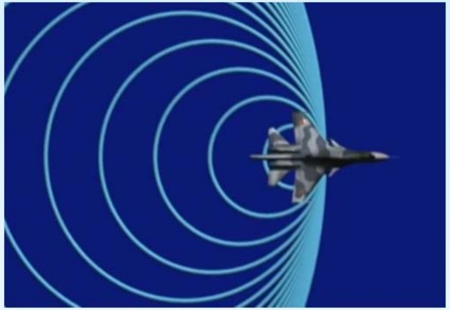
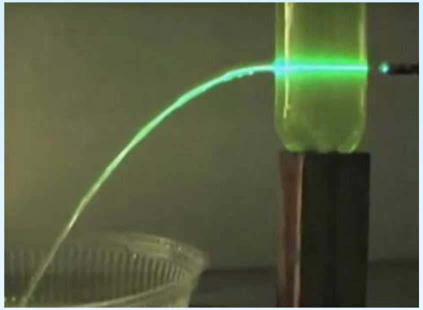
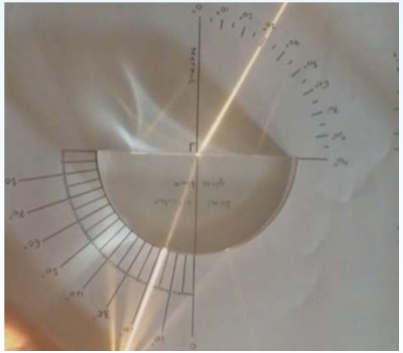
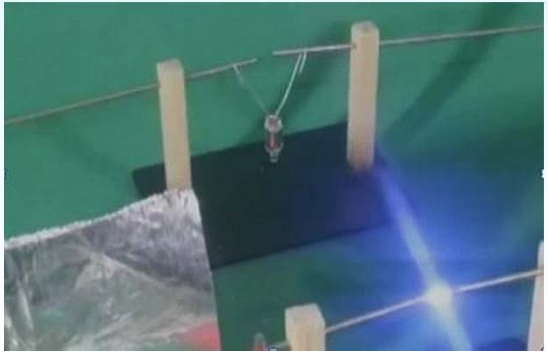
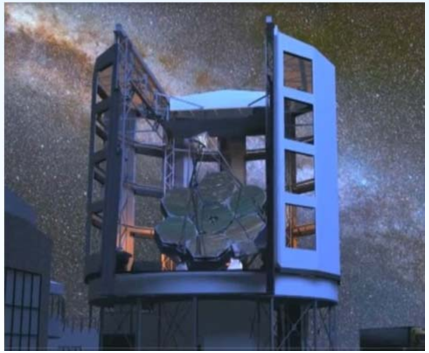
Wavemechanics and optics

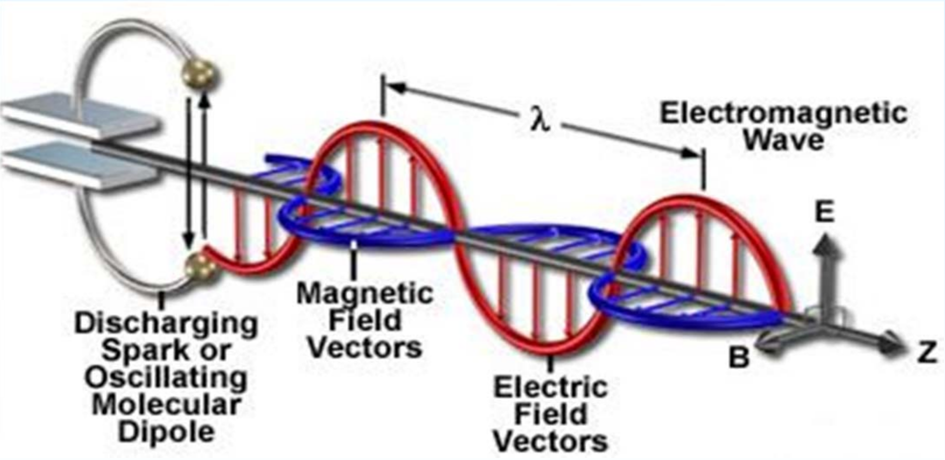



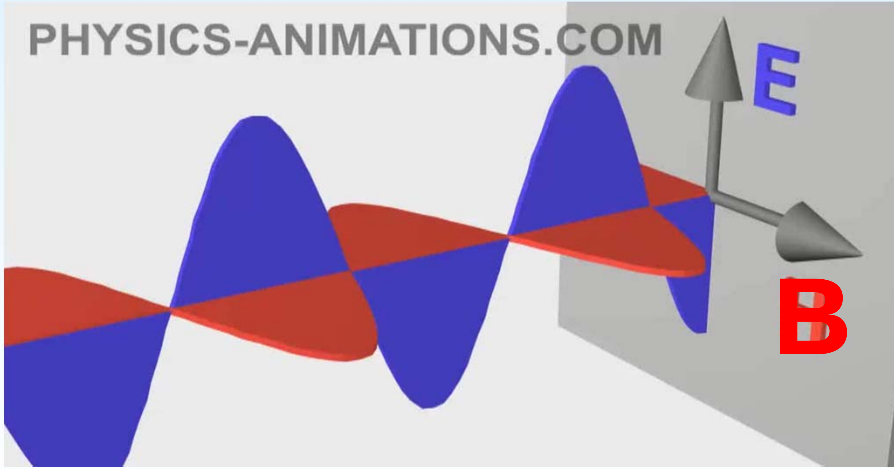




Chapter 32 – Electromagnetic waves

Electromagnetic waves

The electromagnetic wave consists of an electric and a magnetic field.

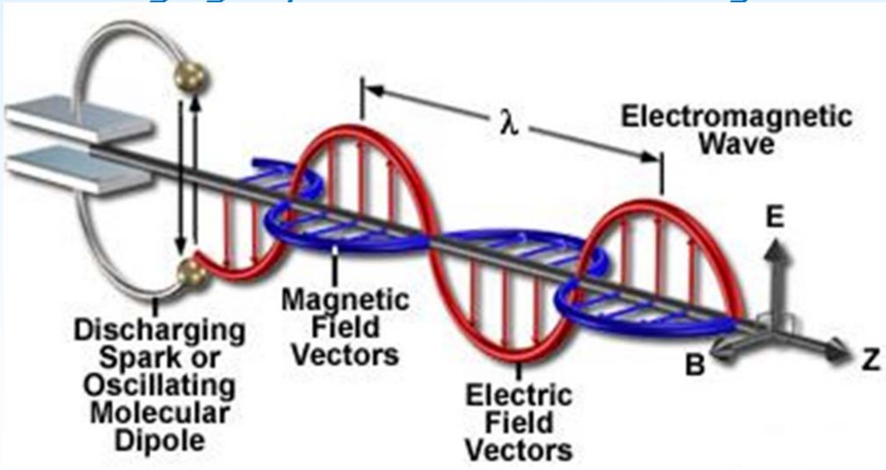




Electromagnetic waves

- Electromagnetic waves are produced by the movement of charged particles.
- An **electromagnetic wave** is capable of transmitting energy through **a vacuum**.

An electromagnetic wave can be generated by a discharging capacitor or an oscillating molecular dipole.



The field is strongest at 90 degrees to the moving charge and zero in the direction of the moving charge.

As the current oscillates up and down in the spark gap a magnetic field is created that oscillates in a horizontal plane.

The changing magnetic field, in turn, induces an electric field so that a series of electrical and magnetic oscillations combine to produce a formation that propagates as an electromagnetic wave.

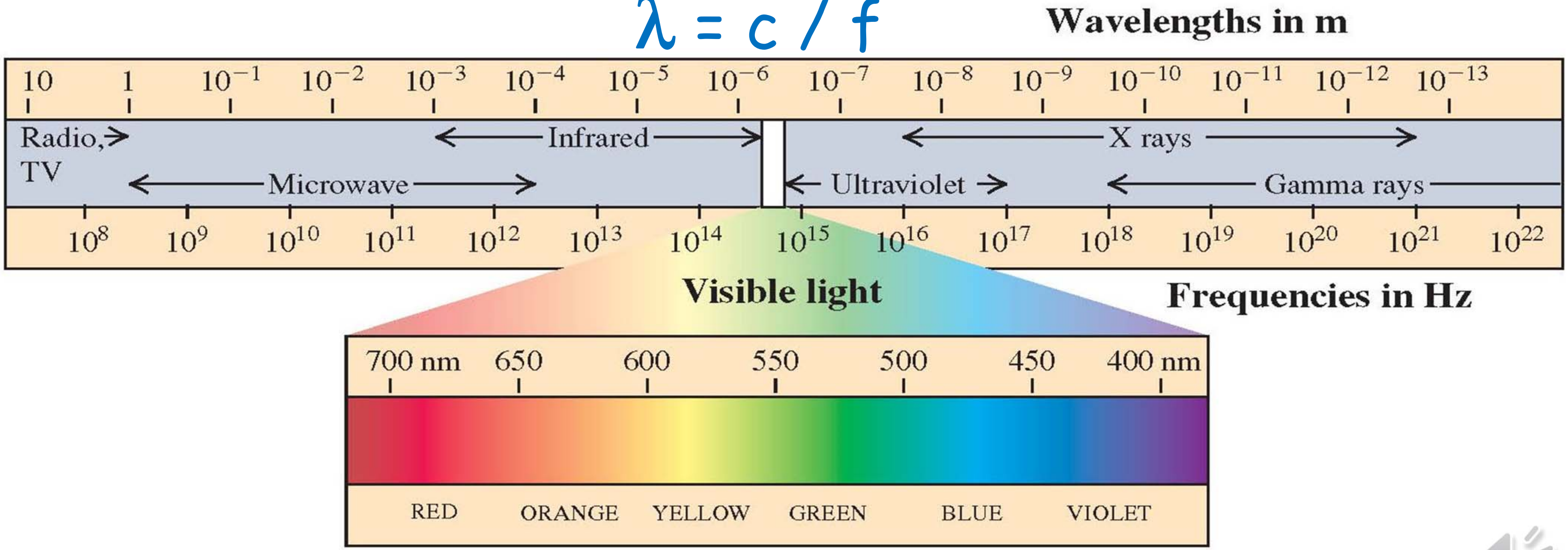
Electromagnetic waves

Hertz experiment demonstrated how moving charges creates an electromagnetic field

https://www.youtube.com/watch?v=9gDFll6Ge7g

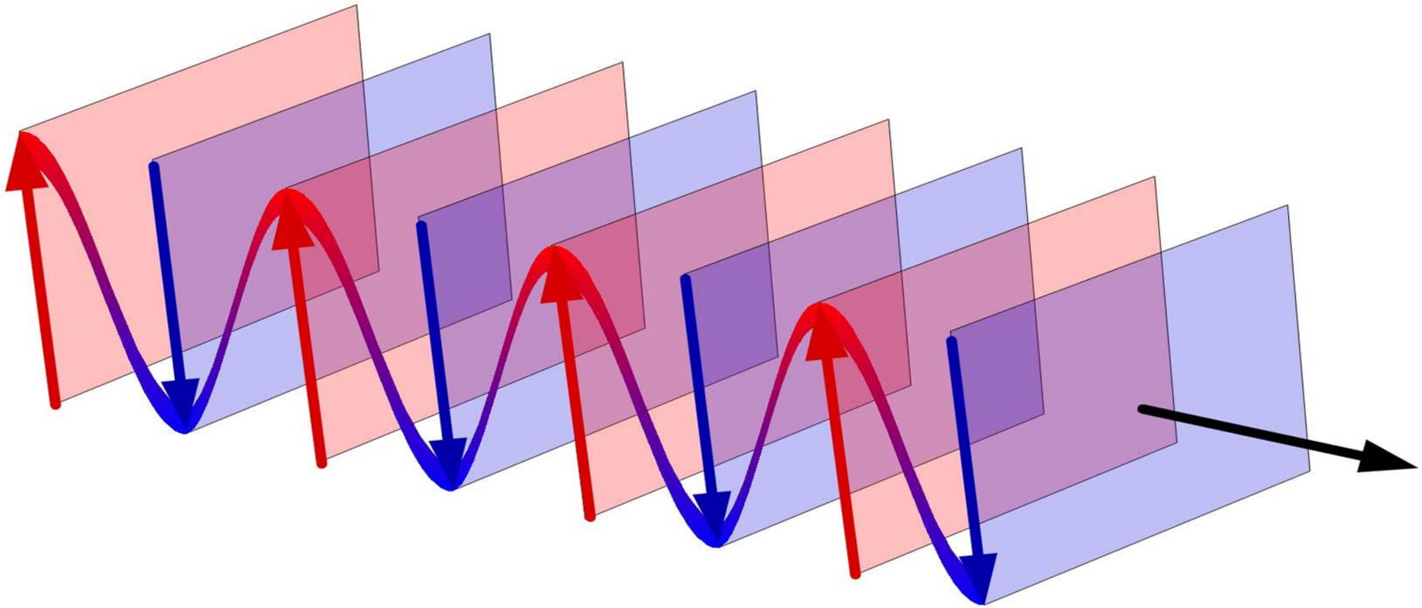
The electromagnetic spectrum

$$\lambda = c / f$$



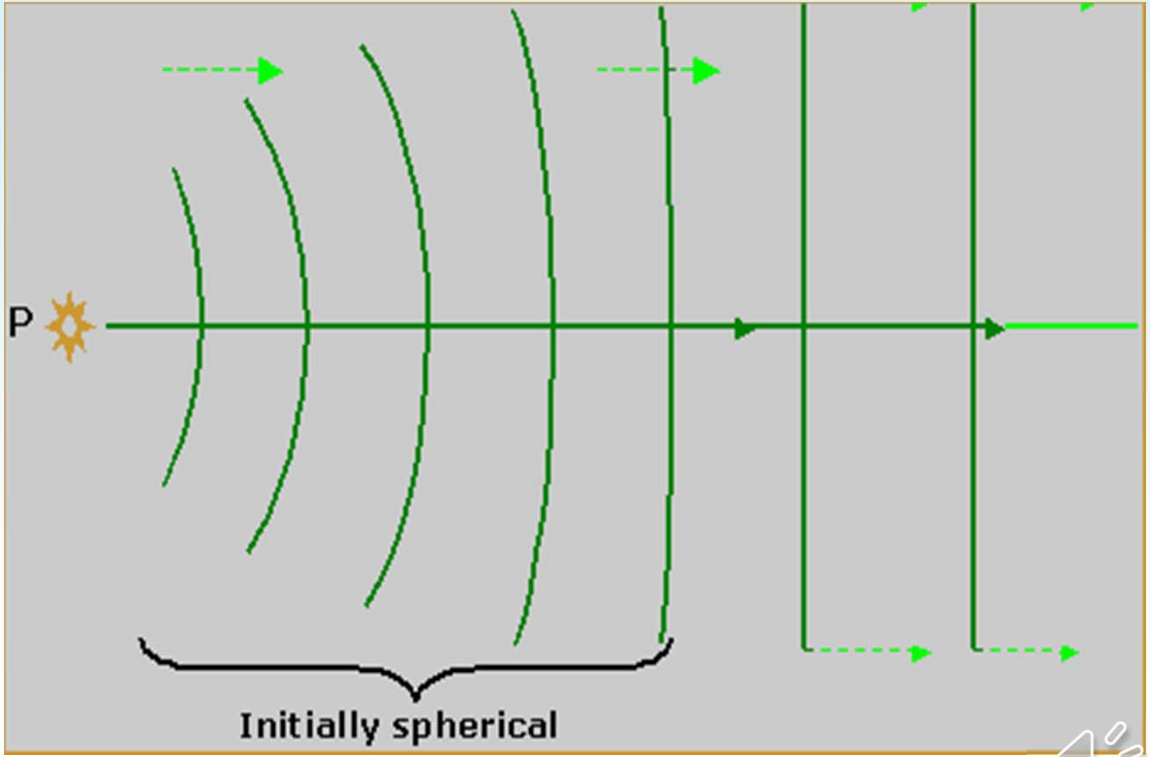
Electromagnetic waves

Wavefronts: surfaces with constant phase



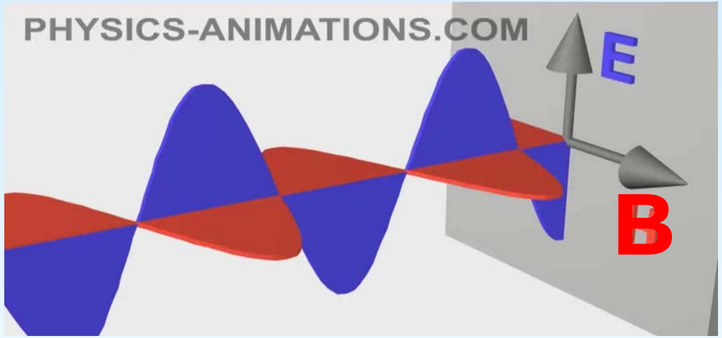
Electromagnetic waves

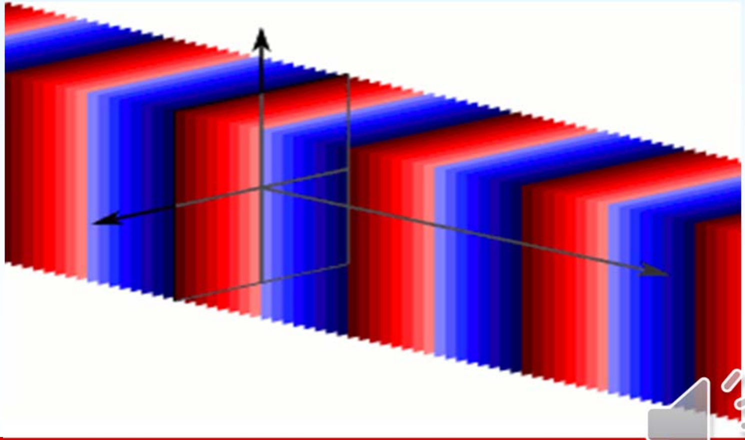
Wavefronts depends on the distance to the source



Electromagnetic waves

- Electromagnetic waves are transverse because the E and B fields are perpendicular to the direction of propagation.
- A plane wave is a wave with constant frequency whose wave fronts are infinite parallel planes with constant peak-to-peak amplitude.
- At a certain point and time, all the E and B vectors in the plane are of the same size.
- Completely flat waves do not exist because only an infinitely large wave can be flat. But many waves are approximately flat in a localized area of space.





Electromagnetic waves

For flat electromagnetic waves, one can find relations between the magnitude of the magnetic and electric fields from two of Maxwell's equations:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)}$$

Flat wave → $E = cB$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \quad \text{(Ampere's law)}$$

Flat wave → $E = \dfrac{B}{\varepsilon_0 \mu_0 c}$

c = The speed of light.
ε = Permittivity: A mediums ability to form an electric field in itself.
μ = Permeability: A mediums ability to form a magnetic field in itself.

Electromagnetic waves

The speed of light from Maxwell's equations:

$E = cB$ from Faraday´s law

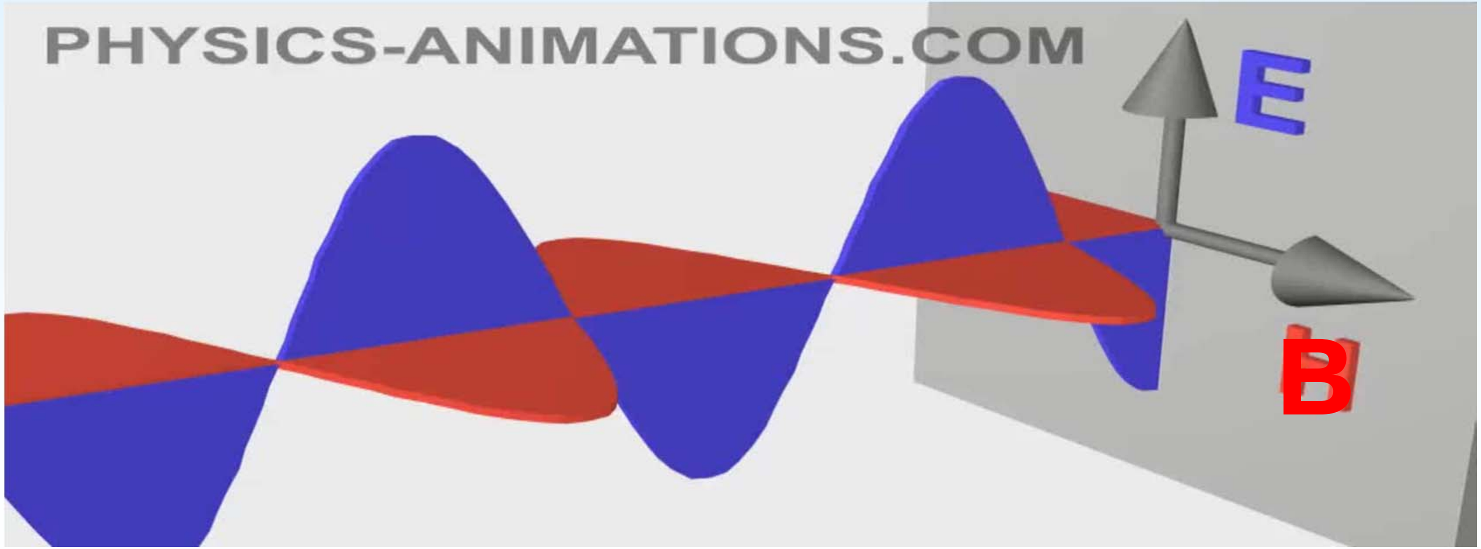
$E = B / (\varepsilon_0 \mu_0 c)$ from Ampere´s law

ε_0 is the permittivity in vacuum $= 8.85 \times 10^{-12}$ F/m

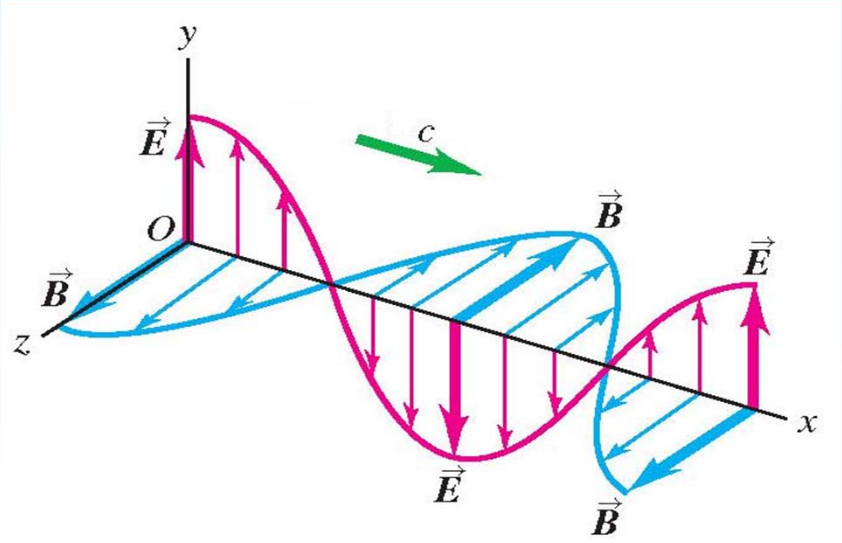
μ_0 is the permeability in vacuum $= 1.26 \times 10^{-6}$ N/A^2

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

The wavefunction



The electromagnetic wavefunction for sinusoidal waves



$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Not the same k

(one is a direction vector and the other is the wave number)

Electromagnetic waves: The wavefunction

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

$c = \lambda / T$

$f = 1 / T$

Amplitude: $E_{max} = c\, B_{max}$

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$c = \lambda / T = (2\pi/k) / (2\pi/\omega) = \omega / k$

Compare wavefunctions

Mechanical waves

$$y(x, t) = A\cos(kx - \omega t)$$

Amplitude: A

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$v = \lambda / T = \omega / k$

Electromagnetic waves

$$\vec{E}(x, t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x, t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Amplitude: $E_{max} = c\, B_{max}$

Wavenumber: $k = \frac{2\pi}{\lambda}$

Angular frequency: $\omega = \frac{2\pi}{T}$

$c = \lambda / T = \omega / k$

Electromagnetic waves: Wavefunction

In a dielectric medium the speed of light is smaller than c !

Electromagnetic waves in matter:

$c \longrightarrow v$

$\mu_0 \longrightarrow \mu$

$\varepsilon_0 \longrightarrow \varepsilon$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon \mu}}$$

Dielectric constant

$K = \varepsilon / \varepsilon_0$

Relative permeability

$K_m = \mu / \mu_0$

Electromagnetic waves: Wavefunction

Electromagnetic waves in vacuum

$E = cB$ from Faraday's law

$E = B / (\varepsilon_0 \mu_0 c)$ from Ampere's law

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Electromagnetic waves in matter

$E = vB$ from Faraday's law

$E = B / (\varepsilon \mu v)$ from Ampere's law

$$v = \frac{1}{\sqrt{\epsilon \mu}}$$

Permability (μ)

Permittivity (ϵ)

$$\frac{c}{v} = n = \frac{\frac{1}{\sqrt{\epsilon_0 \mu_0}}}{\frac{1}{\sqrt{\epsilon \mu}}} = \sqrt{K K_m} \cong \sqrt{K}$$

$$\Rightarrow \quad v = \frac{c}{\sqrt{K K_m}}$$

Refractive index (n)

Dielectric constant (K)

Relative permeability (K_m)

$K = \varepsilon / \varepsilon_0$

$K_m = \mu / \mu_0$

Power and intensity

Blue Laser
Power = 1 W

Electromagnetic waves: Power & Intensity

Total energy density (u):
Energy per unit volume due to an electric and magnetic field. Unit: J/m^3

Power (P):
The instantaneous rate at which energy is transfered along a wave.
Unit: W or J/s

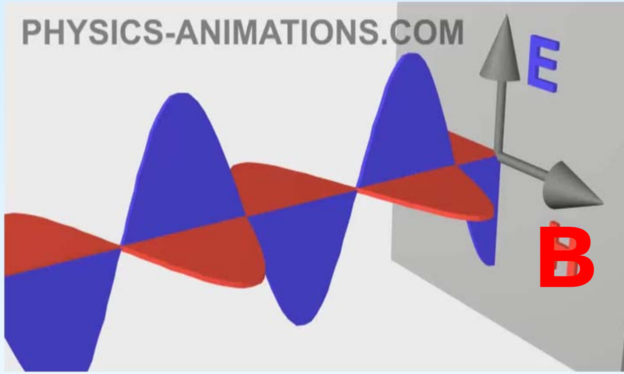
The Poynting vector ($\bar{S}$):
Energy transferred per unit time per unit area = Power per unit area.
Unit: W/m^2

Intensity (I):
Average power per unit area through a surface perpendicular to the wave direction = the average value of S.
Unit: W/m^2

Electromagnetic waves: Power & Intensity

Energydensity (energy per unit volume) due to an electromagnetic field.

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_0}$$



Faraday's law: E = cB ⟹ $B^2 = \varepsilon_0 \mu_0 E^2$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}E)^2 = \epsilon_0 E^2$$

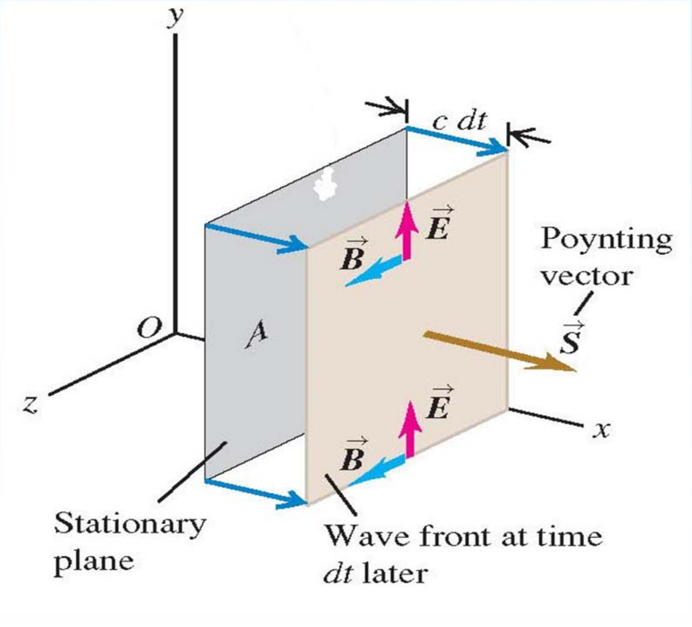
where $E(x,t) = E_{max}\cos(kx - \omega t)$

Energy E-field   Energy B-field

Conclusions: The electric and magnetic fields carry the same amount of energy.
The energy density varies with position and time.

Electromagnetic waves: Power & Intensity

- Energy transfer = energy transferred per unit time per unit area.
- S = Power per unit area = Energy transfer = Energy flow



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \text{(Poynting vector in vacuum)}$$

Sinusoidal waves:

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t)$$
$$= \frac{1}{\mu_0}[\hat{j}E_{max}\cos(kx-\omega t)] \times [\hat{k}B_{max}\cos(kx-\omega t)]$$

$$S_x(x,t) = \frac{E_{max}B_{max}}{\mu_0}\cos^2(kx-\omega t)$$

Amplitude = maximum energy transfer

Electromagnetic waves: Power & Intensity

Intensity = the average value of S

$$E = c\,B$$

$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx - \omega t)$$

The average of $\cos^2(x) = 1/2$

$$I = S_{av} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c}$$

$$S_{av} = \frac{1}{2}\varepsilon_0 c E_{max}{}^2$$

Electromagnetic waves in matter:

$c \rightarrow v$

$\mu_0 \rightarrow \mu$

$\varepsilon_0 \rightarrow \varepsilon$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \longrightarrow v = \frac{1}{\sqrt{\epsilon\mu}}$$

SUMMARY

Electromagnetic waves

Electromagnetic waves: Summary

Wavefunction:

$$\vec{E}(x,t) = \hat{j}E_{\max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{\max}\cos(kx - \omega t)$$

$$E = cB$$

Speed of light:

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

c = λ/T = ω/k

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

Energy density:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$u_B = \frac{B^2}{2\mu_0}$$

Electromagnetic waves: Summary

Power per unit area:

$$S_x(x,t) = 2S_{av}\cos^2(kx-\omega t)$$

Intensity =
Average power per unit area:

$$S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^{\;2}}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c E_{max}^{\;\;2}$$

Summary of wave functions

String: $y(x,t) = A\cos(kx-\omega t)$ $\quad$ $y(x,t) = 2A\sin(kx)\sin(\omega t)$

Sound: $y(x,t) = A\cos(kx-\omega t)$ $\quad$ $p(x,t) = p_{max}\sin(kx-\omega t)$

EM waves: $E(x,t) = E_{max}\cos(kx-\omega t)$ $\quad$ $B(x,t) = B_{max}\cos(kx-\omega t)$

Power functions

String: $P(x,t) = 2P_{av}\sin^2(kx-\omega t)$

$$P_{av} = \frac{1}{2}\mu(\omega A)^2 v = \frac{1}{2}\sqrt{\mu F}(\omega A)^2$$

Sound: $P(x,t) = 2I\sin^2(kx-\omega t)$

$$I = \frac{1}{2}\rho(\omega A)^2 v = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{p_{max}{}^2}{2\rho v} = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

EM Waves: $S_x(x,t) = 2S_{av}\cos^2(kx-\omega t)$

$$S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c E_{max}{}^2$$