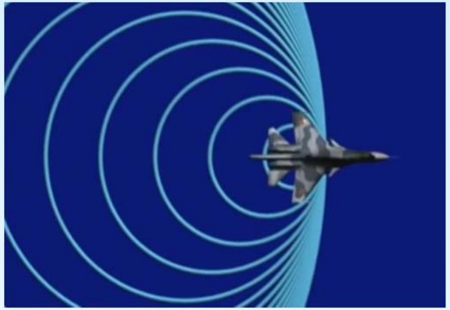
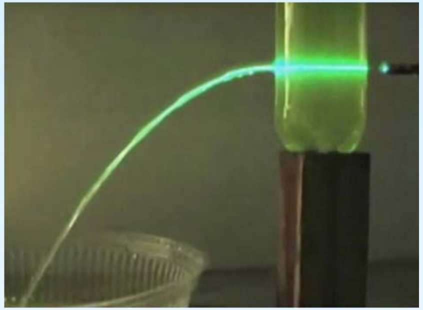
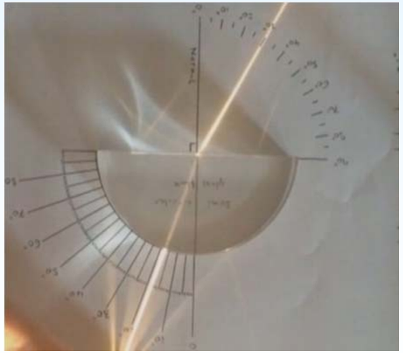
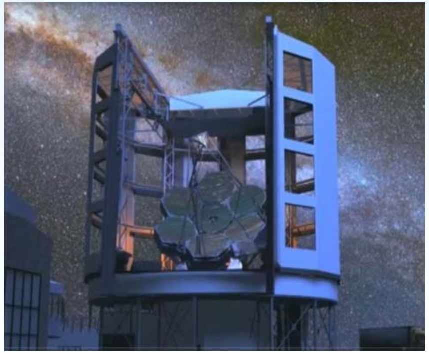
Wavemechanics and optics









Chapter 34 – Optics

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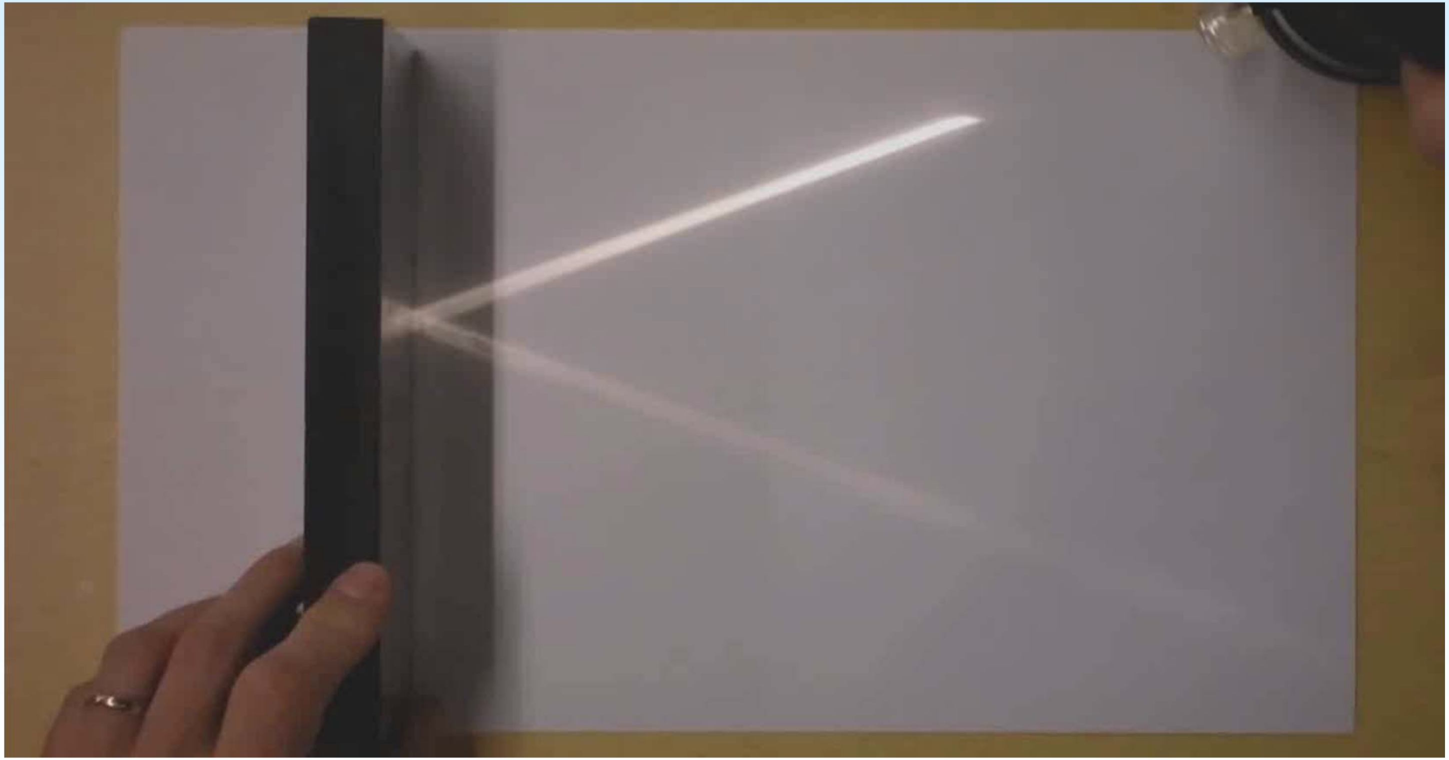
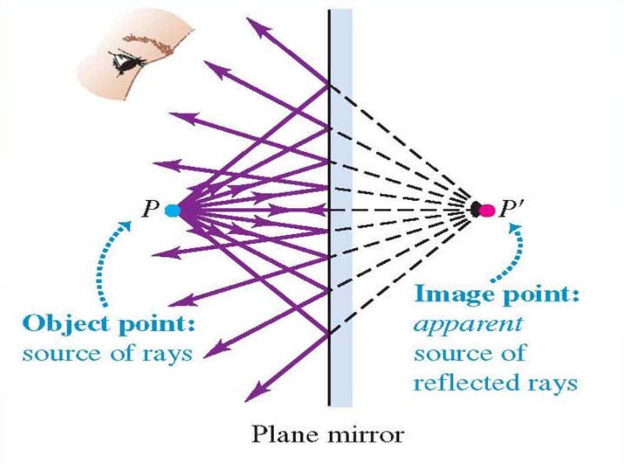
Geometrical optics: Flat mirrors

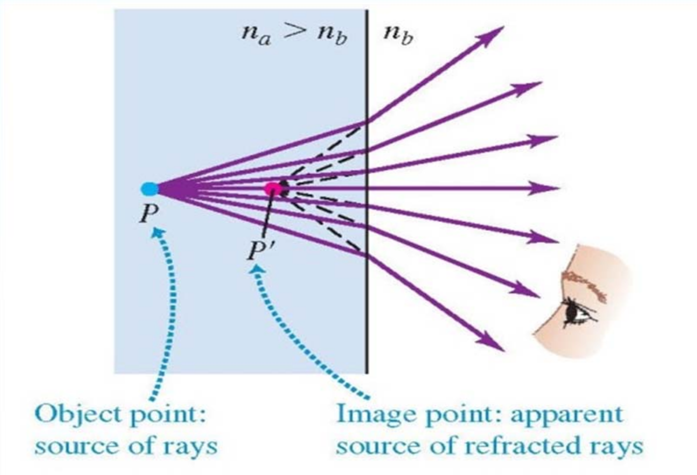


https://www.youtube.com/watch?v=uQE659ICjqQ

Geometrical optics: Flat mirrors

Virtual Images: outgoing rays diverge



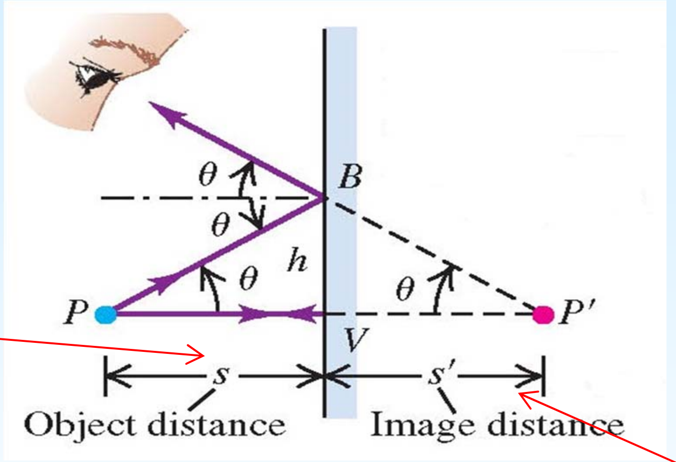


Real Images: outgoing rays converge to an image that can be shown on a screen

Geometrical optics: Flat mirrors

Point object

positive



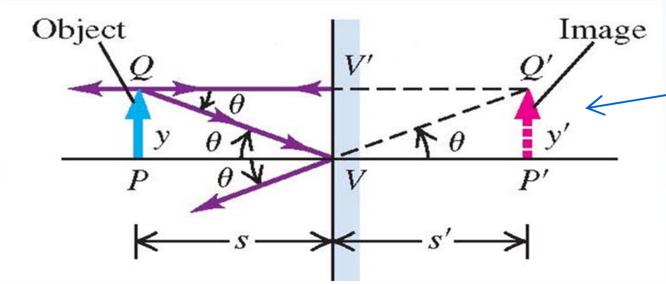
negative

Sign rules:

Object distance (s) – positive if same side as incoming light.

Image distance (s') – positive if same side as outgoing light.

Extended object

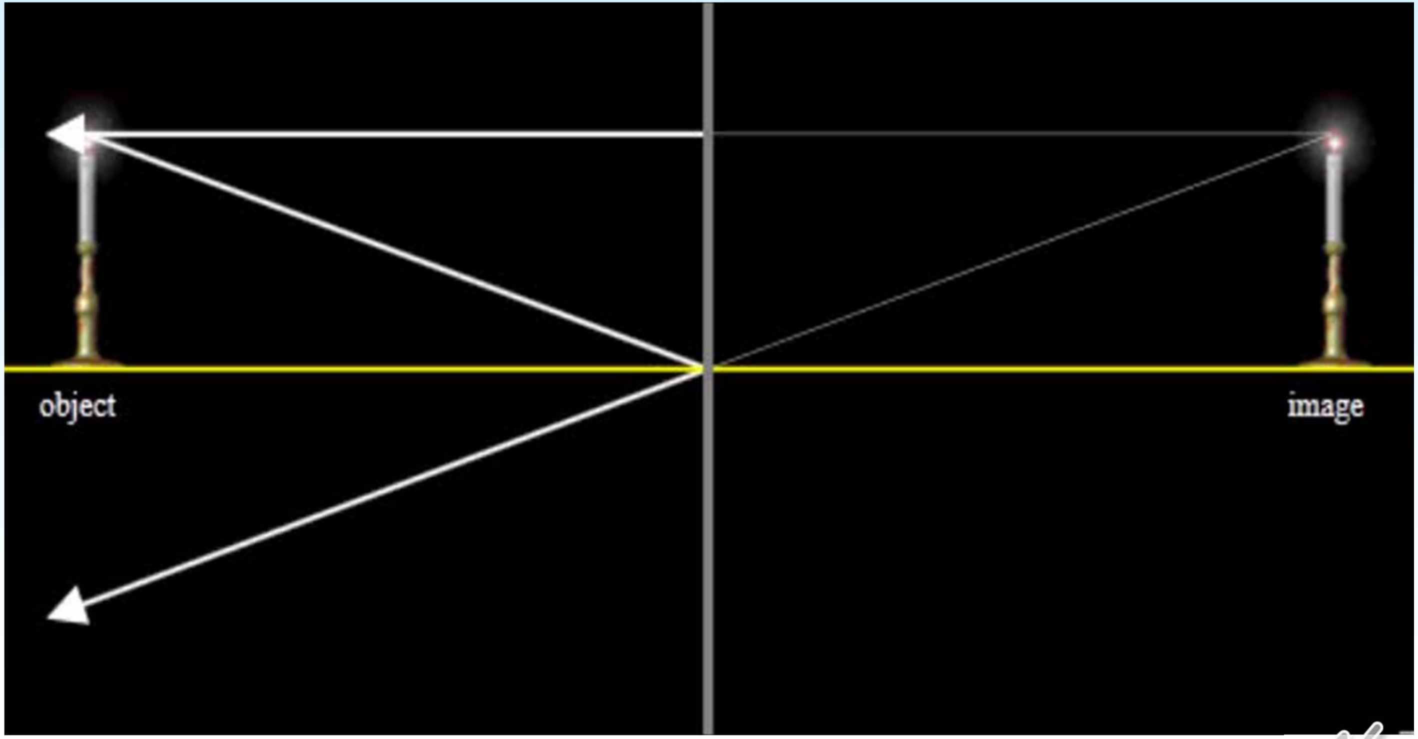


Virtual image

$$m = \frac{y'}{y} \qquad \text{(lateral magnification)}$$

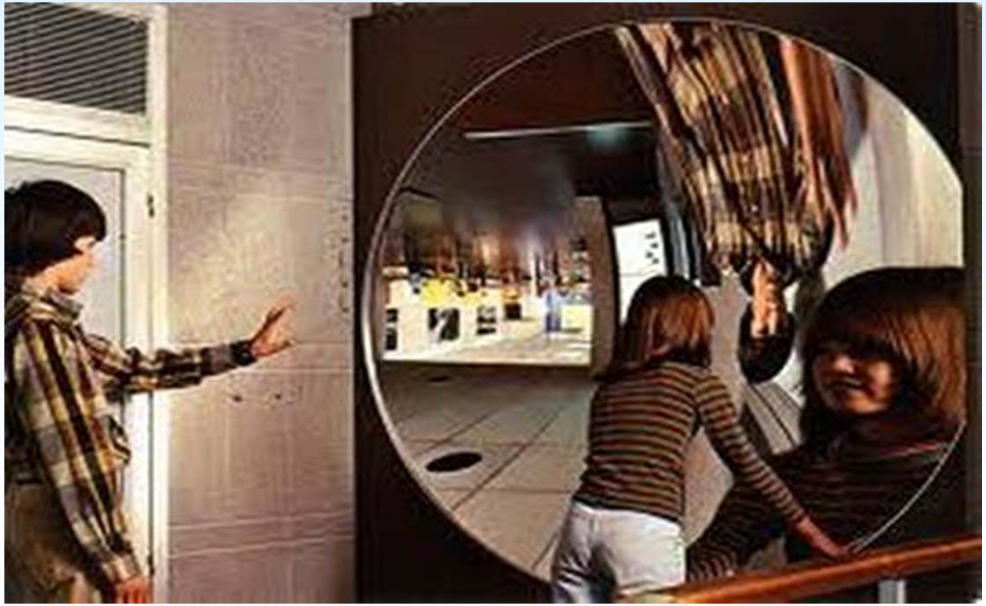
Geometrical optics: Flat mirrors

Simulation of a flat mirror:



http://www.opensourcephysics.org/osp/EJSS/3650/21.htm

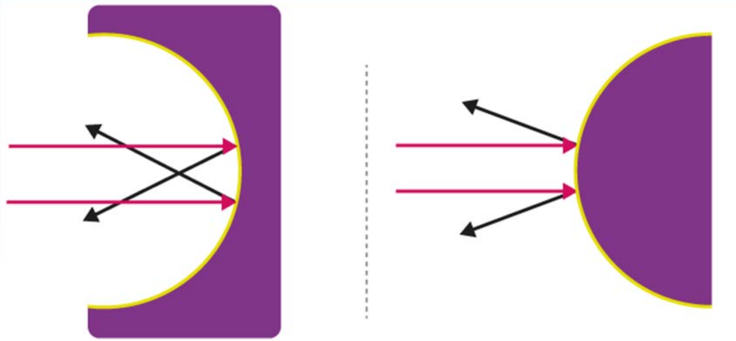
Concave mirrors



Geometrical optics: Concave mirrors

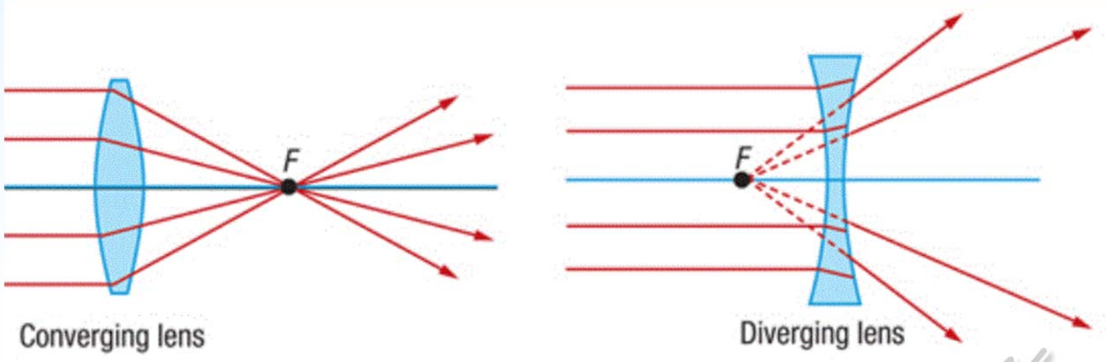
Concave means "hollowed out or rounded inward" and is easily remembered because these surfaces form a "cave".
The opposite is **convex meaning** "curved or rounded outward."

Concave & Convex mirror

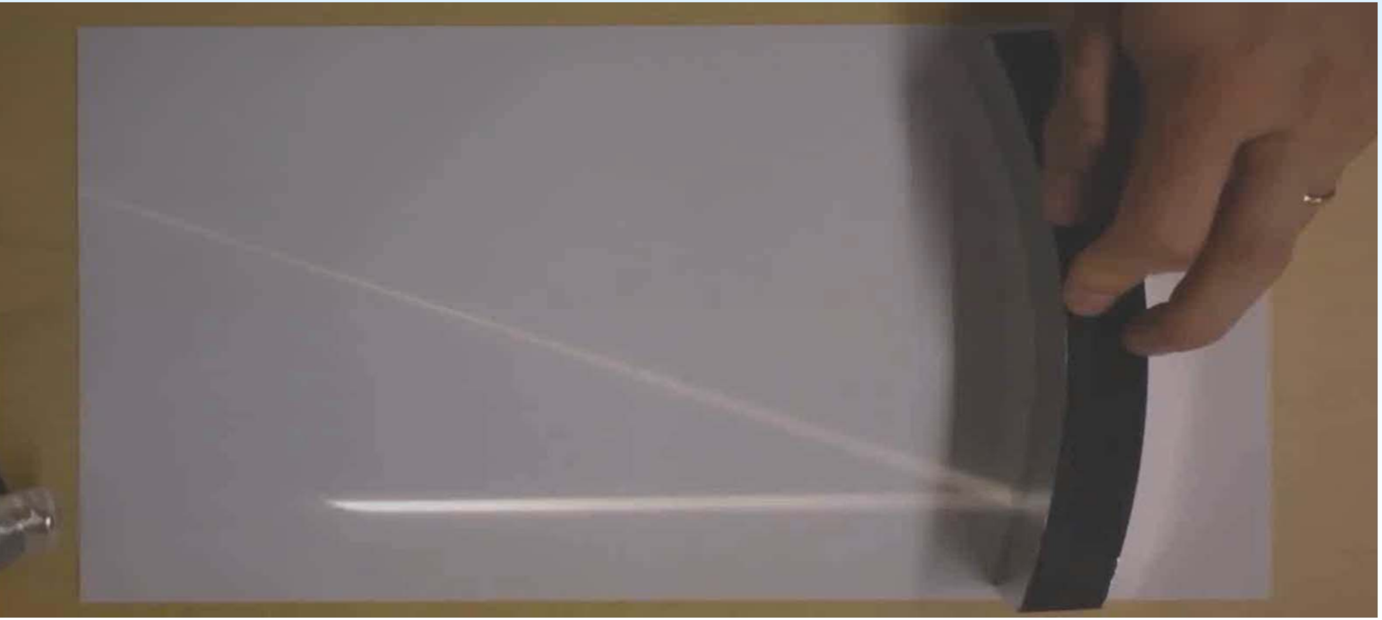


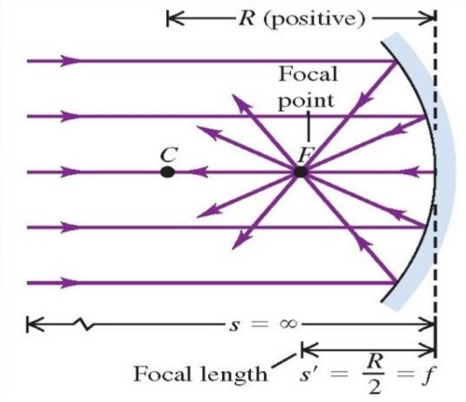
Converging & Diverging

Convex & Concave lens

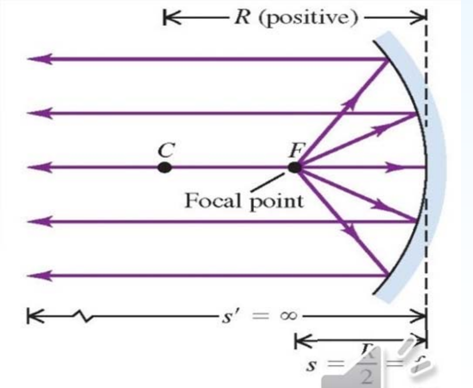


Focal point



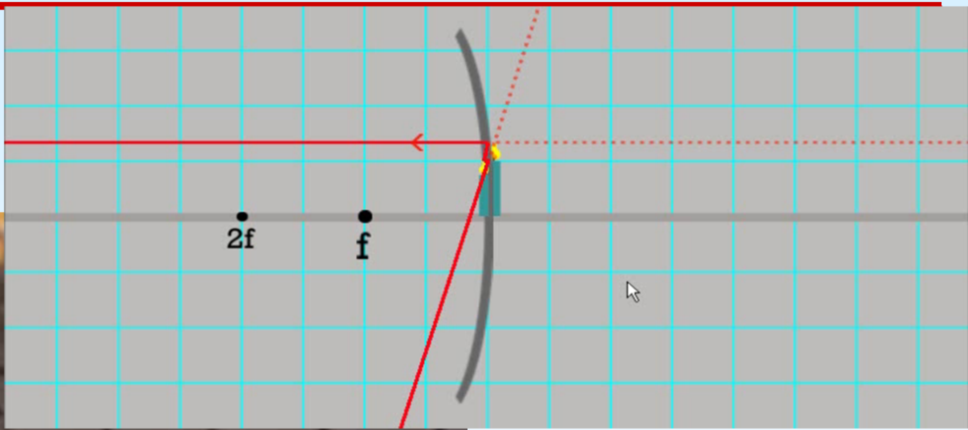


R (positive)
Focal point
C
F
s = ∞
Focal length
s' = R/2 = f



R (positive)
C
F
Focal point
s' = ∞

Geometrical optics: Concave mirrors



Concave mirrors can produce real images

Geometrical optics: Concave mirrors

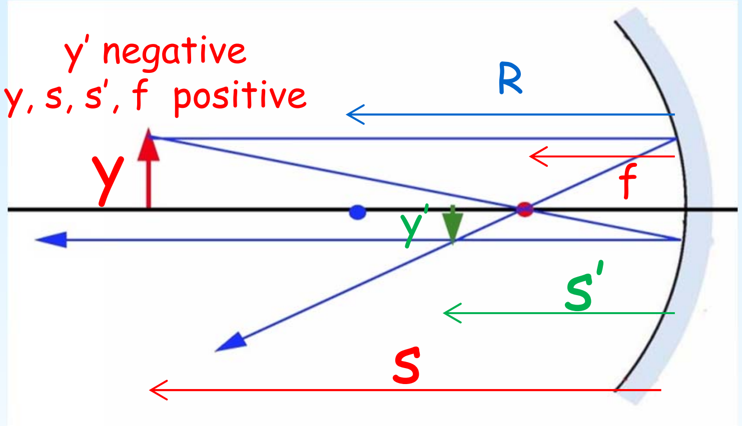
Sign rules:

Positive object distance (s) =
Object is on the side of the incoming light.

Positive image distance (s') =
Image and outgoing light on the same side.

Positive radius of curvature (R) =
Center is on the side of outgoing light.

Positive magnification (m) =
Direction of object and image is the same.



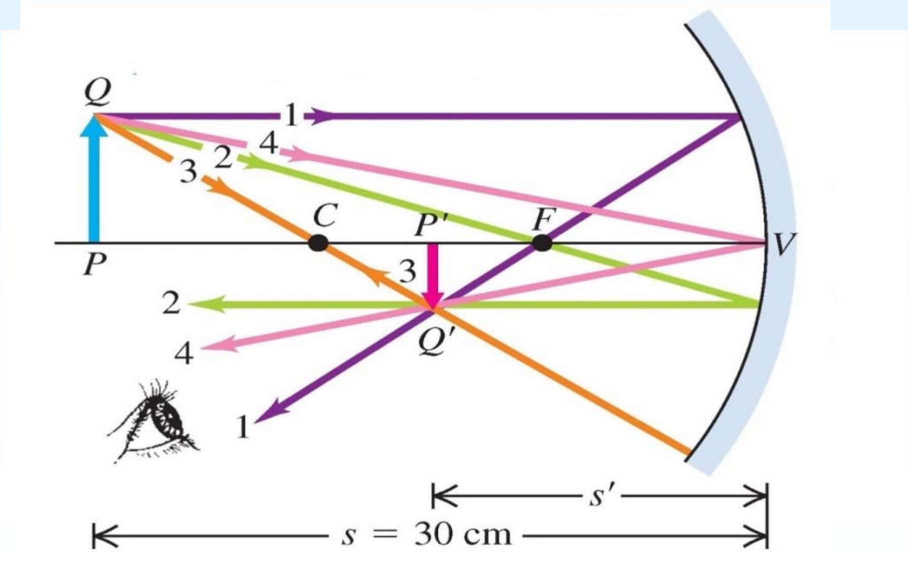
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2}$$

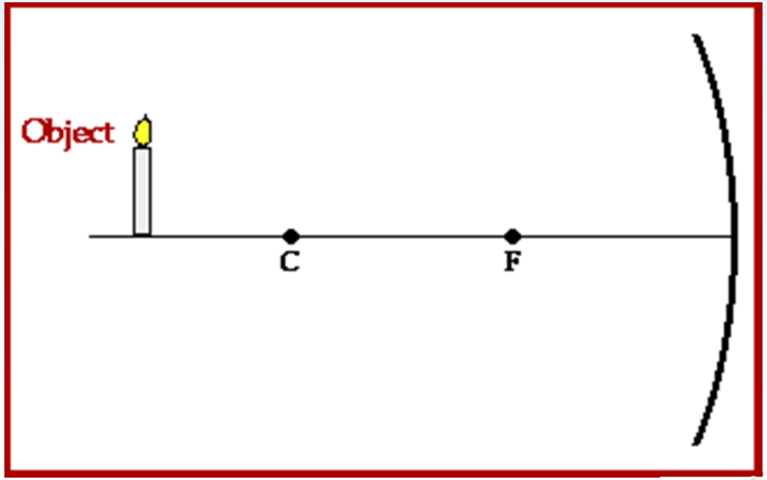
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics: Concave mirrors

An infinite number of rays can be drawn from an object to its image.



But only two rays are needed to determine the location of the image.

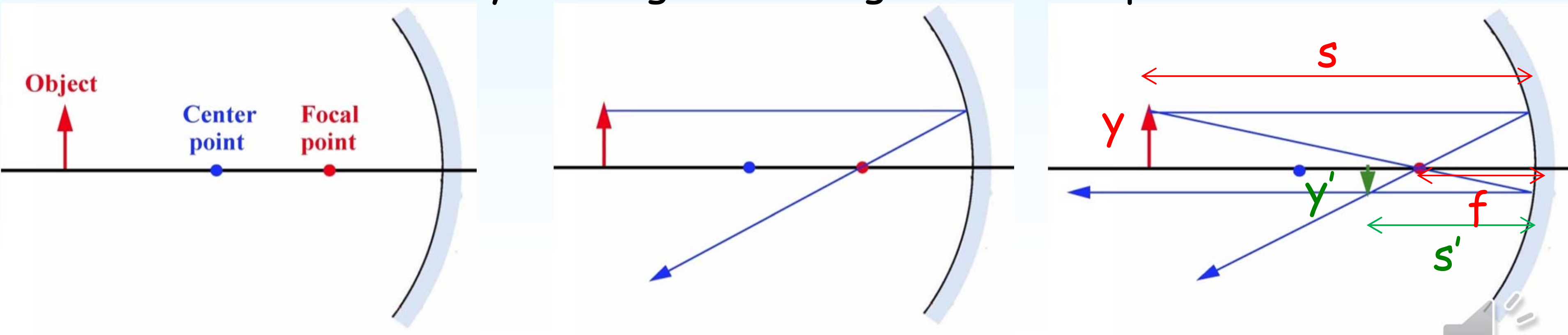


Geometrical optics: Concave mirrors

How to find the image in a concave mirror

The bottom of the object is on the optical axis and so the bottom of the image will also be on the optical axis.

The top of the image can be found with any two rays. Use for example two rays that goes through the focal point.



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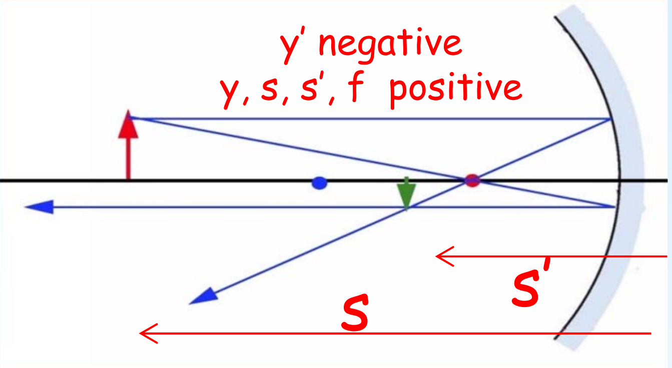
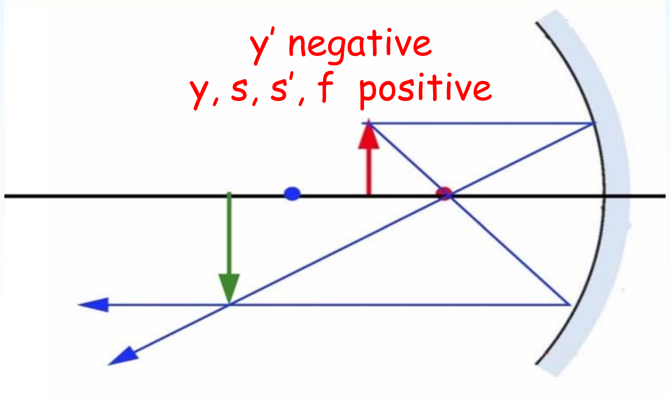
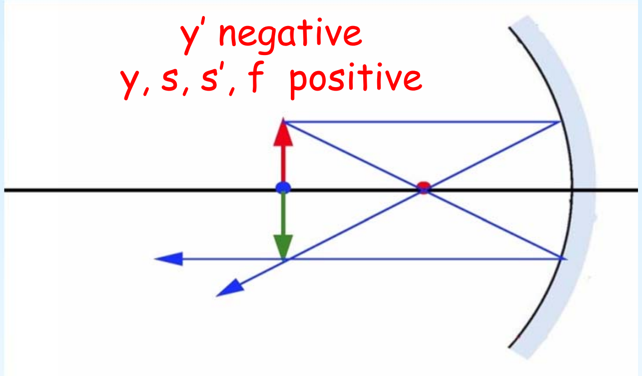
Geometrical optics: Concave mirrors

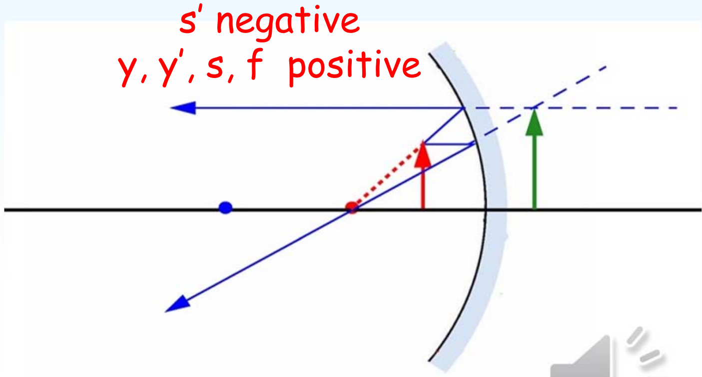




$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

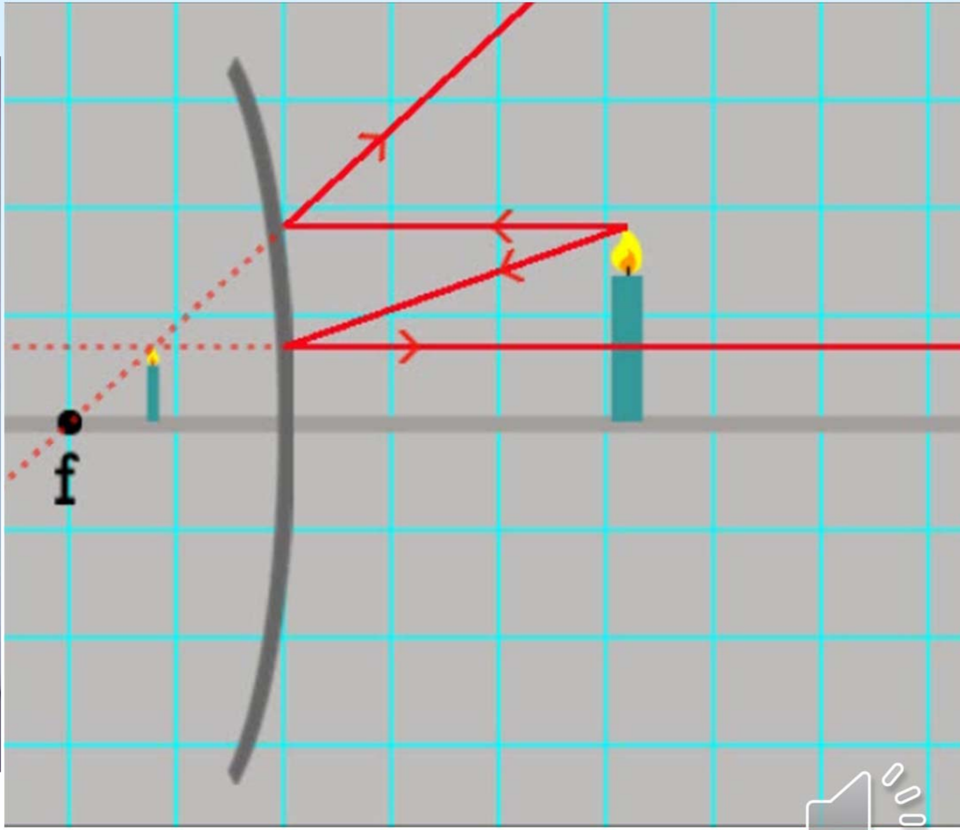
$$m = \frac{y'}{y} = -\frac{s'}{s}$$





Convex mirrors

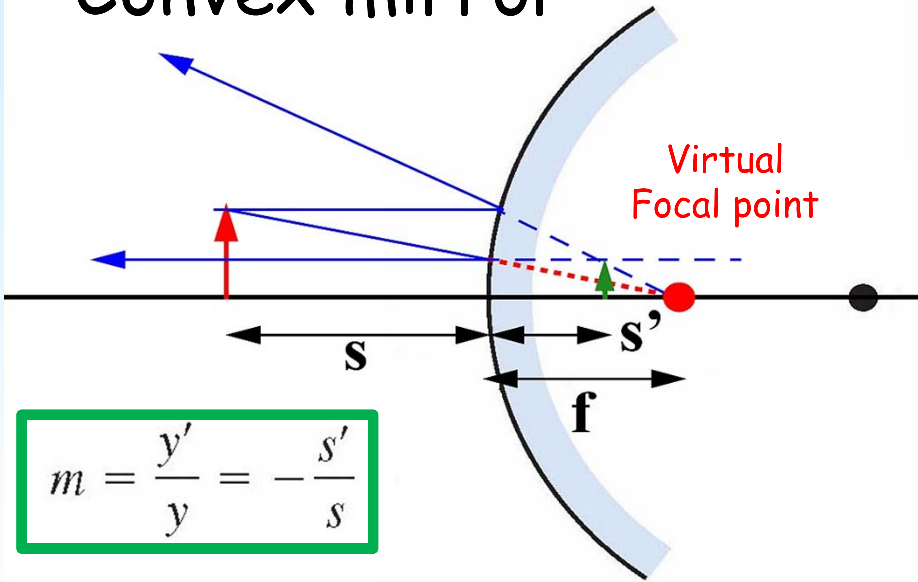
Geometrical optics: Convex mirrors



Geometrical optics: Convex mirrors

https://www.youtube.com/watch?v=J6LQM6re_1s

Convex mirror



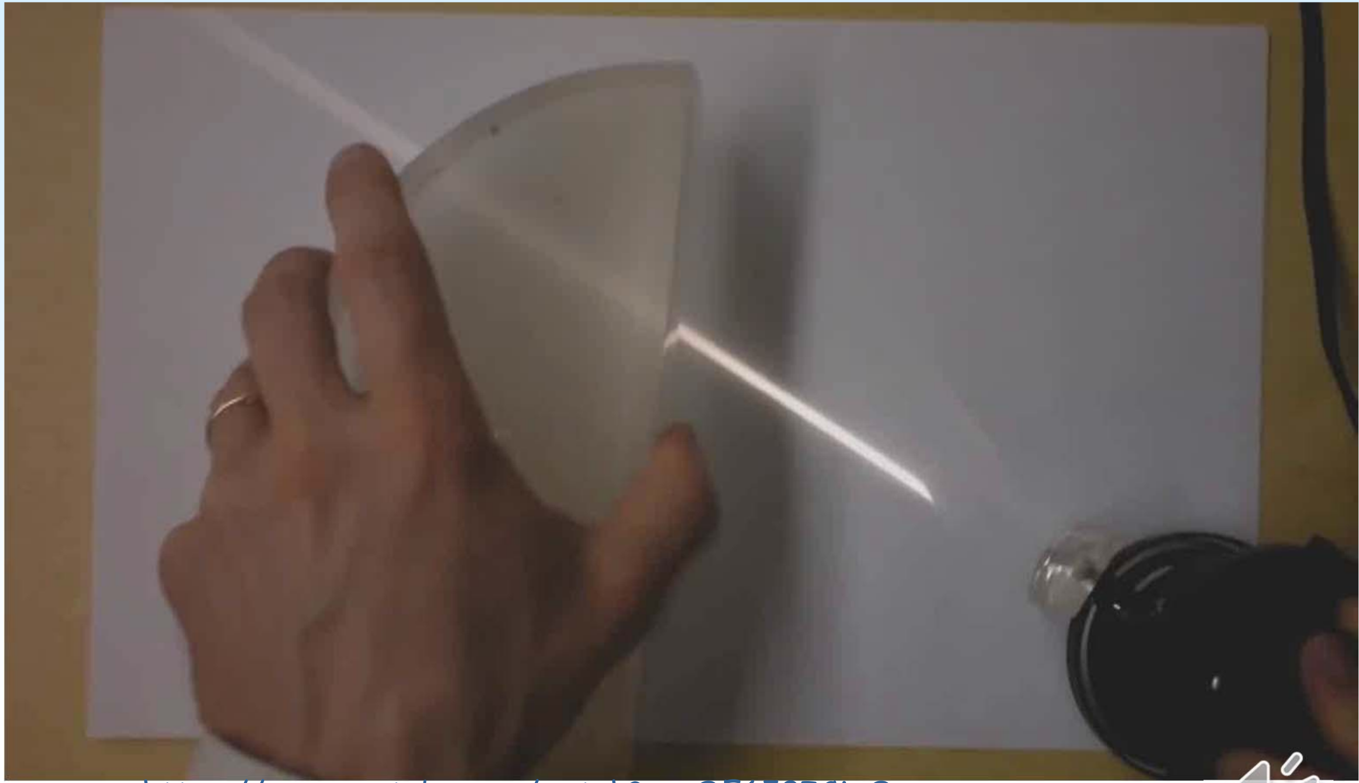
$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

s', f negative
y, y', s positive

Geometrical optics: Spherical surface

Spherical surface



https://www.youtube.com/watch?v=uQE659ICjqQ

Geometrical optics: Spherical surface

Summary - spherical surface

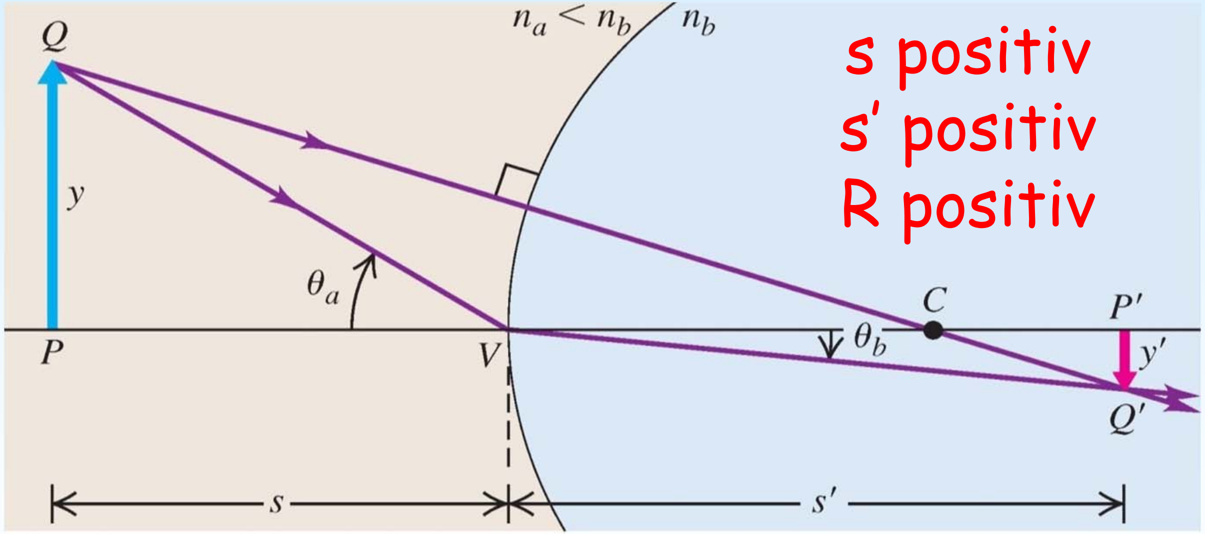
Sign rules:

Positive object distance (s) =
Object is on the side of the incoming light.

Positive image distance (s') =
Image and outgoing light on the same side.

Positive radius (R) =
Center is on the side of outgoing light.

Positive magnification (m) =
Direction of object and image is the same.



s positiv
s' positiv
R positiv

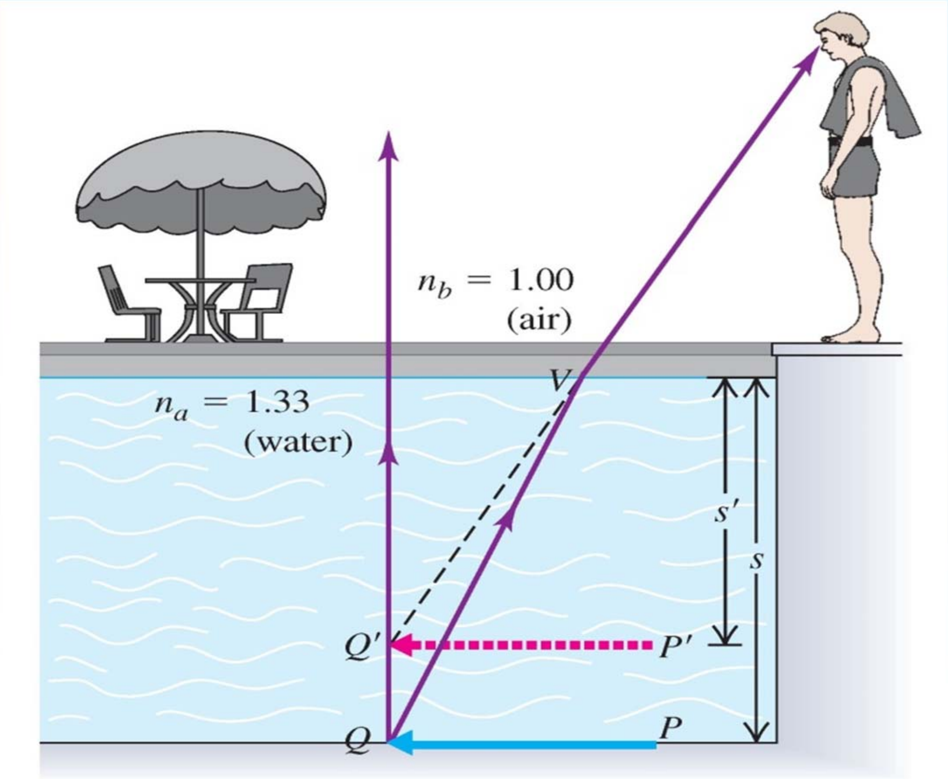
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Flat surfaces

https://www.youtube.com/watch?v=7aU8sX8cFNs

Geometrical optics: Flat surface



Special case: Flat surface

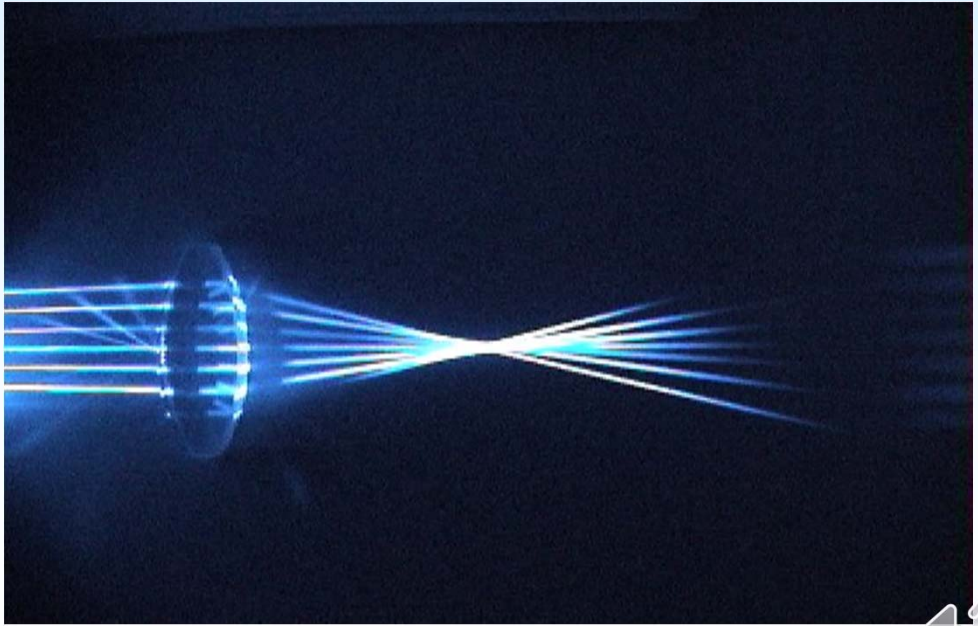
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} = 0$$

∞

$n_a/s = -n_b/s'$

$-s'/s = n_b/n_a$

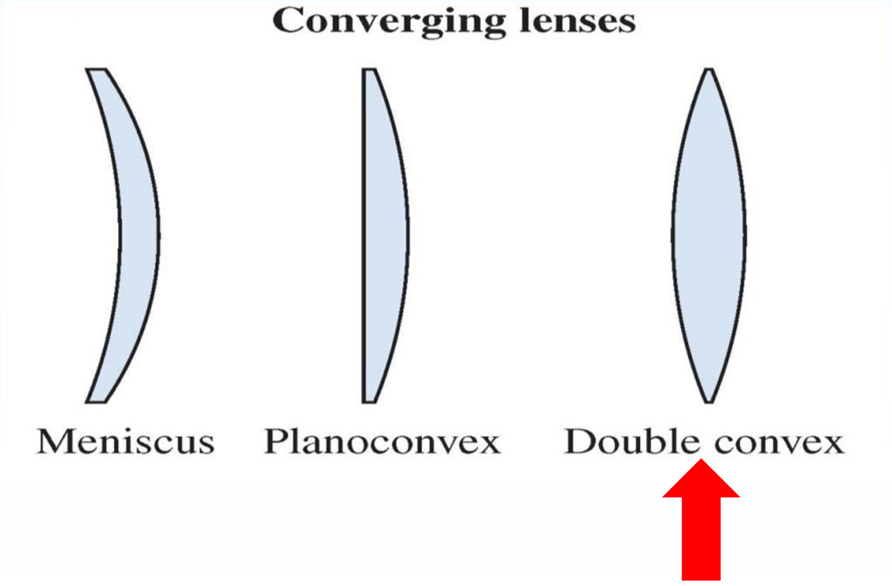
Convex lenses

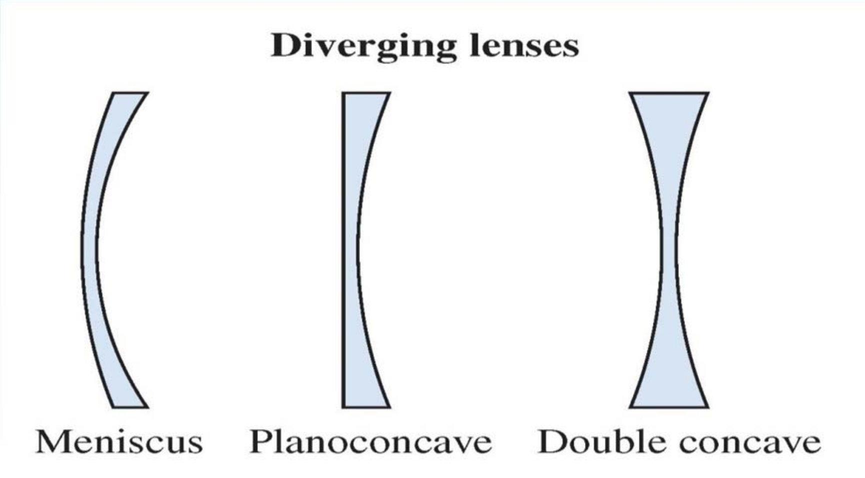


Different types of lenses

A lens thicker in the middle than in the edges is convergent.

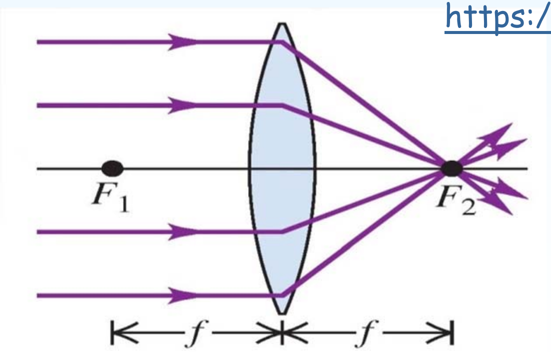
A lens thinner in the middle than in the edges is divergent.

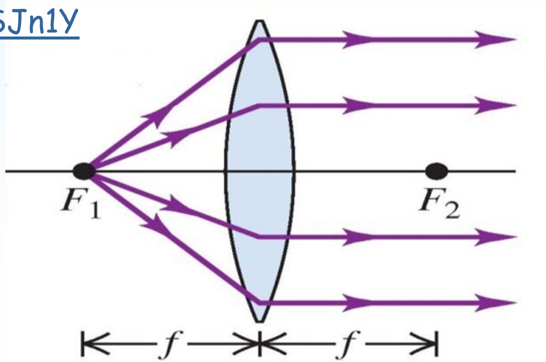




Geometrical optics: Convex lenses

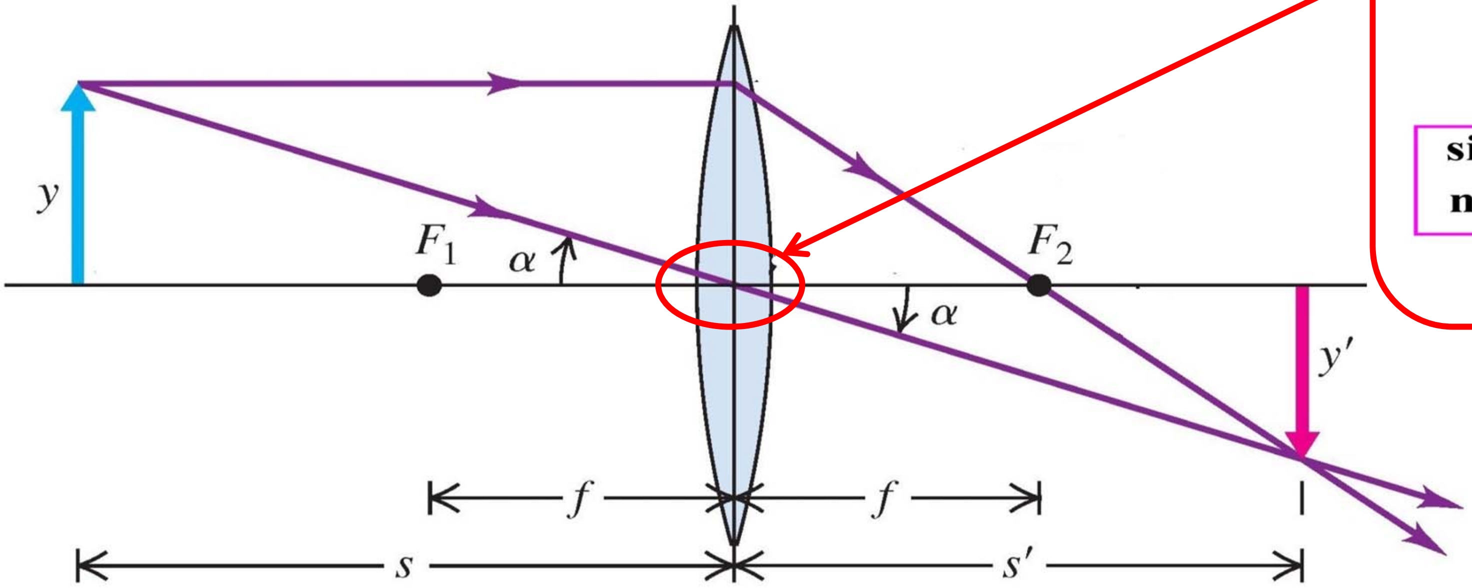
https://www.youtube.com/watch?v=4zuB_dSJn1Y

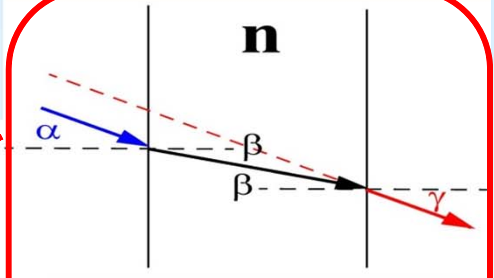




Geometrical optics: Convex lenses

Two useful rays:

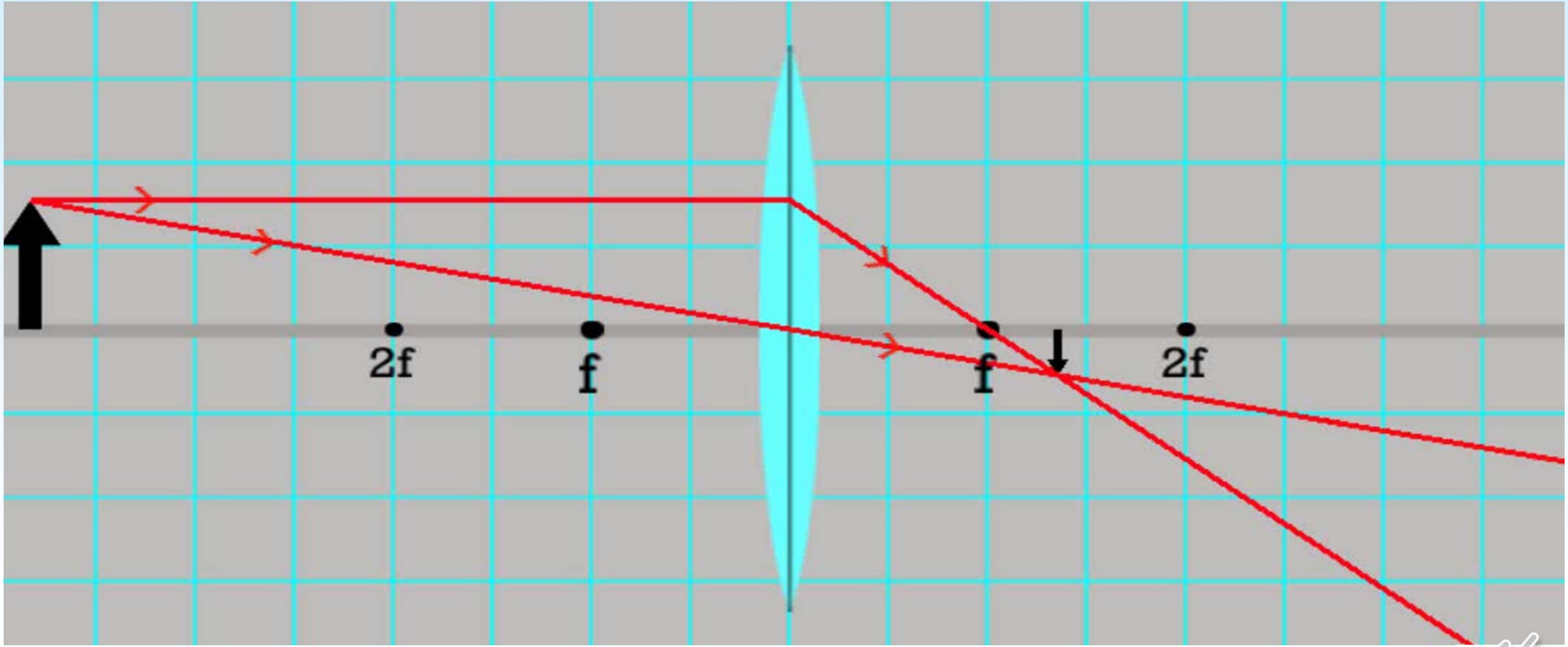




$$\sin(\alpha) = n \sin(\beta)$$
$$n \sin(\beta) = \sin(\gamma)$$

$$\alpha = \gamma$$

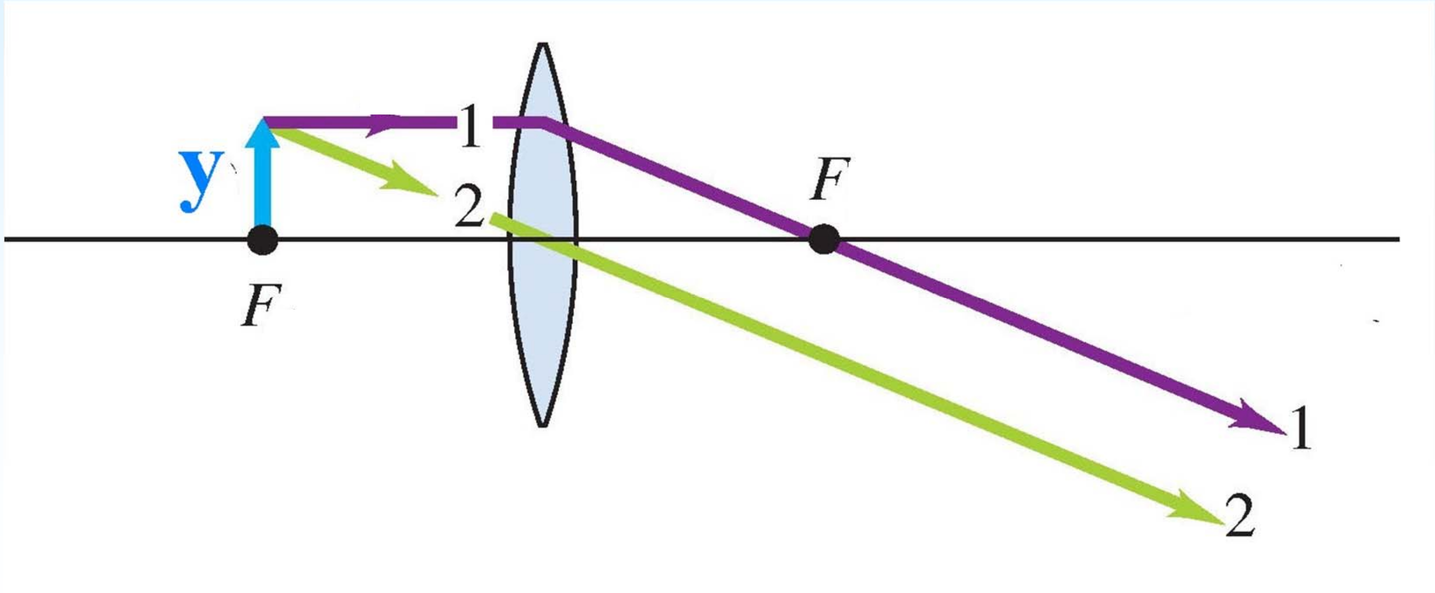
Geometrical optics: Convex lenses



http://simbucket.com/lensesandmirrors/

Geometrical optics: Convex lenses

An object placed at the focal point seems to be infinitely far away



Geometrical optics: Convex lenses

<u>Sign rules:</u>

Positive object distance (s)
Object and incoming light is on the same side.

Positive image distance (s')
Image and outgoing light is on the same side

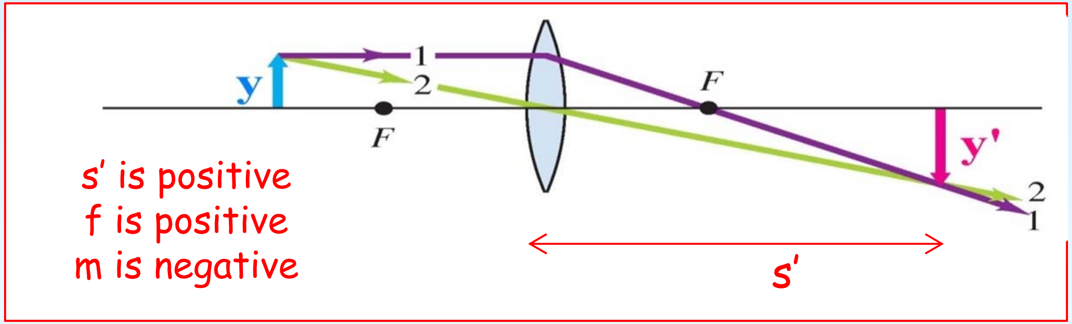
Positive focal length (f)
Converging (convex) lenses

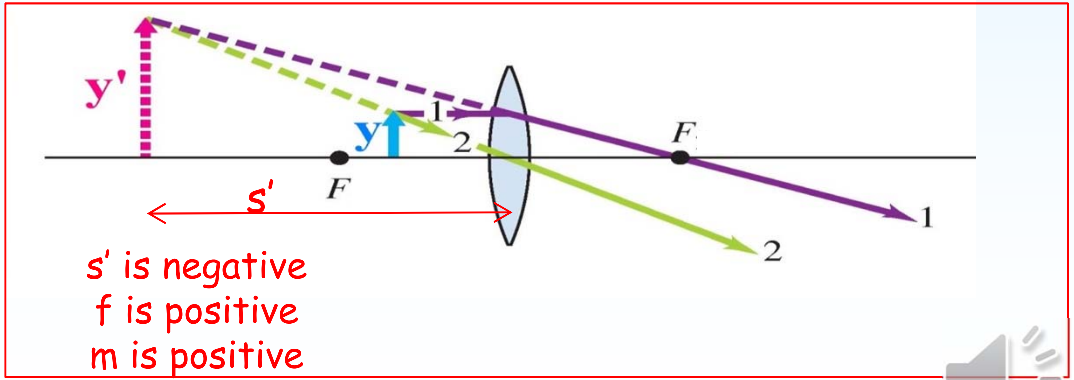
Positive magnification (m)
Same direction of object and image.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Summary of convex lenses





Geometrical optics: Convex lenses

Gauss' formula

Newton's formula

$$f = \frac{s\,s'}{s + s'}$$

Formula collection

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

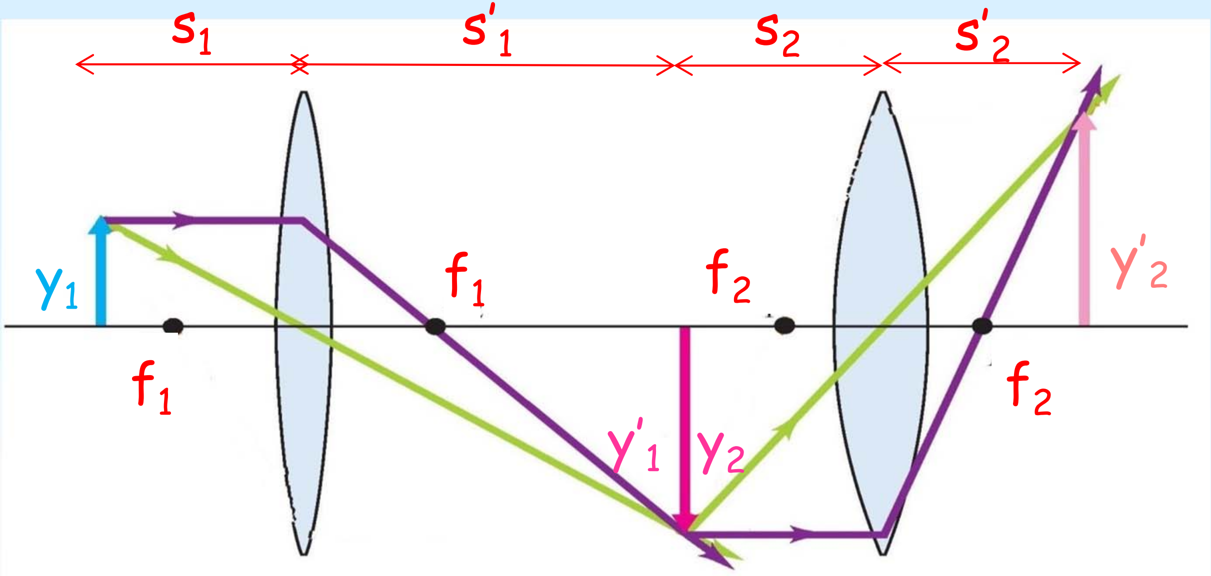
$$s = \frac{s'f}{s' - f}$$

$$s' = \frac{sf}{s - f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$m = -\frac{f}{s - f}$$

Geometrical optics: Convex lenses



$$\frac{1}{f_1} = \frac{1}{s_1} + \frac{1}{s'_1}$$

$$m_1 = -\frac{s'_1}{s_1}$$

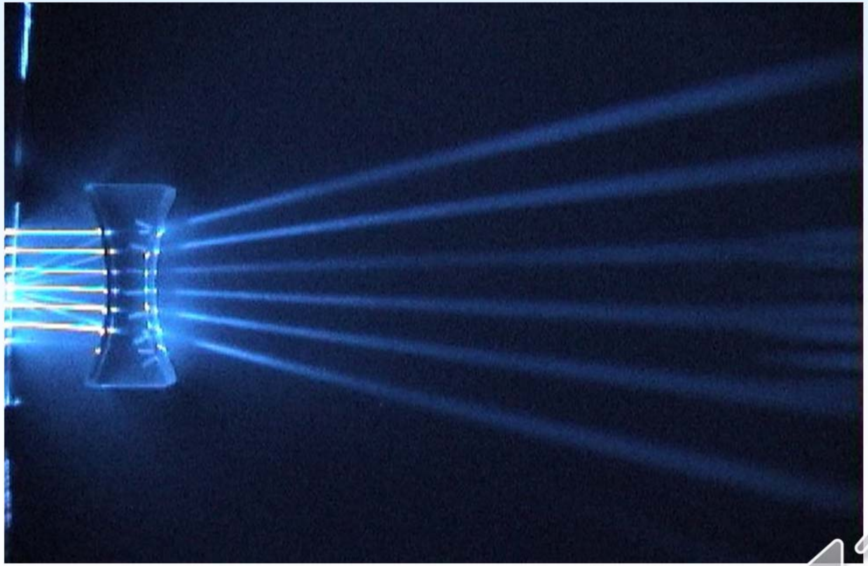
$$\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{s'_2}$$

$$m_2 = -\frac{s'_2}{s_2}$$

How to combine the magnification of two lenses:

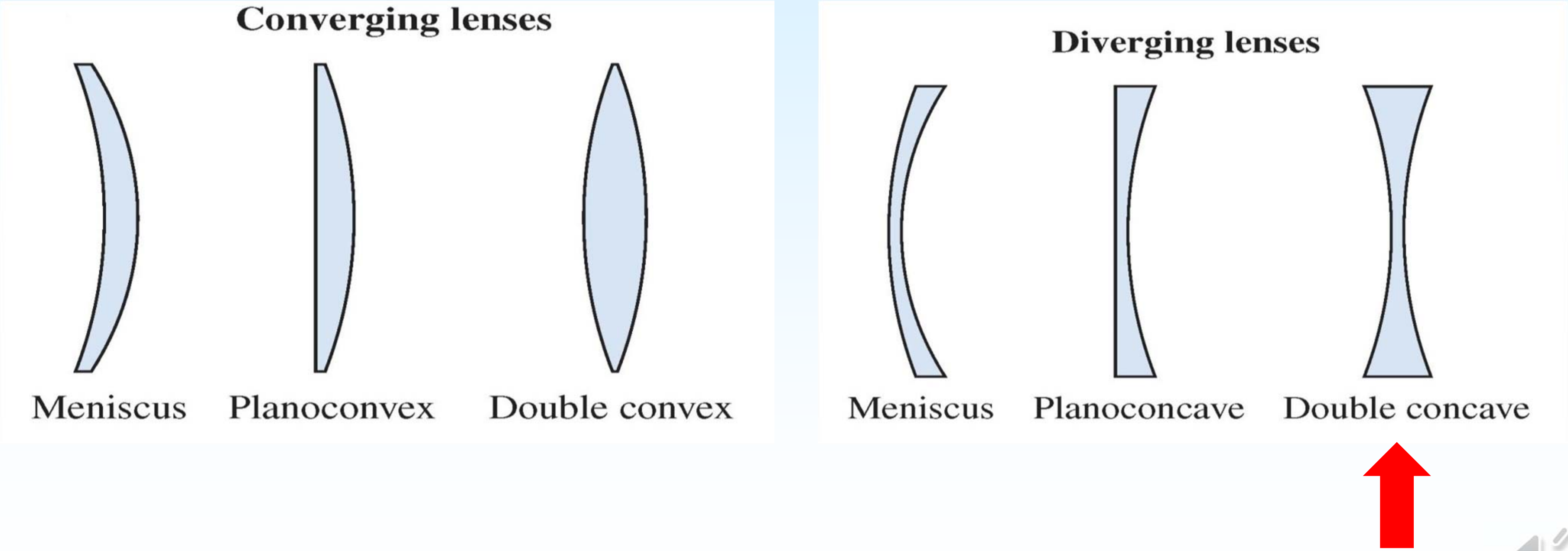
$$\Rightarrow \quad m = m_1 m_2 = \frac{s'_1 s'_2}{s_1 s_2}$$

Concave lenses

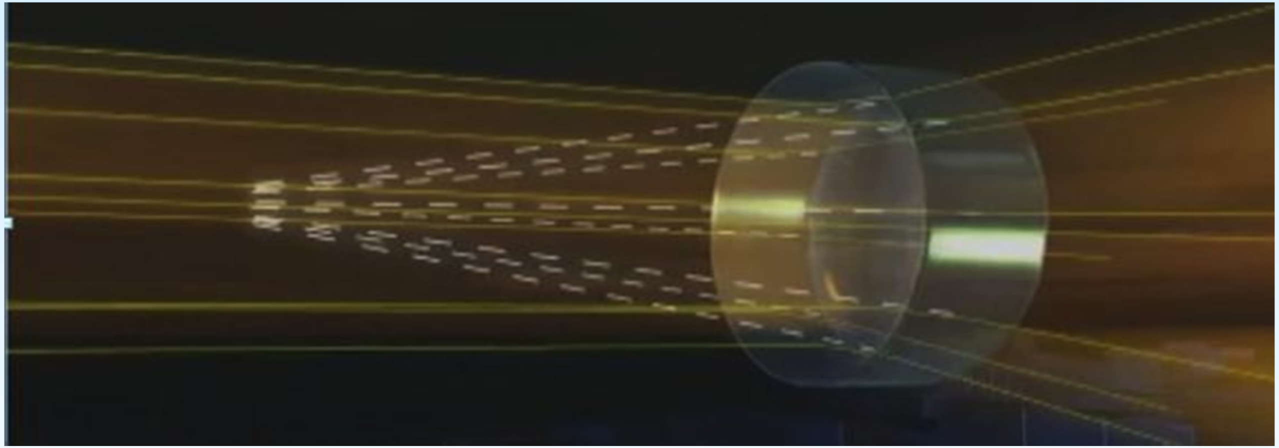


Geometrical optics: Concave lenses

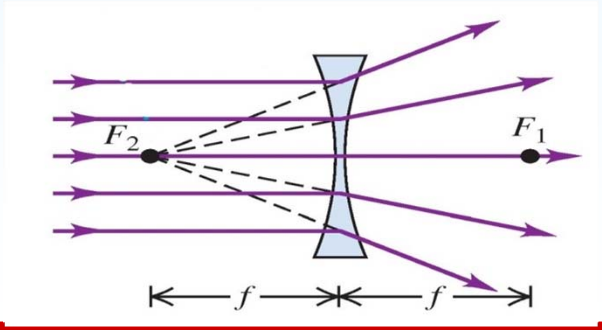
Different types of lenses

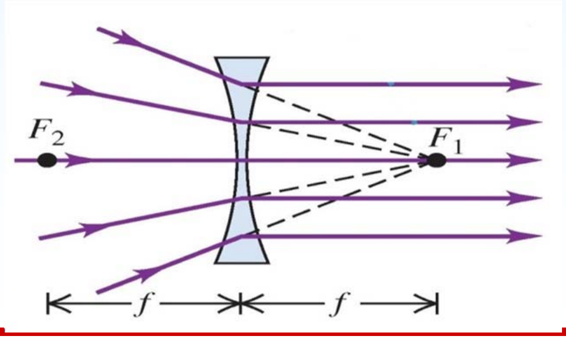


Geometrical optics: Concave lenses

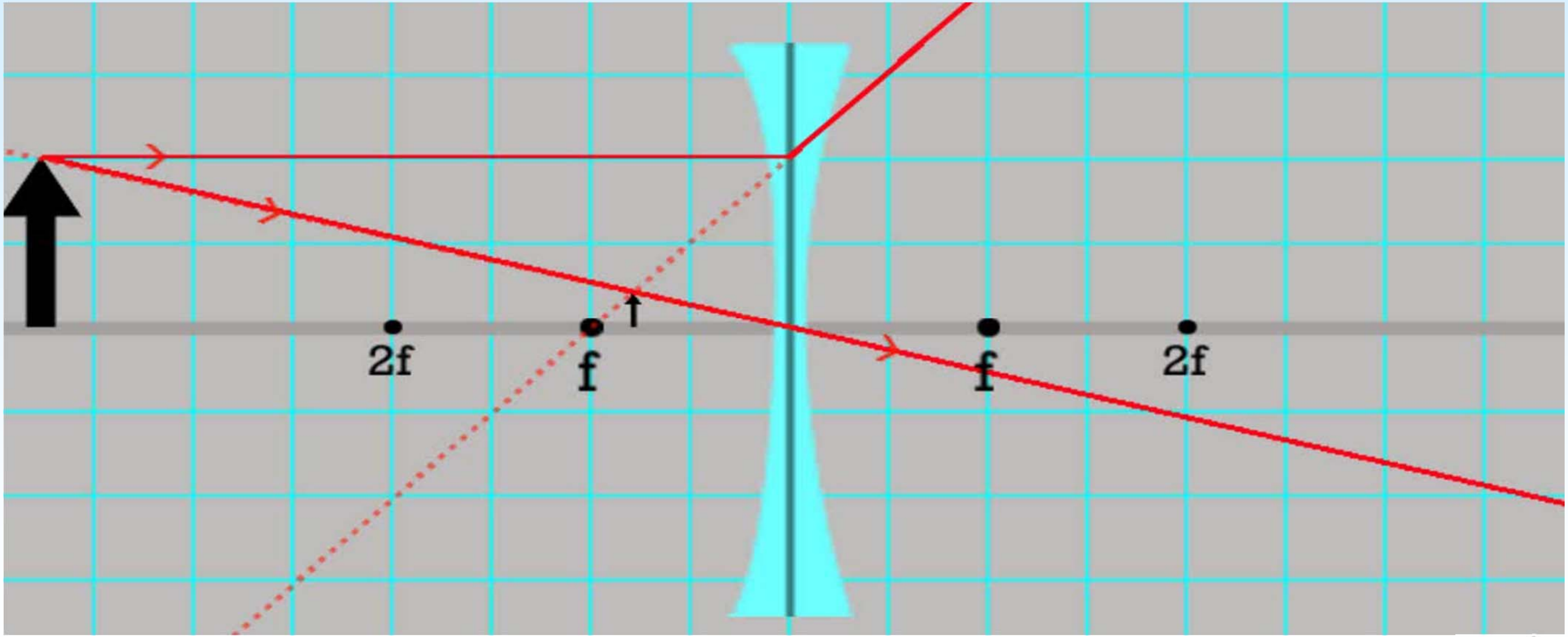


https://www.youtube.com/watch?v=4zuB_dSJn1Y





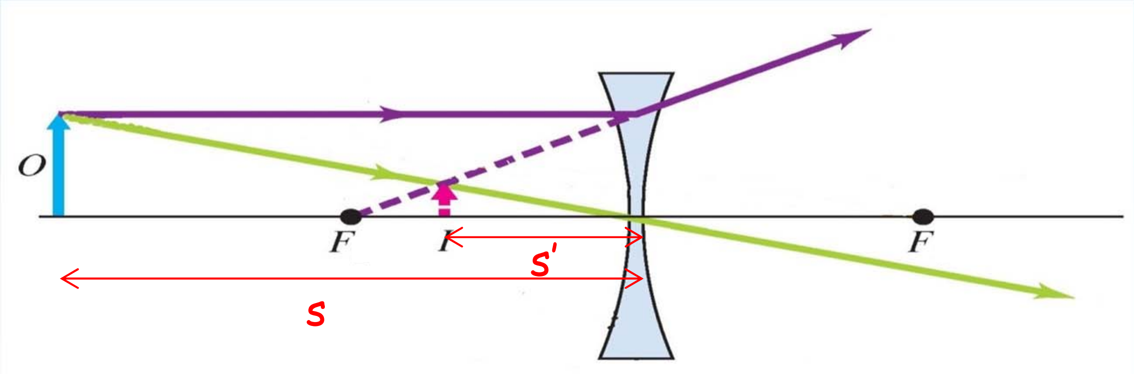
Geometrical optics: Concave lenses



http://simbucket.com/lensesandmirrors/

Geometrical optics: Concave lenses

The lens formula for concave lenses



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s}$$

f is negative for diverging lenses

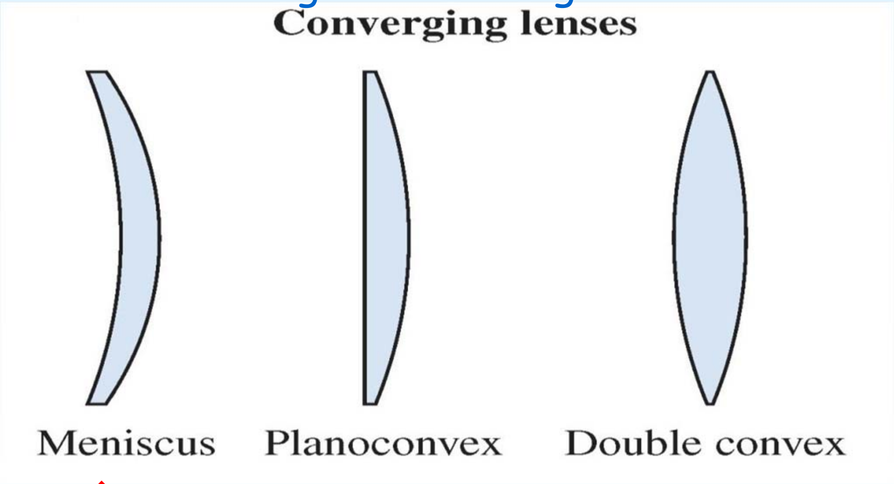
s' is negative for diveregent lenses

m is always positive

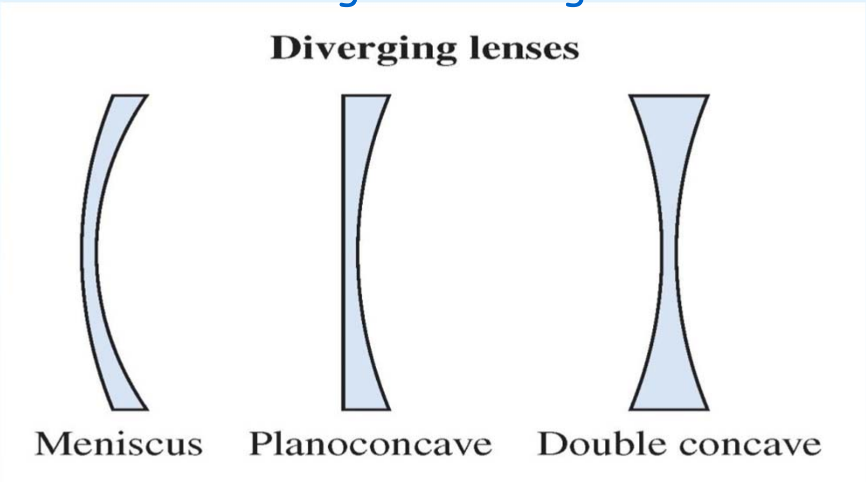
The lensmaker's equation

Different types of lenses

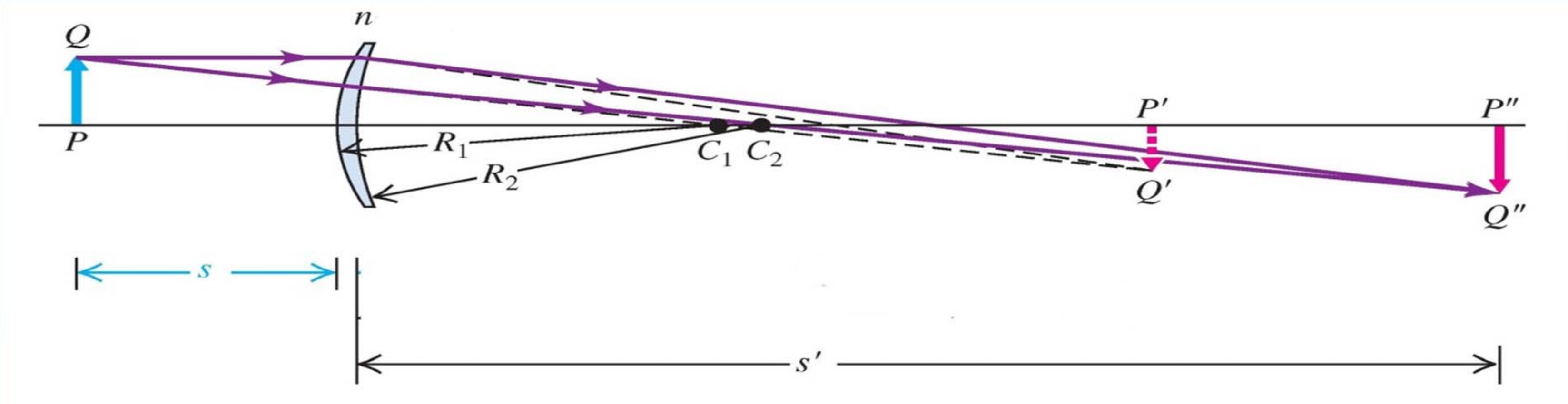
A lens thicker in the middle than in the edges is convergent.



A lens thinner in the middle than in the edges is divergent.



Optics: The lensmaker's equation



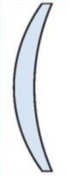
The lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{s} + \frac{1}{s'}$$

Optics: The lensmaker's equation

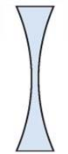
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad m = -\frac{s'}{s} \qquad m = \frac{y'}{y}$$

Sign rule for the radius (R) says it is positive if center is on same side as outgoing light.



f = positive R_1 = positive R_2 = positive s′ = positive or negative

f = positive R_1 = positive R_2 = negative s′ = positive or negative



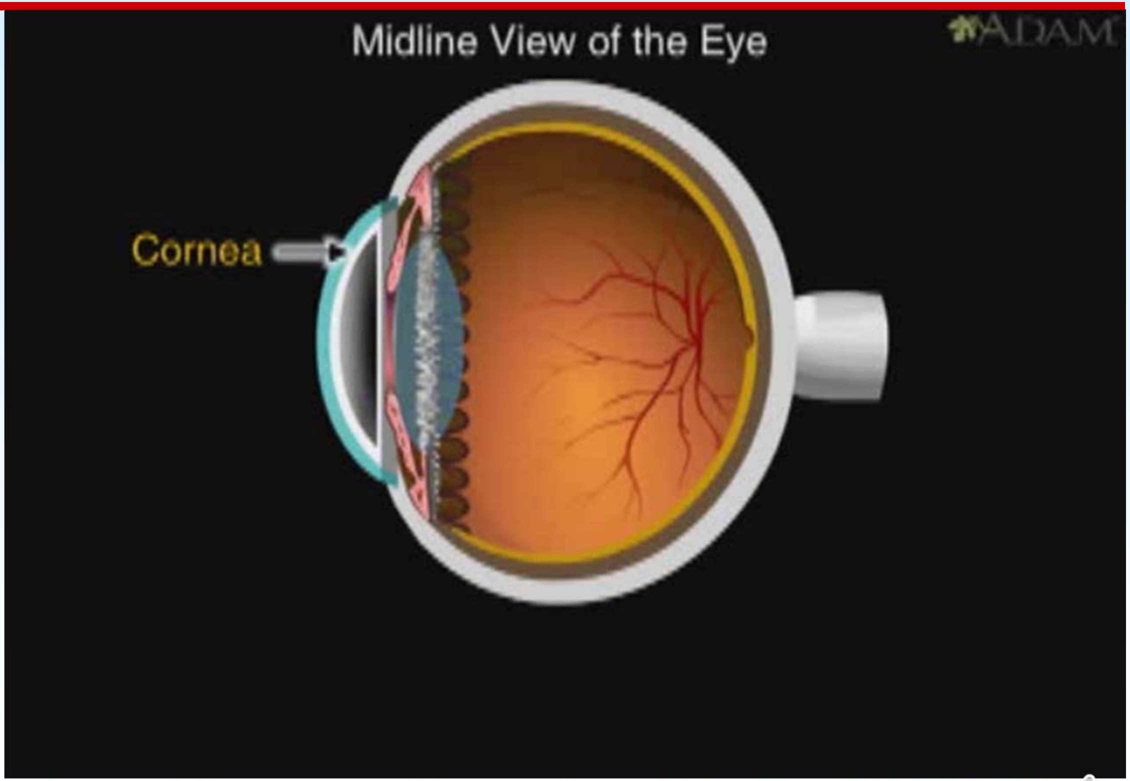
f = negative R_1 = negative R_2 = positive s′ = negative

The Eye

In 1936, 9% of Swedish recruits were nearsighted.
In 2009, 38% of Swedish recruits were nearsighted.

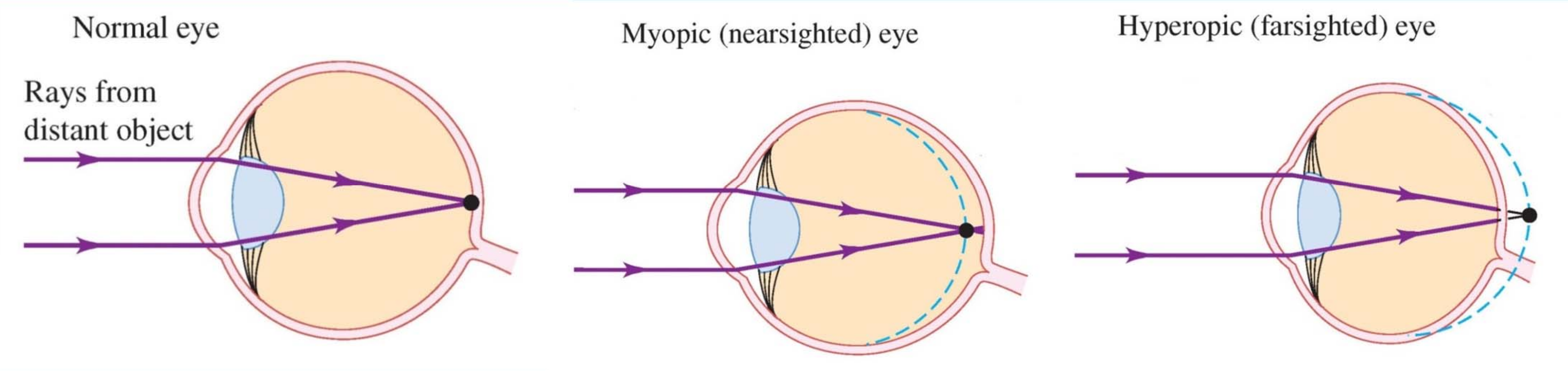
The reason: Time spent outdoors (exposure to daylight).

The function of the eye



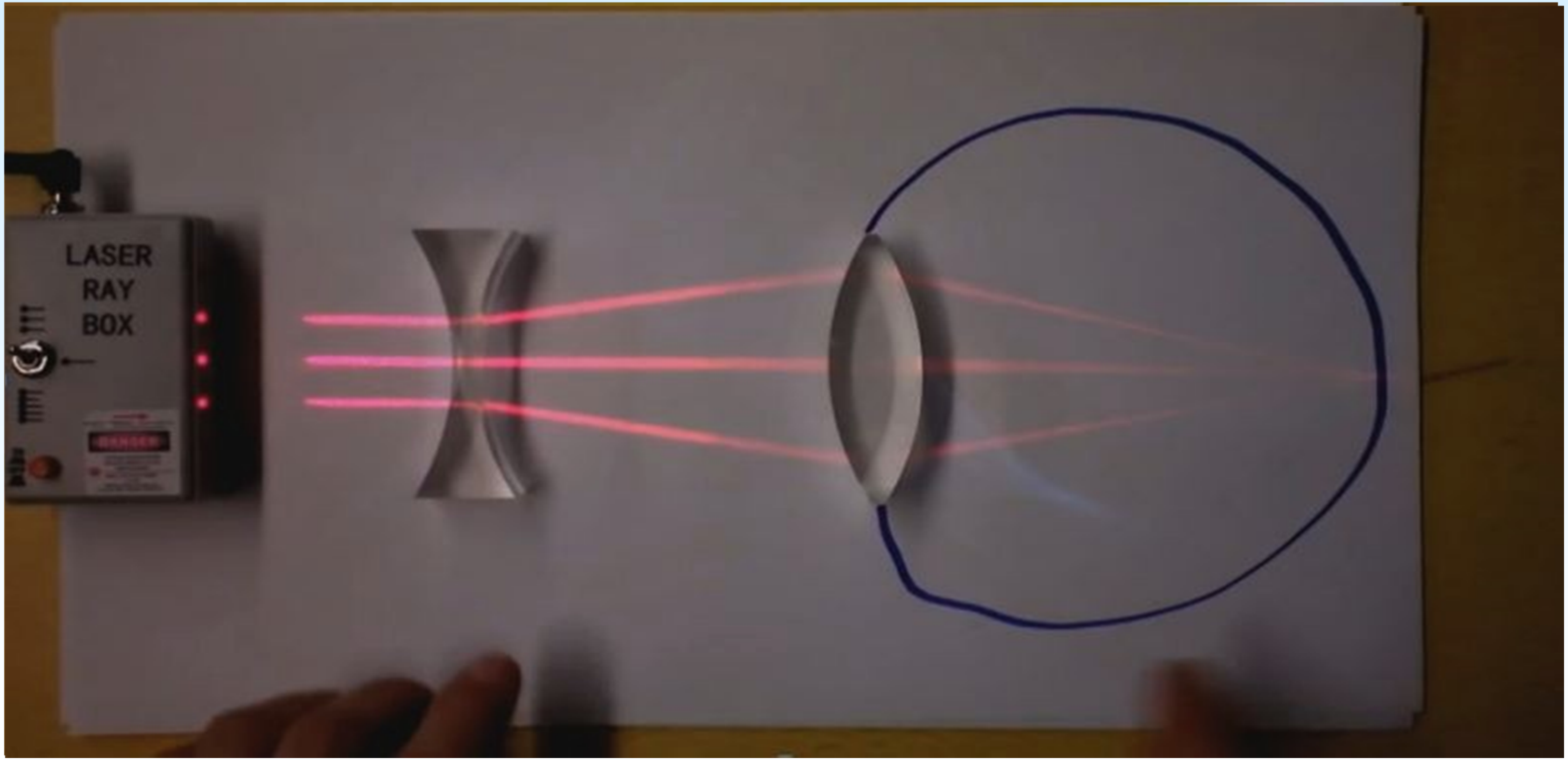
https://www.youtube.com/watch?v=YcedXDN6a88

Geometrical optics: The eye



- **Near point**: Closest distance to the eye at which people can see clear (7cm at age 10 to 40cm at age 50 for normal eye).
- **Normal reading distance:** Assumed to be 25 cm when designing correction lenses.
- **The far point:** The longest distance to the eye at which people can see clearly.
- **Lens power = 1/f (unit diopter = m^{-1})** is the quantity used for correction lenses.

Geometrical optics: The eye

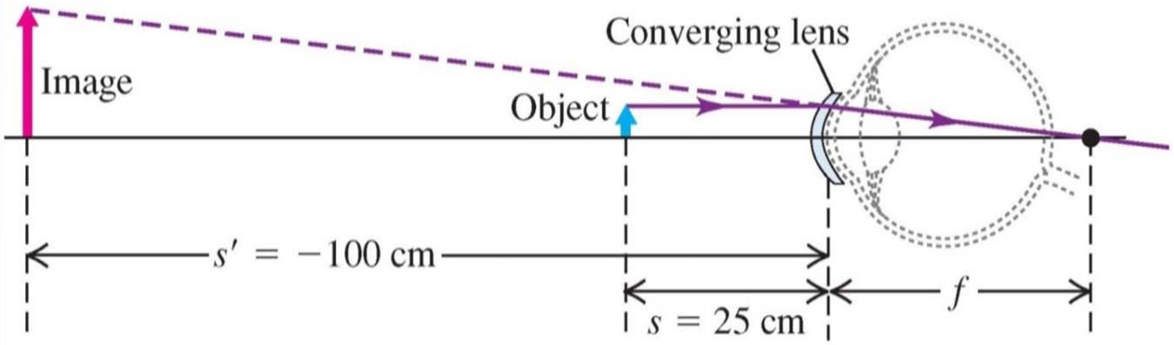


https://www.youtube.com/watch?v=VDehC_Txa1U

Geometrical optics: Problems

The near point of a farsighted (hyperopic) eye is at 100 cm.

What lens power is needed to move the near point to 25 cm ?



When the person puts an object at s = 25 cm from the correcting lens we want the image to end up at s′ = 100 cm because this is the nearest point the eye can see sharply.

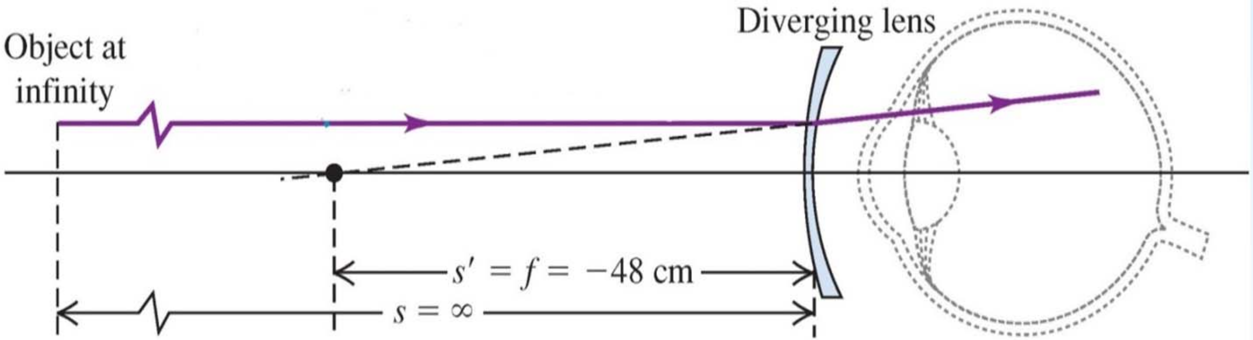
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25\text{ cm}} + \frac{1}{-100\text{ cm}}$$

$$f = +33\text{ cm}$$

Lens power = 1/f =
= 1/0.33 m^{-1} = 3 diopter

Geometrical optics: Problems

A nearsighted (myopic) eye has the far point at a distance of 50 cm. What lens power is needed to correct the eye if the lens is 2 cm in front of the eye ?



$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-48\text{ cm}}$$

$$f = -48\text{ cm}$$

The lens should move the far point from 50 cm to infinity.

The correcting lens should therefore have s = infinity for s′ = -50+2 = -48 cm.

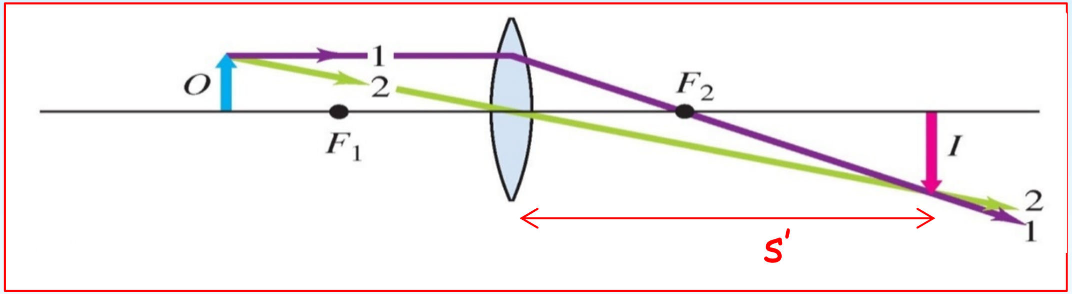
Lens power = 1/f =
= $-1/0.48\ m^{-1}$ = -2.1 diopter

The magnifying glass

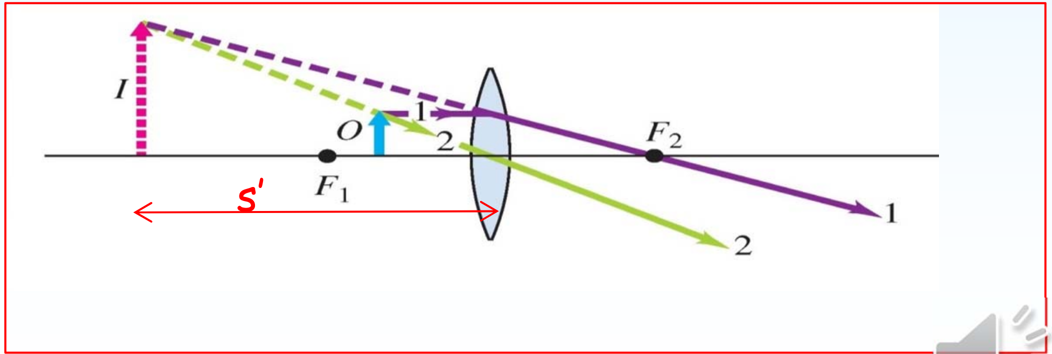
Optics: The magnifying glass

A magnifying glass is a convex lens.

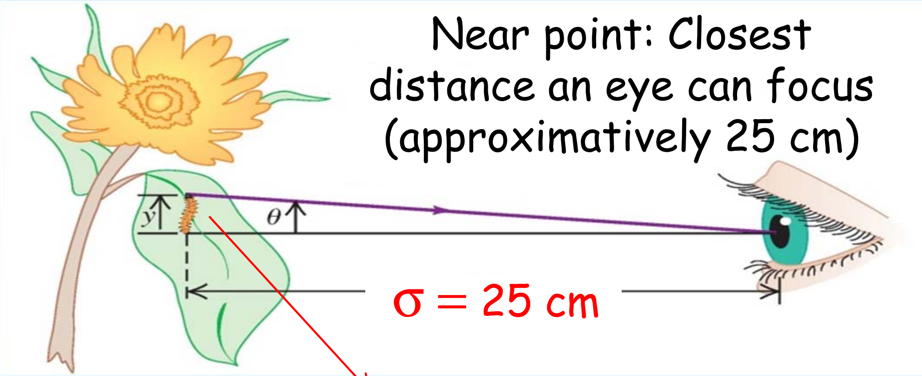
If you hold a magnifying glass far away from the eye (arms lengths distance) you can see a magnified and up-side down image.

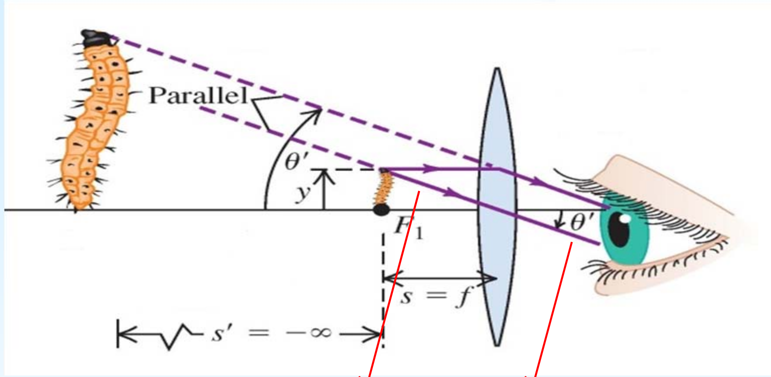


The normal use of a magnifying glass is to put the object between the focal point and the glass to get a magnified up-right image.



Optics: The magnifying glass





Angle without magnifying glass

$$\tan(\theta) \approx \theta = \frac{y}{\sigma} \approx \frac{y}{25\ cm}$$

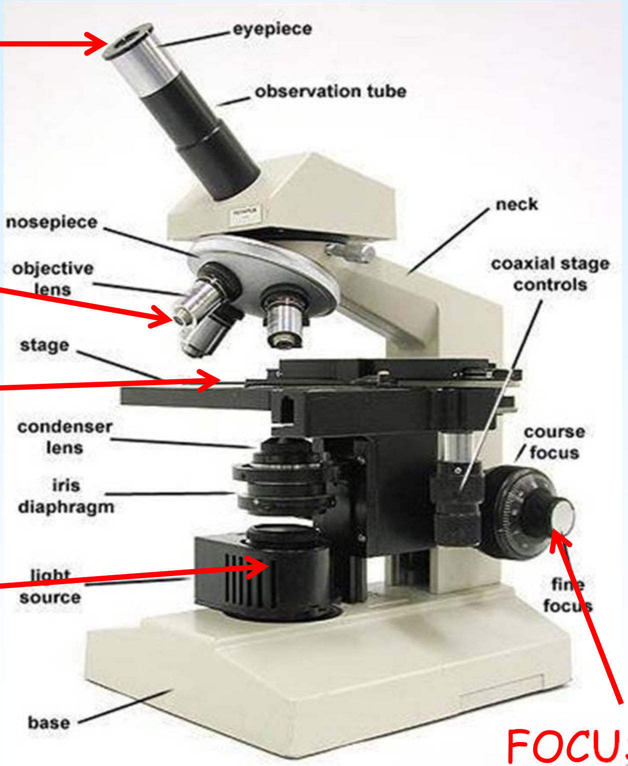
Angle with magnifying glass

$$\tan(\theta') \approx \theta' = \frac{y}{f}$$

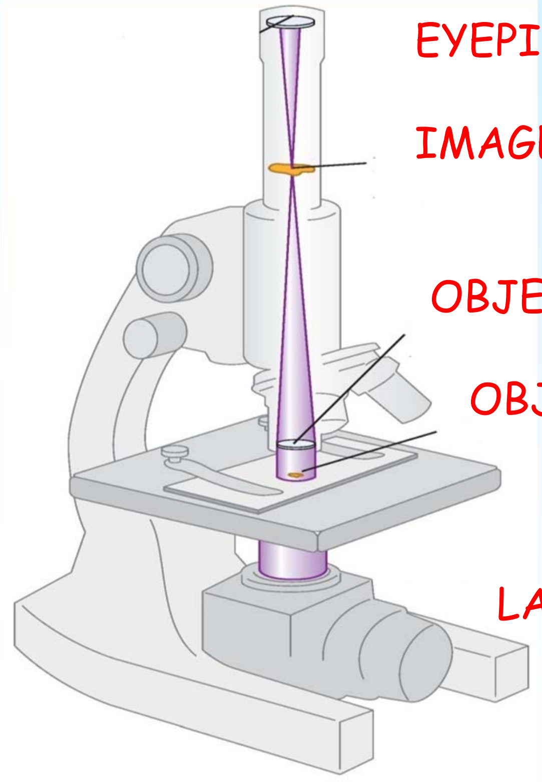
When the object is at the focal point one uses angular magnification (M) instead of lateral magnification (m).

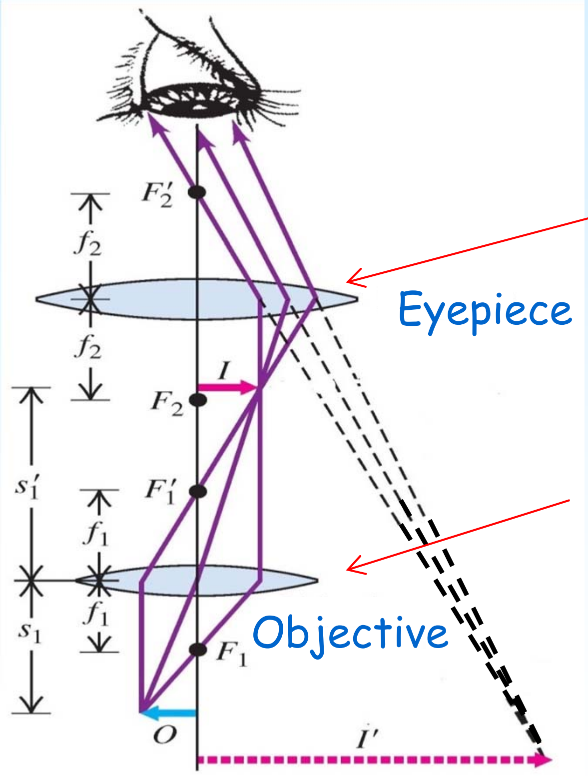
$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/\sigma} = \frac{\sigma}{f} = \frac{25\ cm}{f}$$

The Microscope



Geometrical optics: The microscope

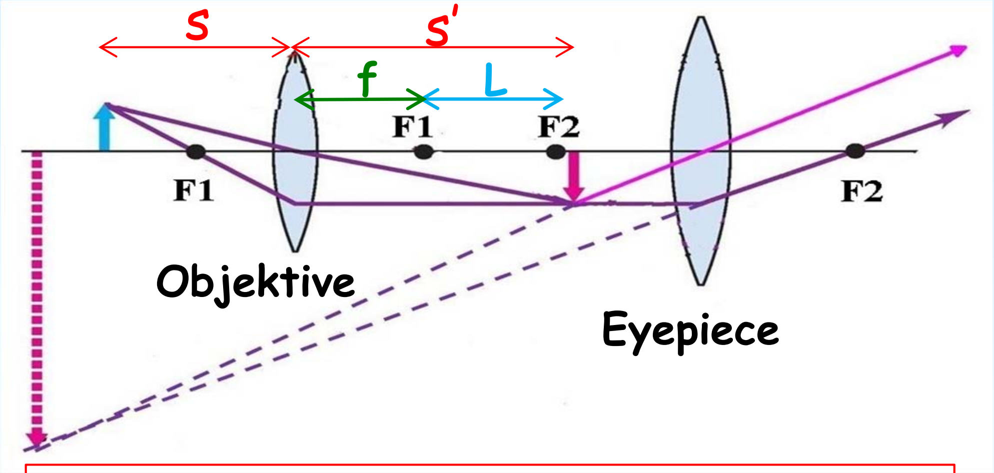




Magnifying glass
(f is a couple of cm)

Creates magnified image close to the focal point of the eye piece
(f < 1 cm)

Geometrical optics: The microscope



EYEPIECE

Angular magnification of magnifying glass:

$$M = \frac{\sigma}{f} \qquad where\ \sigma = 25\ cm$$

OBJEKTIVE

$$s' \approx f + L$$

$$m = -\frac{s'}{s} = -\frac{s' - f}{f} \approx -\frac{f + L - f}{f} = -\frac{L}{f}$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \Rightarrow s = \frac{s'f}{s' - f}$$

MICROSCOPE

Magnification:

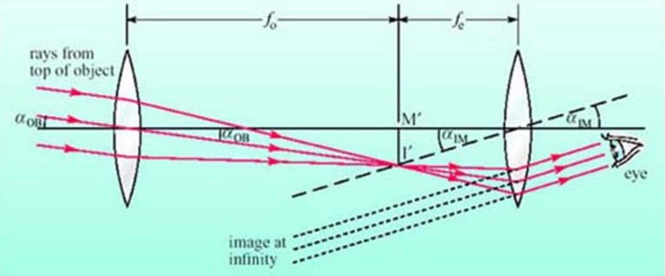
$$M = m_1 M_2 = -\frac{s'_1}{s_1}\frac{\sigma}{f_2} = -\frac{L\sigma}{f_1 f_2}$$

σ is the nearpoint distance which is typically 25 cm

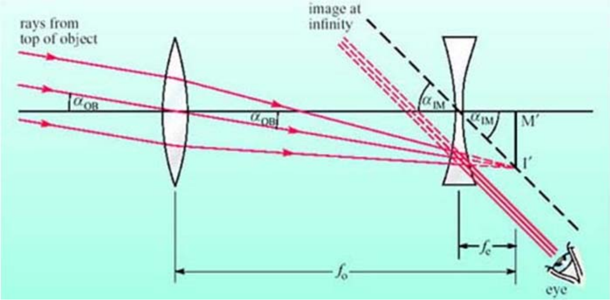
The Telescope

Different types of telescopes

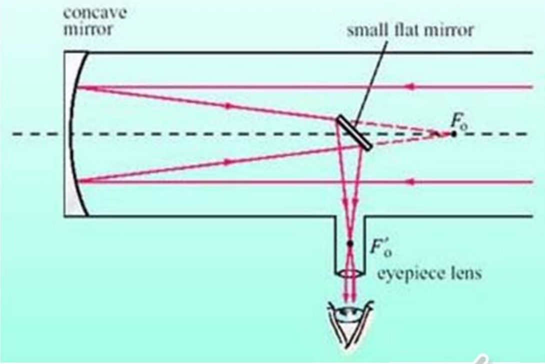
Keplerian Refracting Telescope



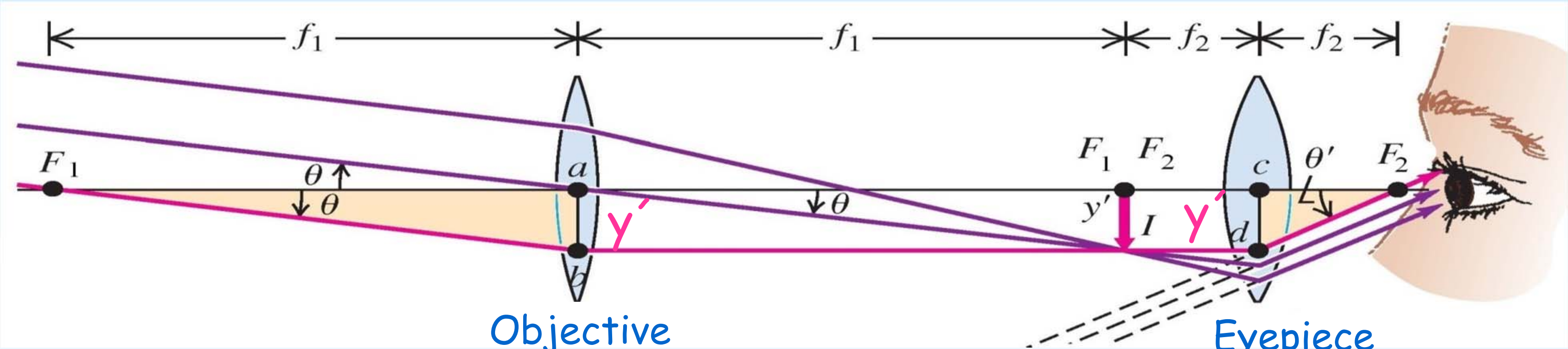
Galilean Refracting Telescope



Newtonian Reflecting Telescope



Geometrical optics: The Telescope



Objective

Eyepiece

The object is infinitely far away so the image will be at the focal point of the lens.

$$\tan(\theta) = \theta = \frac{-y'}{f_1}$$

The eye piece works as a magnifying glass with the image y′ in its focal point.

$$\tan(\theta') = \theta' = \frac{y'}{f_2}$$

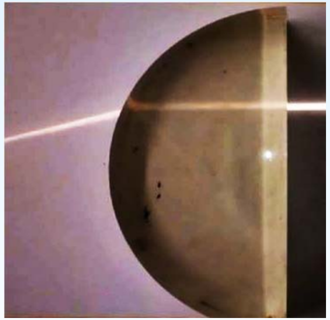
The angular magnification of a telescope is defined as the ratio of the angle of the image to that of the incoming light.

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}$$

SUMMARY

Geometrical optics

Mirrors and lenses

Concave mirror

Convex mirror

Spherical surface

Convex lens

Concave lens

Formulas

Concave mirror

Convex mirror

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

$$f = \frac{R}{2}$$

Spherical surface

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}$$

Convex lens

Concave lens

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Geometrical optics: Summary

Sign rules for mirrors:

Positive object distance (s) =
Object is on the side of the incoming light.

Positive image distance (s') =
Image and outgoing light on the same side.

Positive radius (R) =
Center is on the side of outgoing light.

Positive magnification (m) =
Direction of object and image is the same.

Sign rules for lenses:

Positive object distance (s)
Object and incoming light is on the same side.

Positive image distance (s')
Image and outgoing light is on the same side.

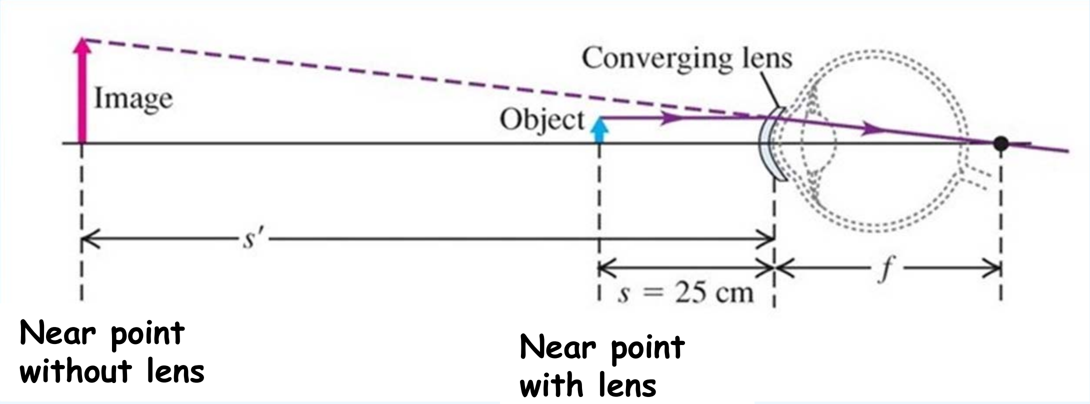
Positive focal length (f)
Converging (convex) lenses.

Positive magnification (m)
Same direction of object and image.

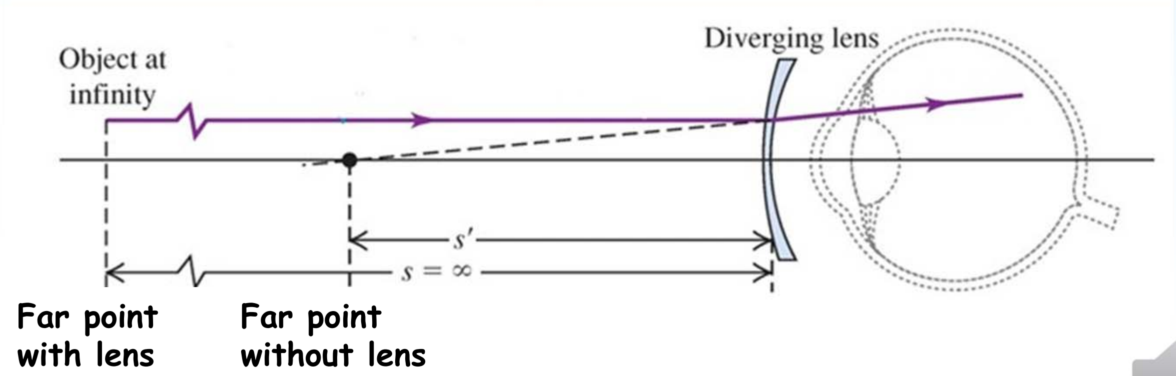
Eye, microscope and telescope

Geometrical optics: The eye

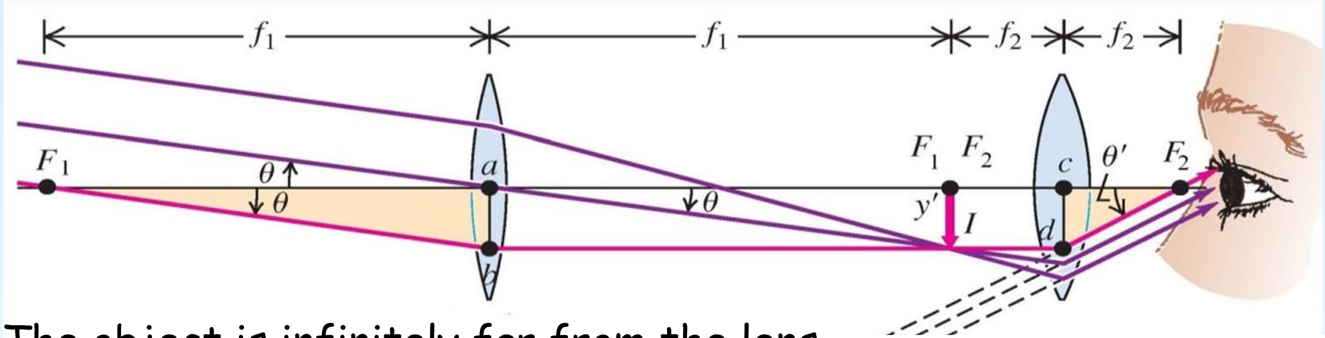
Farsighted



Nearsighted



Summary: Microscope & Telescope

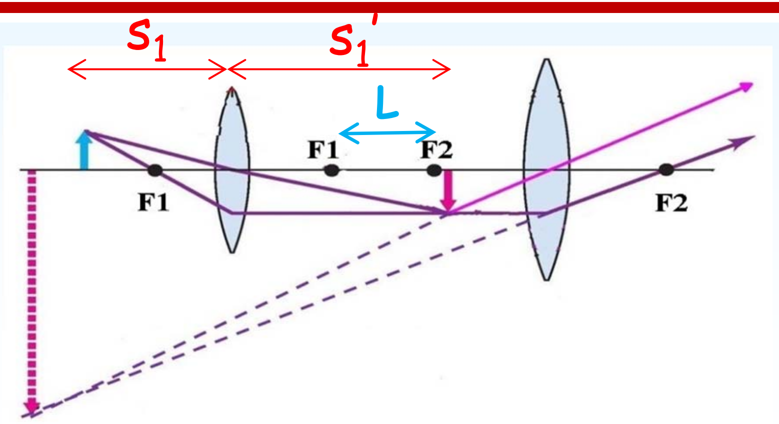


The object is infinitely far from the lens

Telescope

$$M = -\frac{f_1}{f_2}$$

Large f_1 & Small f_2



The object is close to the lens

Microscope

$$M = m_1 M_2 = -\frac{s'_1}{s_1}\frac{\sigma}{f_2} = -\frac{L\sigma}{f_1 f_2}$$

σ is the near point (typically 25 cm)

Small f_1 & Small f_2