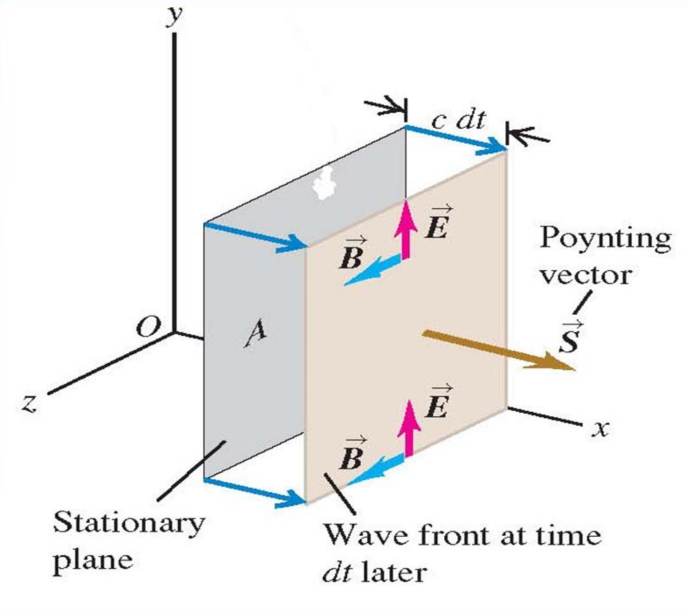
where $E(x, t) = E_{max} \cos(kx - \omega t)$

Energy E-field Energy B-field

Conclusions: The electric and magnetic fields carry the same amount of energy.
The energy density varies with position and time.

Electromagnetic waves: Power & Intensity

- Energy transfer = energy transferred per unit time per unit area.
- S = Power per unit area = Energy transfer = Energy flow



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \text{(Poynting vector in vacuum)}$$

Sinusoidal waves:

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t)$$

$$= \frac{1}{\mu_0}[\hat{\jmath}E_{\max}\cos(kx - \omega t)] \times [\hat{k}B_{\max}\cos(kx - \omega t)]$$

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx - \omega t)$$

Amplitude = maximum energy transfer

Electromagnetic waves: Power & Intensity

Intensity = the average value of S

$$E = c\,B$$

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx - \omega t)$$

The average of $\cos^2(x) = 1/2$

$$I = S_{av} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c}$$

Electromagnetic waves in matter:

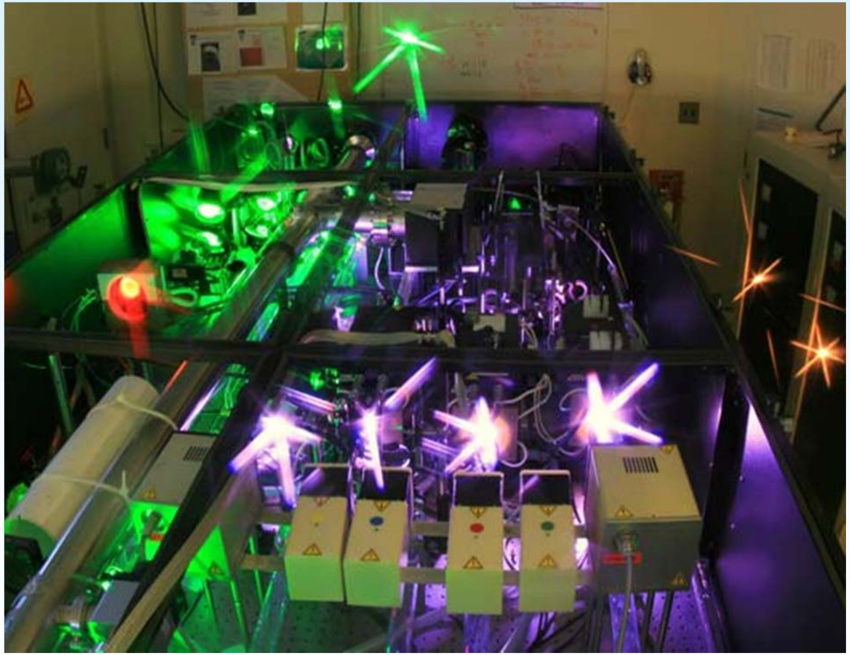
$c \rightarrow v$

$\mu_0 \rightarrow \mu$

$\varepsilon_0 \rightarrow \varepsilon$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$$

The Hercules Petawatt Laser

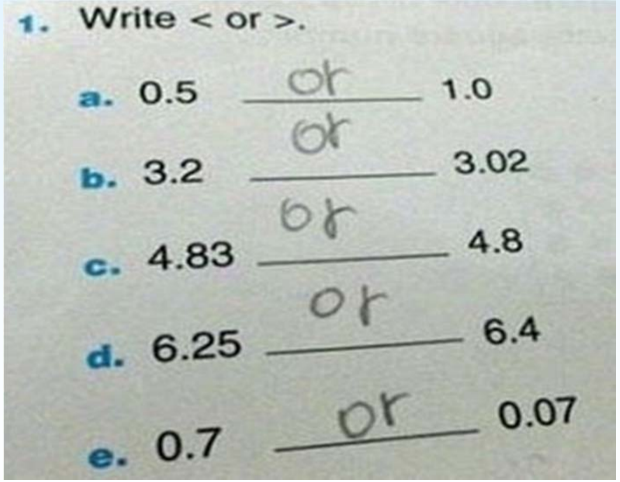


Power = 300 TW = 3×10^{14} W

Intensity = 2×10^{22} W/cm^2

To get the same intensity from the sun's light, you need to focus all the sun's rays that hit the earth on a grain of sand .....

Part 6. Problems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a. 0.5 __or__ 1.0

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d. 6.25 __or__ 6.4

e. 0.7 __or__ 0.07

Electromagnetic waves: problems

A sinusoidal electromagnetic wave has E_{max} = 100 V/m. What is B_{max} ?
What is the maximum energy density?

Given:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$
$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{100\ \mathrm{V/m}}{3.00 \times 10^8\ \mathrm{m/s}} = 3.33 \times 10^{-7}\ \mathrm{T}$$

$$E(x,t) = E_{max}\cos(kx - \omega t)$$

$$u(x,t) = \varepsilon_0 E^2 = \varepsilon_0 E_{max}{}^2 \cos^2(kx - \omega t)$$

$$u_{max} = \varepsilon_0 E_{max}{}^2 = (8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(100\ \mathrm{N/C})^2 = 8.85 \times 10^{-8}\ \mathrm{N/m^2}$$

Electromagnetic waves: problems

A sinusoidal electromagnetic wave has E_{max} = 100 V/m and B_{max} = 3.33x10^{-7} T.

What is the intensity?

Given:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

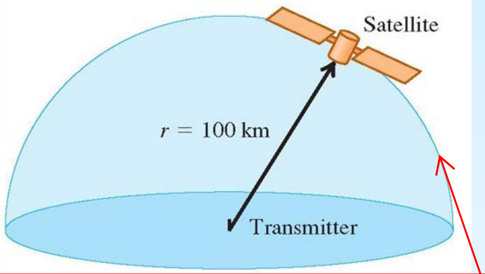
$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

$$S_x(x,t) = \frac{E_{\max} B_{\max}}{\mu_0} \cos^2(kx - \omega t)$$

$$I = S_{av} = \frac{E_{\max} B_{\max}}{2\mu_0}$$

$$I = S_{av} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{(100\ \mathrm{V/m})(3.33 \times 10^{-7}\ \mathrm{T})}{2 \times 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}} = 13.2\ \mathrm{W/m^2}$$

Electromagnetic waves: problems



A radio station sends out a sinusoidal wave with an average power of 50 kW. What will be the amplitude of the wave if it is detected by a satellite 100 km away?

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$
$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

Area: $A = 2\pi R^2$

I from method 1: $I = \frac{P}{A} = \frac{P}{2\pi R^2}$

$B_{max} = \frac{E_{max}}{c}$

I from method 2: $I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c}$

Amplitude for E: $\frac{E_{max}{}^2}{2\mu_0 c} = \frac{P}{2\pi R^2}$ ⇒ $E_{max} = \sqrt{\frac{P\mu_0 c}{\pi R^2}} = 2.45\ x\ 10^{-2}\ V/m$

Amplitude for B: $B_{max} = \frac{E_{max}}{c} = 8.17 \times 10^{-11}\ \mathrm{T}$

Part 7. Summary

Electromagnetic waves: Summary

Wavefunction:

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

$$E = cB$$

Speed of light:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

c = λ/T = ω/k

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

Energy density:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$u_B = \frac{B^2}{2\mu_0}$$

Electromagnetic waves: Summary

Power per unit area:

$$S_x(x,t) = 2S_{av}\cos^2(kx-\omega t)$$

Intensity =
Average power per unit area:

$$S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c E_{max}{}^2$$