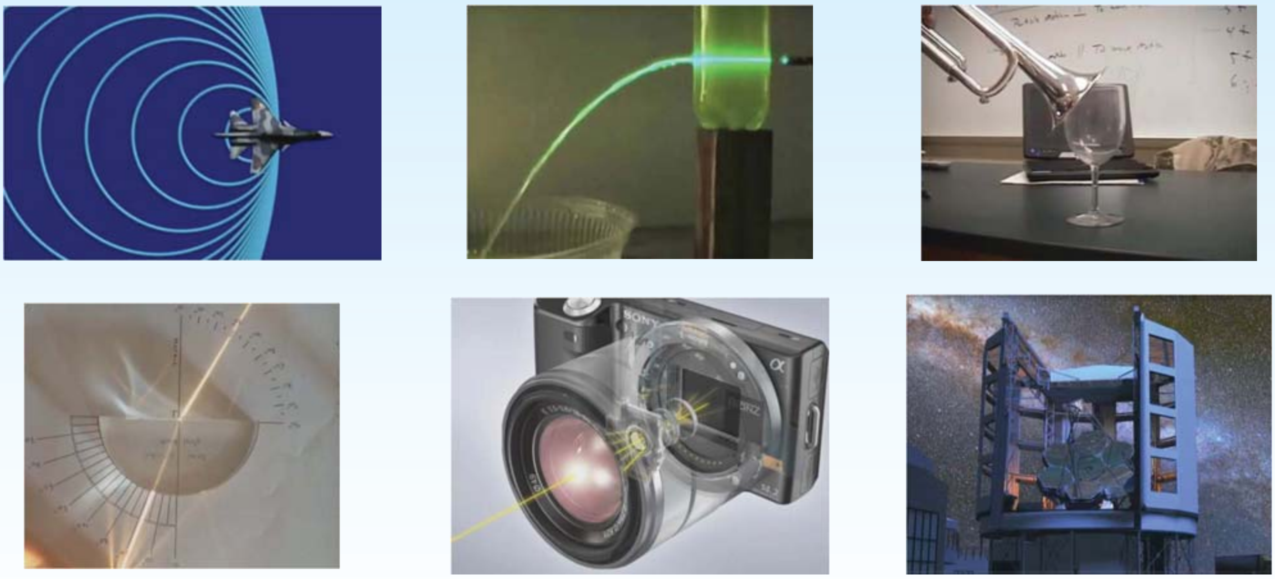
Vågrörelselära och optik



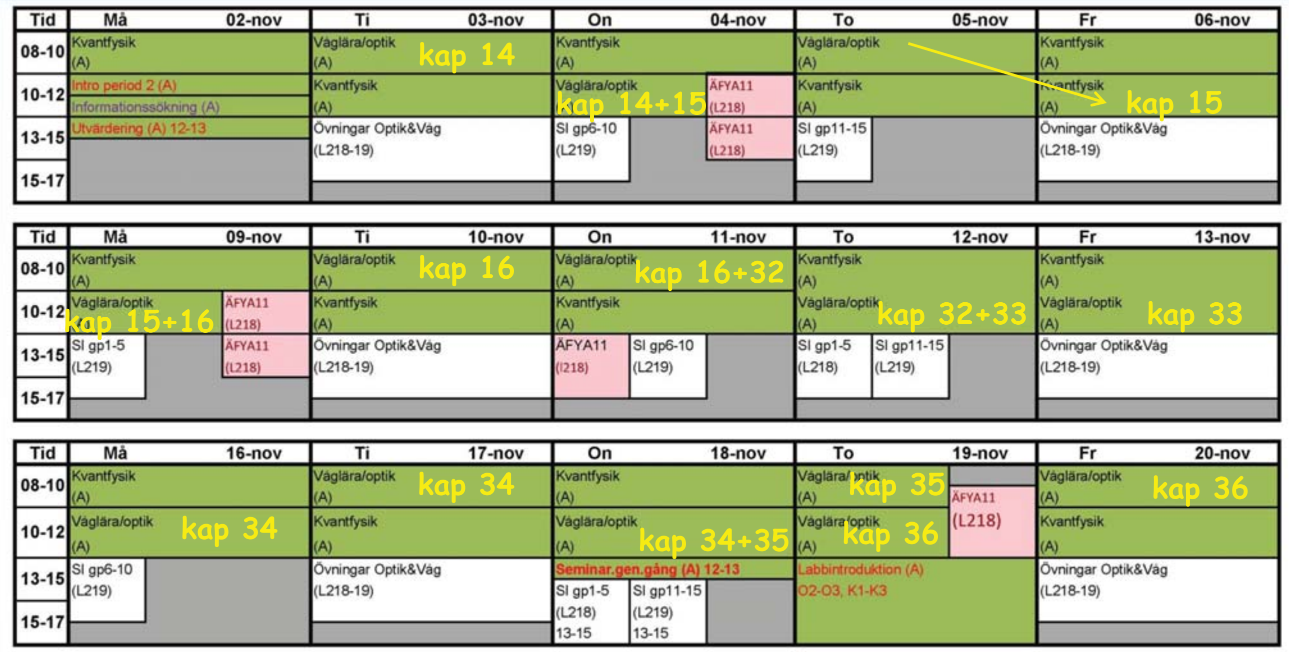
Kapitel 35 - Interferens

Vågrörelselära och optik

Kurslitteratur: University Physics by Young & Friedman

Harmonisk oscillator:	Kapitel 14.1 – 14.4
Mekaniska vågor:	Kapitel 15.1 – 15.8
Ljud och hörande:	Kapitel 16.1 – 16.9
Elektromagnetiska vågor:	Kapitel 32.1 & 32.3 & 32.4
Ljusets natur:	Kapitel 33.1 – 33.4 & 33.7
Stråloptik:	Kapitel 34.1 – 34.8
Interferens:	Kapitel 35.1 – 35.5
Diffraktion:	Kapitel 36.1 - 36.5 & 36.7

Vågrörelselära och optik



Tid	Må 02-nov	Ti 03-nov	On 04-nov	To 05-nov	Fr 06-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 14	Kvantfysik (A)	Våglära/optik (A)	Kvantfysik (A)
10-12	Intro period 2 (A) Informationssökning (A)	Kvantfysik (A)	Våglära/optik (A) kap 14+15 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A) kap 15
13-15	Utvärdering (A) 12-13	Övningar Optik&Våg (L218-19)	SI gp6-10 (L219) ÄFYA11 (L218)	SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

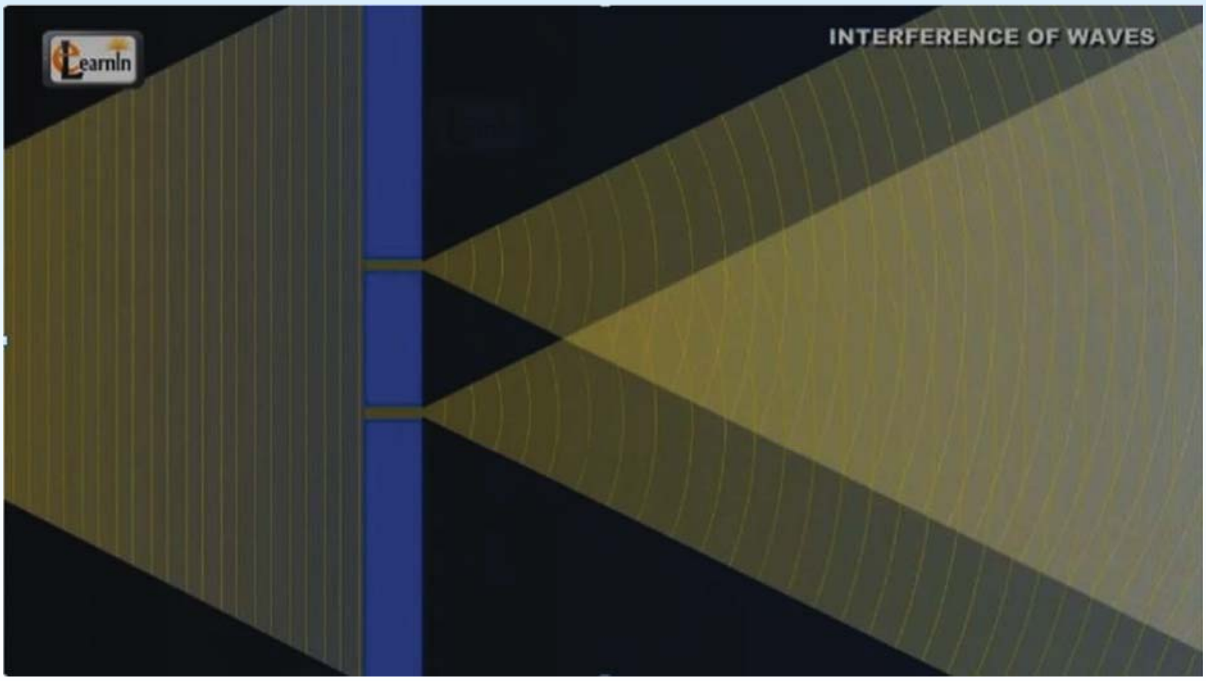
Tid	Må 09-nov	Ti 10-nov	On 11-nov	To 12-nov	Fr 13-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 16	Våglära/optik (A) kap 16+32	Kvantfysik (A)	Kvantfysik (A)
10-12	Våglära/optik kap 15+16 ÄFYA11 (L218)	Kvantfysik (A)	Kvantfysik (A)	Våglära/optik (A) kap 32+33	Våglära/optik (A) kap 33
13-15	SI gp1-5 (L219) ÄFYA11 (L218)	Övningar Optik&Våg (L218-19)	ÄFYA11 (l218) SI gp6-10 (L219)	SI gp1-5 (L218) SI gp11-15 (L219)	Övningar Optik&Våg (L218-19)
15-17					

Tid	Må 16-nov	Ti 17-nov	On 18-nov	To 19-nov	Fr 20-nov
08-10	Kvantfysik (A)	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 35 ÄFYA11 (L218)	Våglära/optik (A) kap 36
10-12	Våglära/optik (A) kap 34	Kvantfysik (A)	Våglära/optik (A) kap 34+35	Våglära/optik (A) kap 36	Kvantfysik (A)
13-15	SI gp6-10 (L219)	Övningar Optik&Våg (L218-19)	Seminar.gen.gång (A) 12-13 SI gp1-5 (L218) 13-15 SI gp11-15 (L219) 13-15	Labbintroduktion (A) O2-O3, K1-K3	Övningar Optik&Våg (L218-19)
15-17					

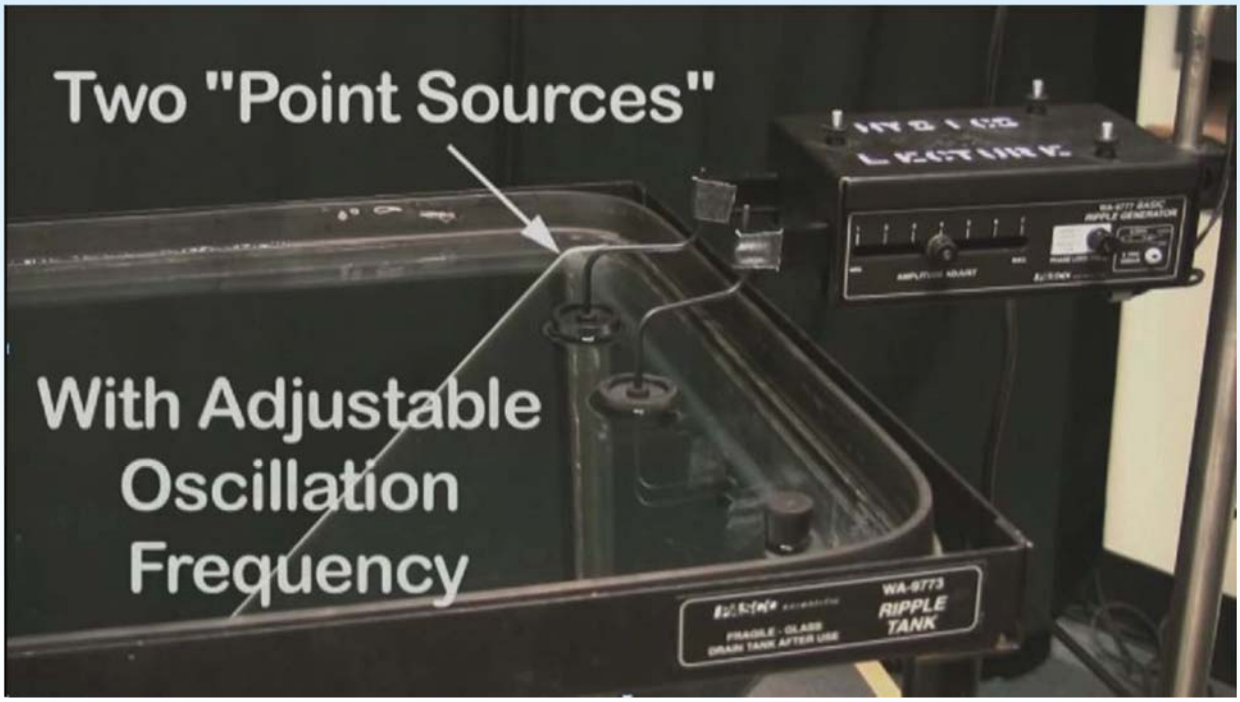
Interference

What is interference ?

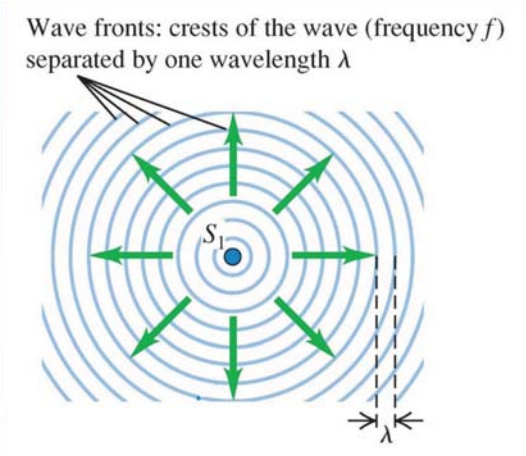
Interference



Interference



Interference



Interference: Wave overlap in space

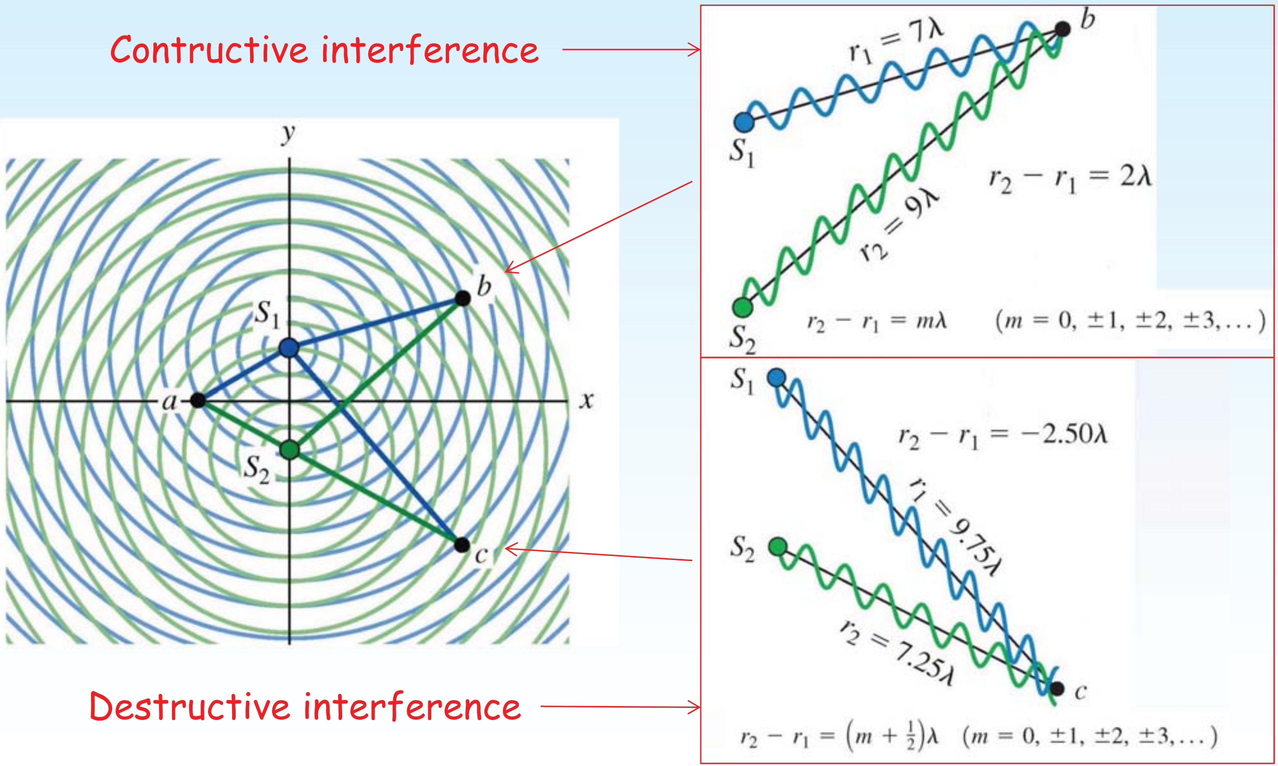
Coherent sources: Same frequency (or wavelength) and constant phase relationship (not necessarily in phase).

The principle of superposition states:

When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

Interference

Contructive interference

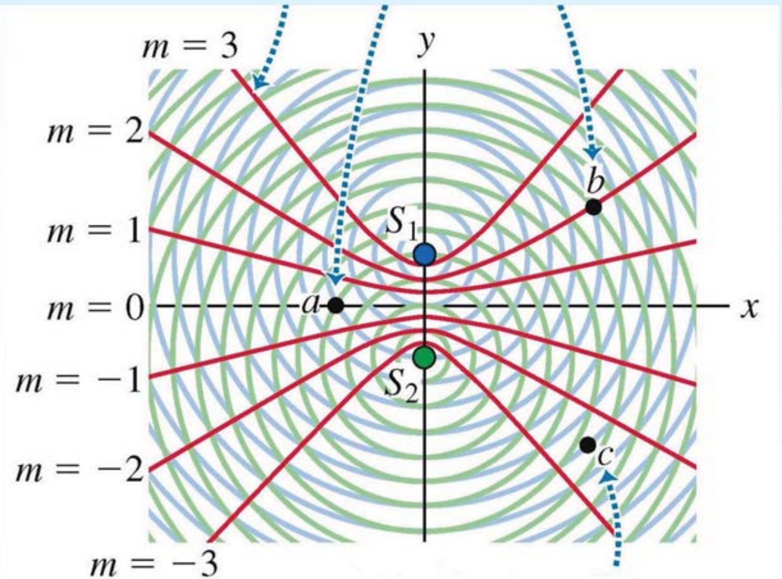


Destructive interference

Interference

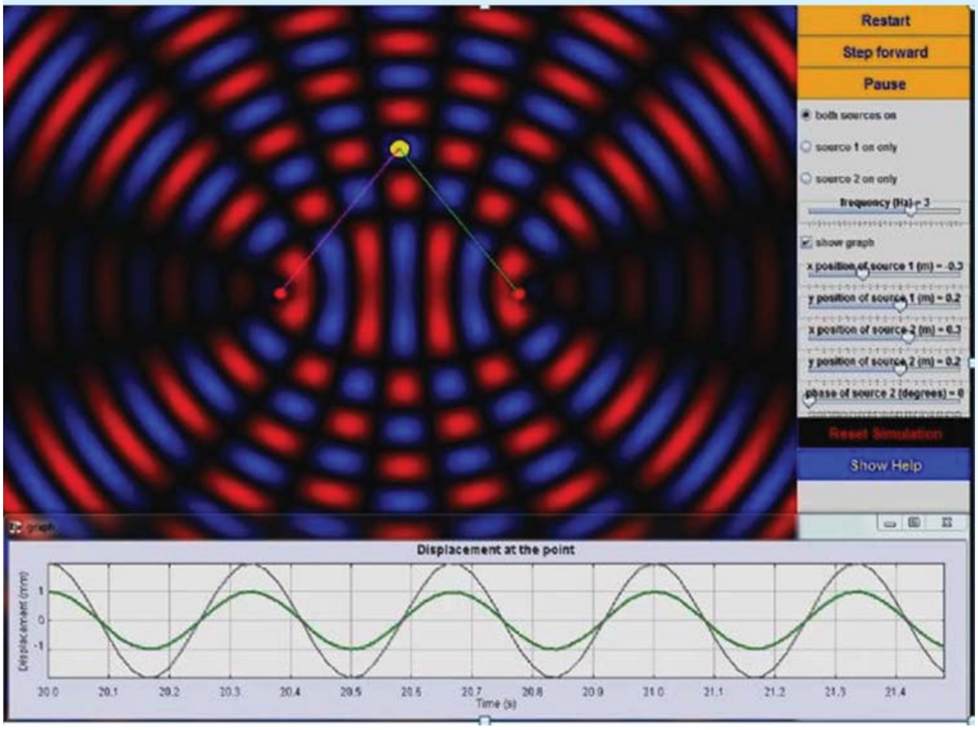
Antinodal curves = Contructive interference

Contructive interference



Destructive interference

Interference



● both sources on
○ source 1 on only
○ source 2 on only
frequency (Hz) = 3
☑ show graph
x position of source 1 (m) = -0.3
y position of source 1 (m) = 0.2
x position of source 2 (m) = 0.3
y position of source 2 (m) = 0.2
phase of source 2 (degrees) = 0

Black:

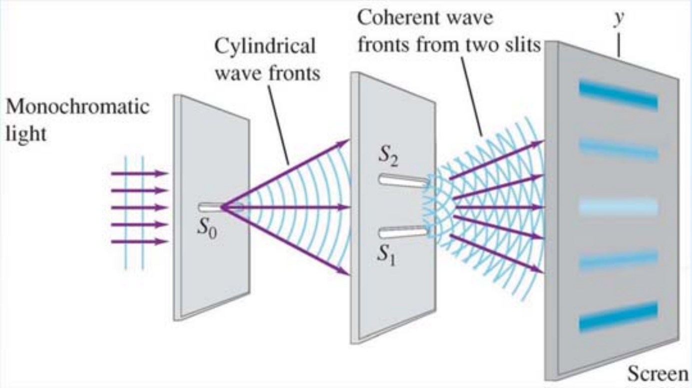
Amplitude = zero

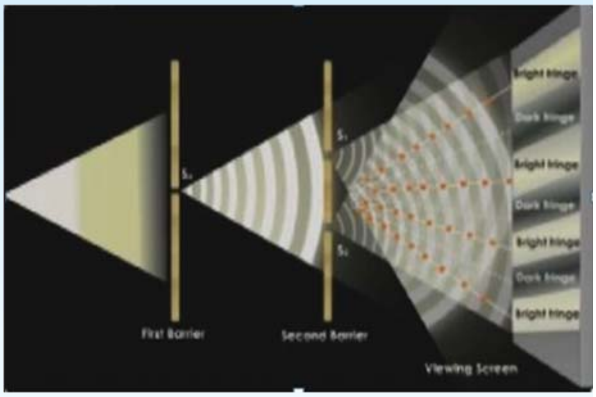
Red:
Amplitude > 0

Blue:
Amplitude < 0

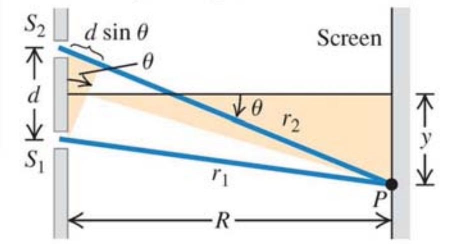
http://www.opensourcephysics.org/items/detail.cfm?ID=9989

Interference

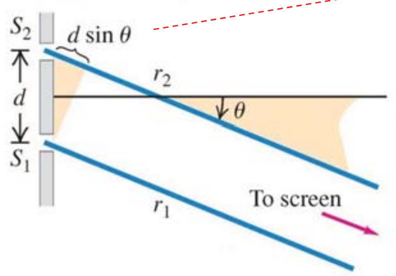




Actual geometry (seen from the side)



Approximate geometry



$$r_2 - r_1 = d\sin\theta$$

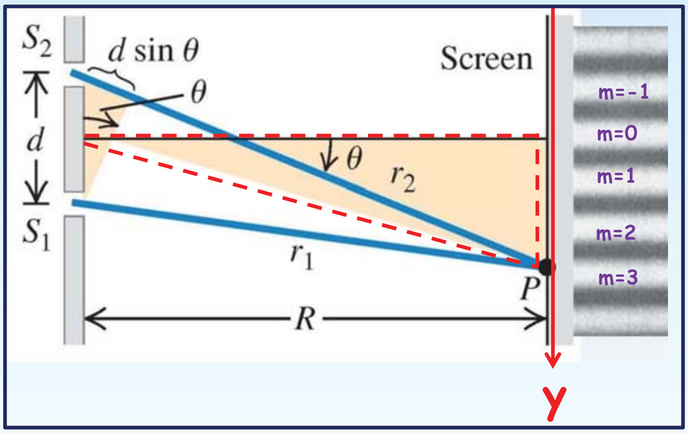
Contructive

$$d\sin\theta = m\lambda$$

Destructive

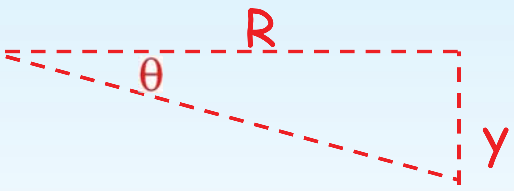
$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

Interference



Geometry:

$$r_2 - r_1 = d\sin(\theta) \approx d\,\theta$$



$$y = R\tan(\theta) \approx R\,\theta \approx R\,(r_2 - r_1)/d$$

Contructive interference:

$$r_2 - r_1 = m\,\lambda$$

$$y_m = R\frac{m\lambda}{d}$$

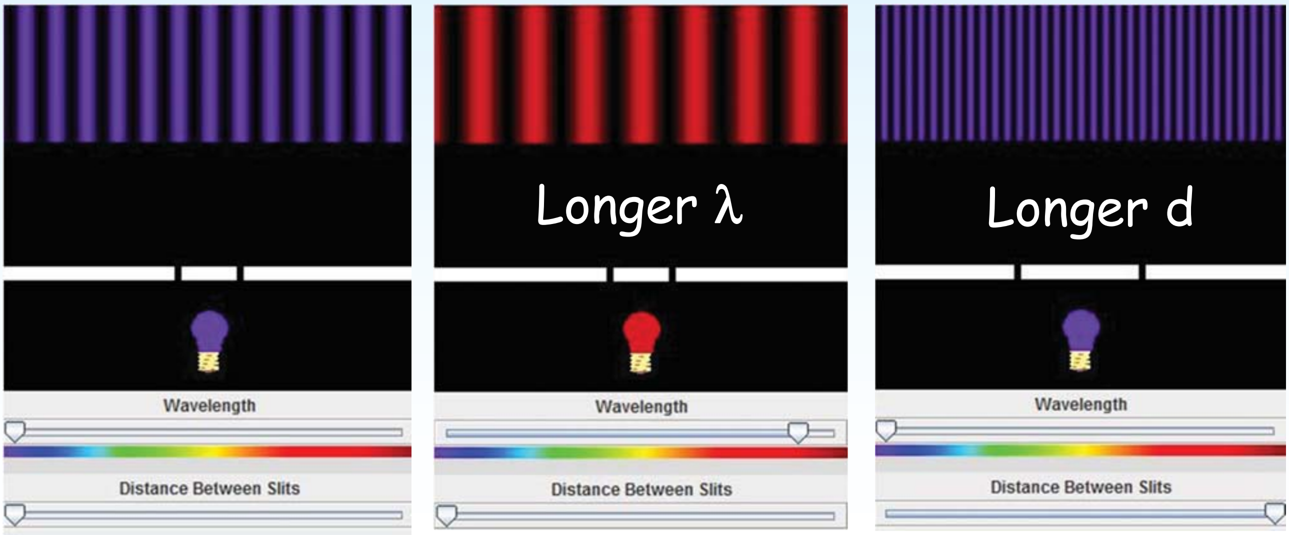
Interference

Constructive interference

$$y_m \approx m \cdot (R\,\lambda / d)$$

Destructive interference

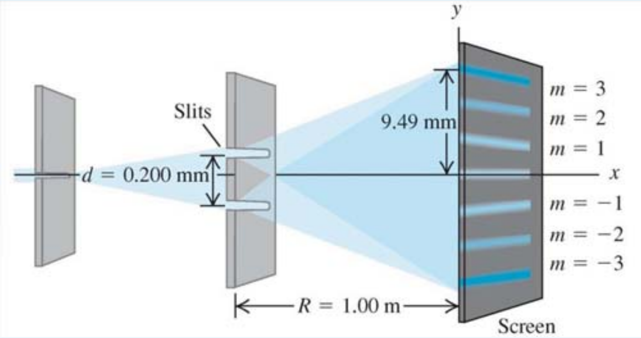
$$y_m \approx (m + ½) \cdot (R\,\lambda / d)$$



Interference

Problem solving

Interference



The $m = 3$ bright fringe in the figure is 9.49 mm from the central fringe. Find the wavelength of the light.

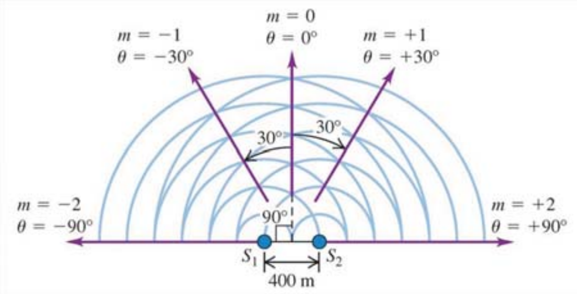
$$y_m = R\frac{m\lambda}{d}$$

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3}\ \text{m})(0.200 \times 10^{-3}\ \text{m})}{(3)(1.00\ \text{m})}$$

$$= 633 \times 10^{-9}\ \text{m} = 633\ \text{nm}$$

Interference

consider two identical vertical antennas 400 m apart, operating at 1500 kHz $= 1.5 \times 10^6$ Hz (near the top end of the AM broadcast band) and oscillating in phase. At distances much greater than 400 m, in what directions is the intensity from the two antennas greatest?



$$d\sin\theta = m\lambda$$

$$d = 400\ \text{m}$$

$$\lambda = c/f = 200\ \text{m}$$

$$\sin\theta = \frac{m\lambda}{d} = \frac{m(200\ \text{m})}{400\ \text{m}} = \frac{m}{2}$$

$$\theta = 0,\ \pm 30°,\ \pm 90°$$

Interference

Intensity of the light after interference

Interference

Power per unit area:

$$S_x(x, t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx - \omega t)$$

$E = c\,B$

Intensity = the average value of S

The average of $\cos^2(x) = 1/2$

$$I = S_{av} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{1}{2}\epsilon_0 c E_{\max}^2$$

How to eliminate μ_0 $\quad c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \quad \varepsilon_0\,\mu_0 = 1 / c^2 \quad \mu_0 = 1 / \varepsilon_0\, c^2$

Intensity of an electromagnetic wave: $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$

Interference

Intensity of an electromagnetic wave: $I = \frac{1}{2}\epsilon_0 c E_{\max}^{\ 2}$

where E_{max} is the amplitude of the electric field

Strategy:

Calculate the amplitude of the electric field after the superposition of two interfering waves. Use phasors to calculate this new E_{max}.

Put the new E_{max} into the formula: $I = \frac{1}{2}\epsilon_0 c E_{\max}^{\ 2}$

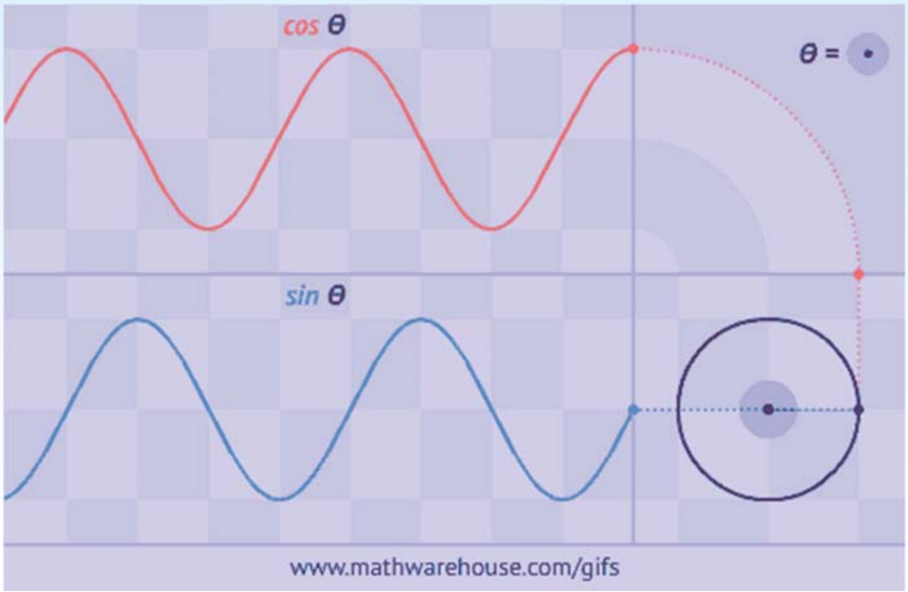
Derive a relationship between intensity and d, y and R.

Interference

Phasors

Interference

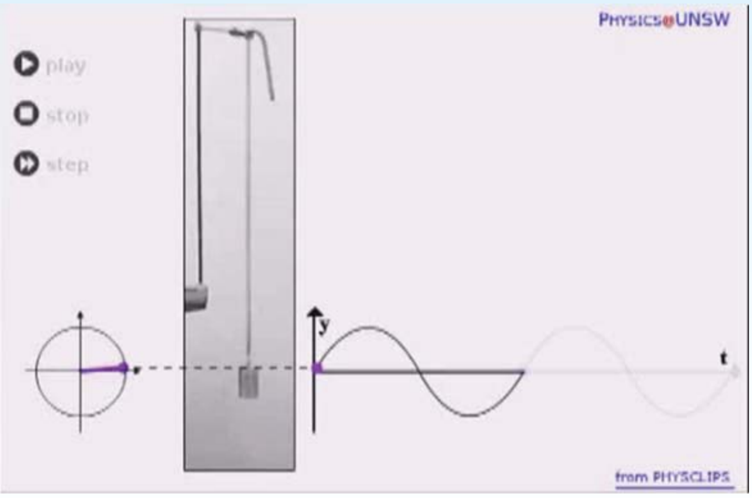
Phasors for a cos- and sin-function



www.mathwarehouse.com/gifs

Interference

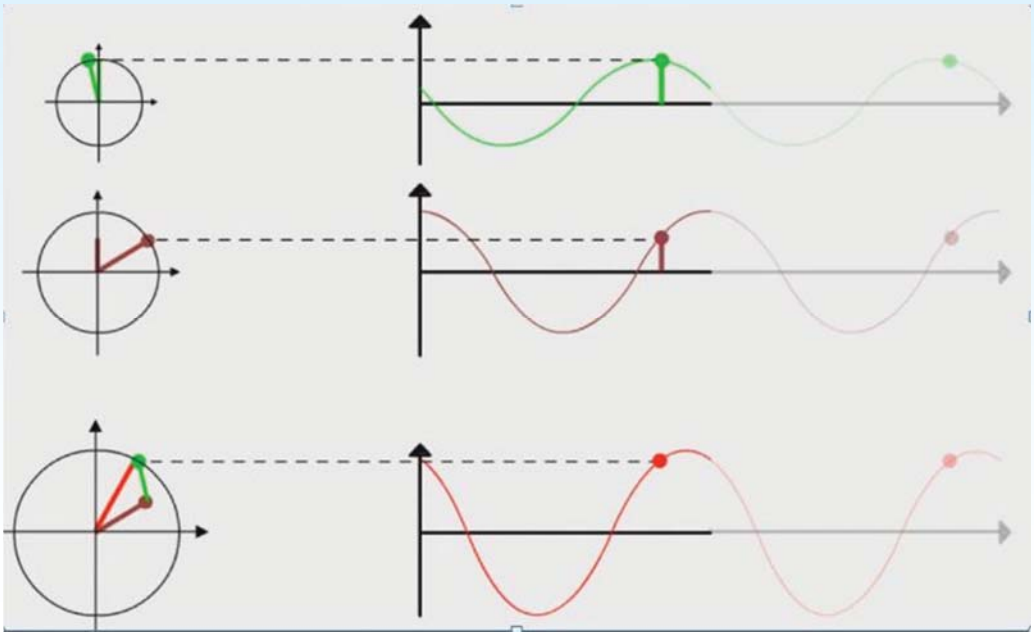
Phasor for a harmonic oscillation



from PHYSCLIPS

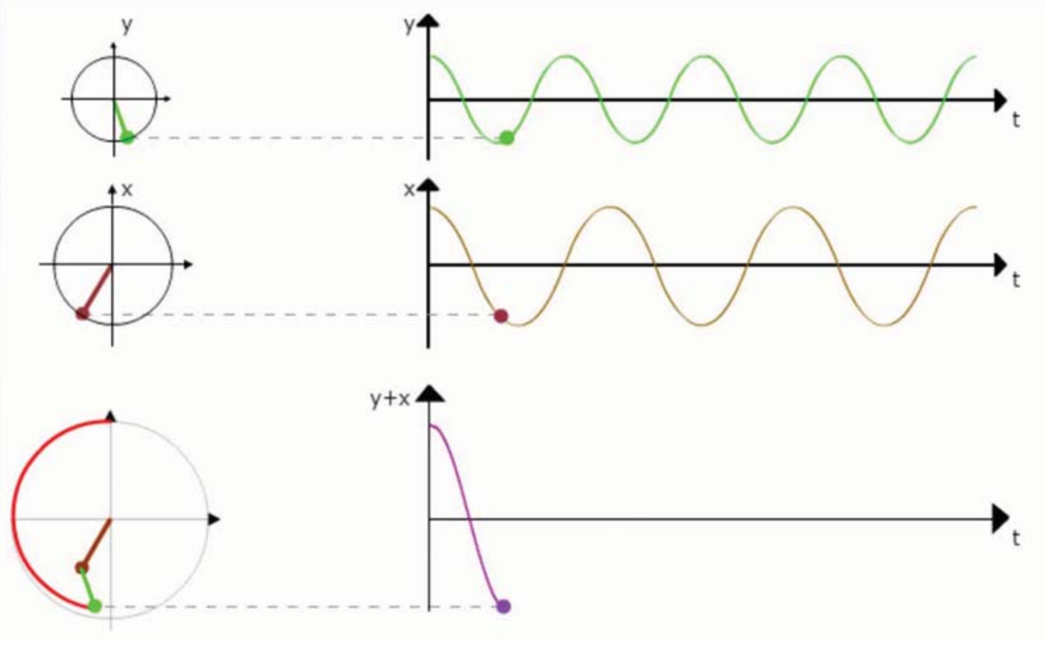
Interference

By adding phasors as vectors one can obtain the combined wave from two waves with the same frequency that are out of phase



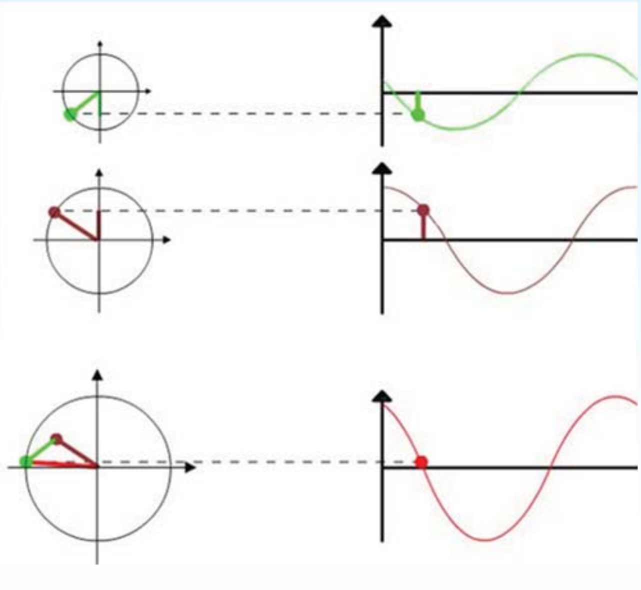
Interference

By adding phasors as vectors one can obtain the combined wave from two waves with different frequency that are out of phase

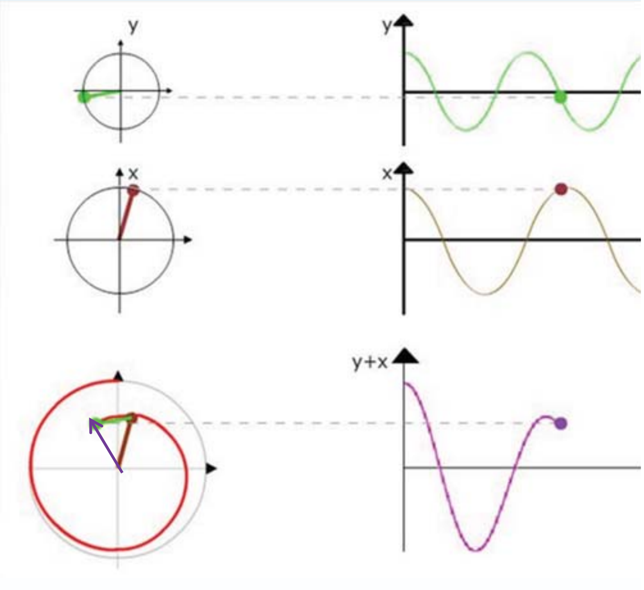


Interference

same frequency
different phase



different frequency
different phase



Interference

Combining two
electric fields by
using phasors to
get the amplitude

Interference

Combine two electric fields with

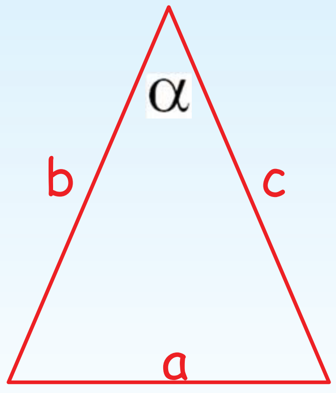
1. The same amplitude – E
2. The same frequency – ω
3. Different phase - ϕ

$$E_1(t) = E\cos(\omega t + \phi)$$

$$E_2(t) = E\cos\omega t$$

Interference

Trigonometry



The law of cosines:

$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$$\cos(\pi - \phi) = -\cos\phi$$

The double angle formula

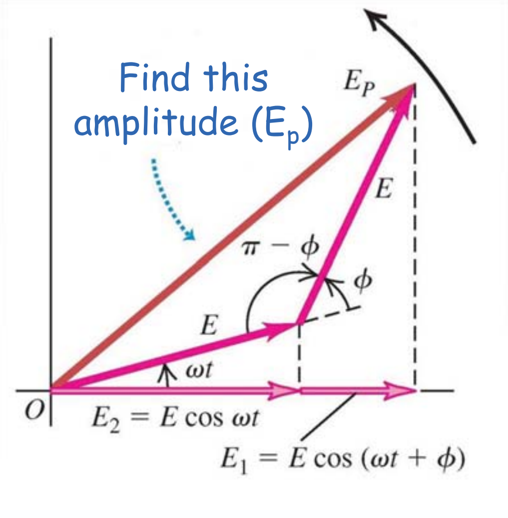
$\cos(2\alpha) = 2\cos^2(\alpha) - 1$

$\cos(\alpha) = 2\cos^2(\alpha/2) - 1$

Interference

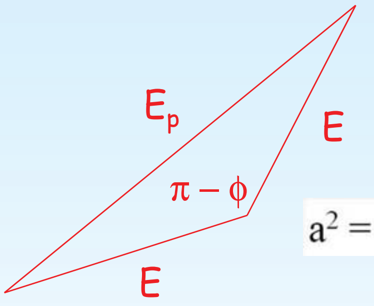
$$E_1(t) = E\cos(\omega t + \phi)$$

$$E_2(t) = E\cos\omega t$$



Find this amplitude (E_p)

Step 1



$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$E_P{}^2 = E^2 + E^2 - 2E^2\cos(\pi - \phi)$$

Step 2

$$\cos(\pi - \phi) = -\cos\phi$$

$$E_P{}^2 = E^2 + E^2 + 2E^2\cos\phi$$

$$E_p{}^2 = 2E^2\,(1+\cos(\phi))$$

Interference

Step3

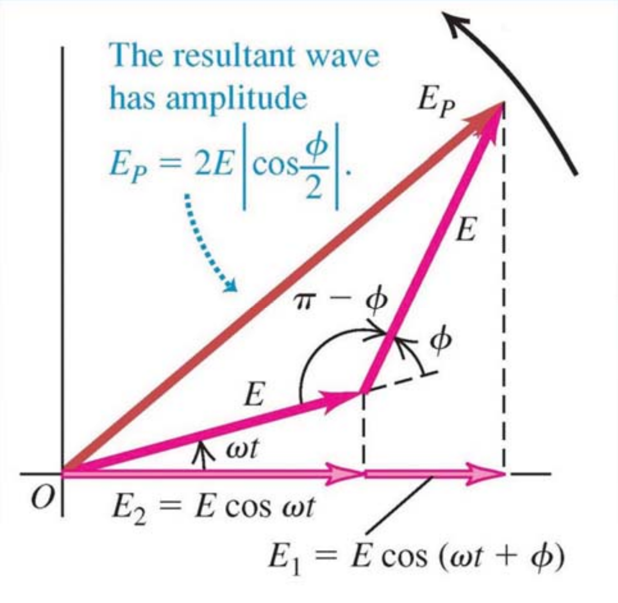
$$\cos(\alpha) = 2\cos^2(\alpha/2) - 1$$

$$E_p{}^2 = 2E^2\,(1+\cos(\phi))$$

$$E_p{}^2 = 2E^2\,(1 + 2\cos^2(\phi/2) - 1)$$

$$E_p{}^2 = 4E^2\cos^2(\phi/2)$$

$$\boxed{E_P = 2E\left|\cos\frac{\phi}{2}\right|}$$



The resultant wave has amplitude $E_P = 2E\left|\cos\frac{\phi}{2}\right|$.

Interference

Amplitude of the waves after interference:

$$E_P = 2E\left|\cos\frac{\phi}{2}\right|$$

Intensity of the waves after interference:

$$I = \frac{1}{2}\epsilon_0 c {E_P}^2 = 2\epsilon_0 c E^2 \cos^2\frac{\phi}{2}$$

The intensity of light (I) is proportional to the square of the amplitude of the electric field (E_p) $\quad I \sim {E_p}^2$

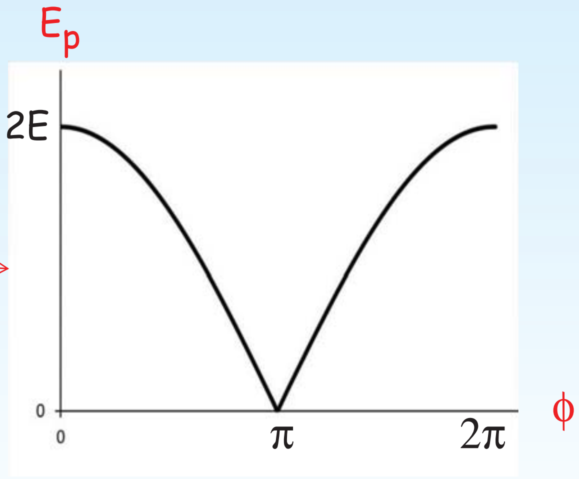
$I = I_0 \cos^2\frac{\phi}{2}$ where $I_0 = 2\epsilon_0 c E^2$ is the maximum intensity.

Interference

Relationship between intensity and d, y and R

Interference

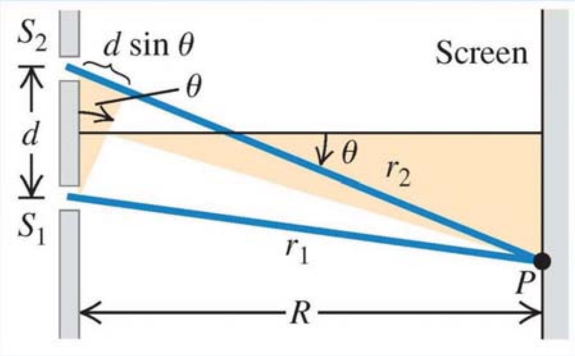
$$E_P = 2E\left|\cos\frac{\phi}{2}\right|$$



Conclusions:

Constructive interference occur when the phase difference is 2π
Destructive interference occur when the phase difference is π

Interference



Path difference

$$r_2 - r_1 = d\sin\theta$$

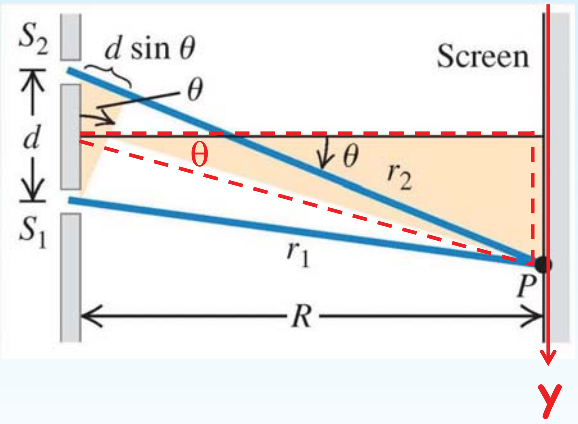
A path difference of one wavelength corresponds to a phase difference of 2π

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

Interference

Introduce y in the formula



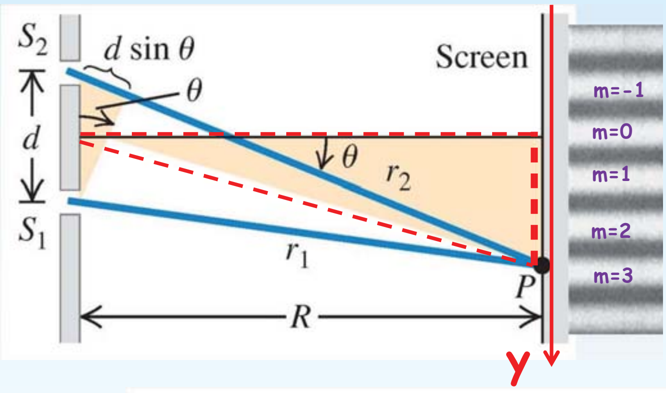
$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

$$\tan(\theta) = y / R \approx \sin(\theta)$$

small θ

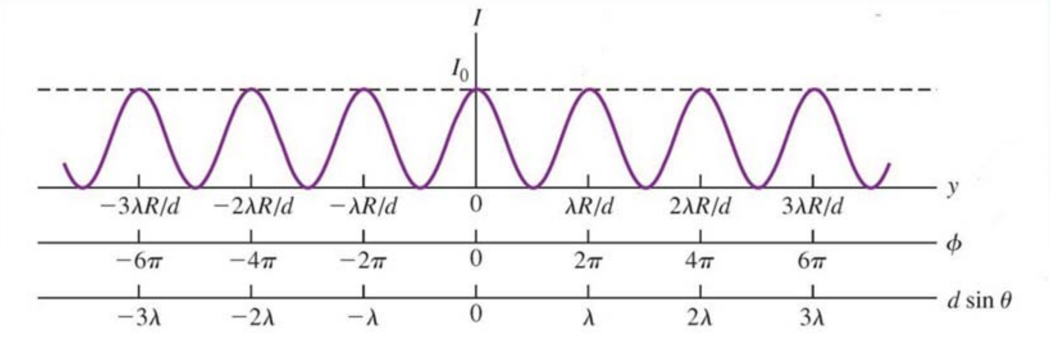
$$\phi = \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d y}{\lambda R}$$

Interference



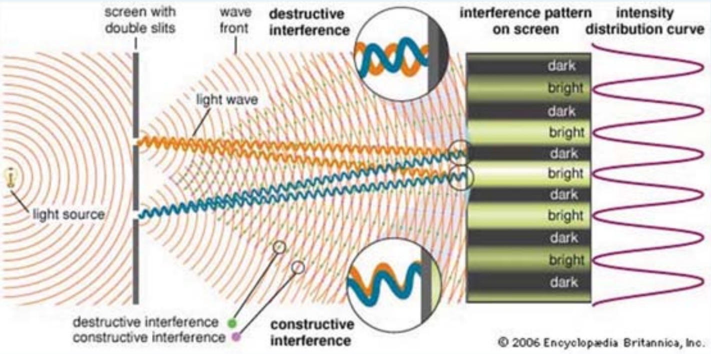
Intensity:

$$I = I_0 \cos^2\frac{\phi}{2} = I_0 \cos^2\left(\frac{\pi d y}{\lambda R}\right)$$



Interference

Summary



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Contructive interference:

$$r_2 - r_1 = d \sin(\theta) = m\,\lambda$$

$$y_m \approx m \cdot (R\,\lambda / d)$$

Intensity:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

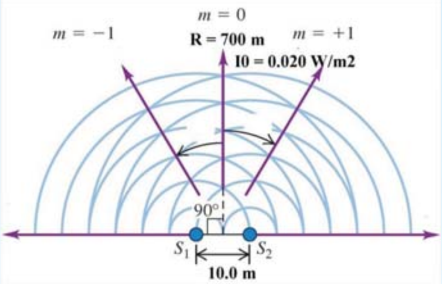
$$\phi = \frac{2\pi d}{\lambda} \sin\theta \approx \frac{2\pi d y}{\lambda R}$$

Interference

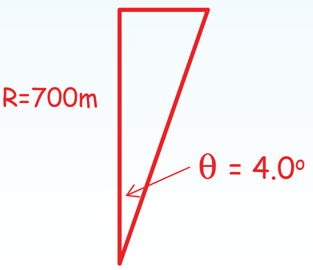
Problem solving

Interference

Suppose two identical radio antennas are moved to be only 10.0 m apart and the broadcast frequency is increased to $f = 60.0$ MHz. At a distance of 700 m from the point midway between the antennas and in the direction $\theta = 0$ the intensity is $I_0 = 0.020\ \mathrm{W/m^2}$. At this same distance, find the intensity in the direction $\theta = 4.0°$



y = 700 tan(4.0°) = 48.9 m



$$I = I_0 \cos^2 \frac{\phi}{2} = I_0 \cos^2\left(\frac{\pi d y}{\lambda R}\right)$$

$\lambda = c/f = 5.00$ m

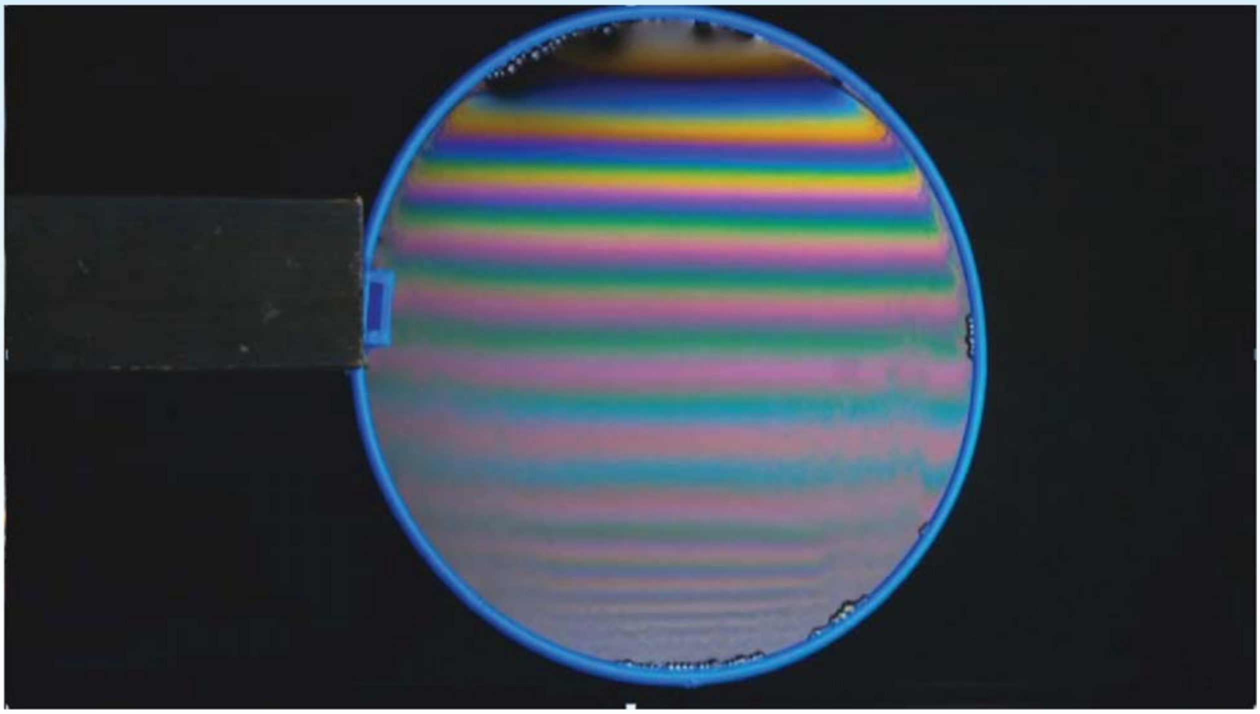
d = 10.0 m

$$I = 0.020 \cos^2(\pi \cdot 10.0 \cdot 48.9 / (5.00 \cdot 700)) = 0.016\ \mathrm{W/m^2}$$

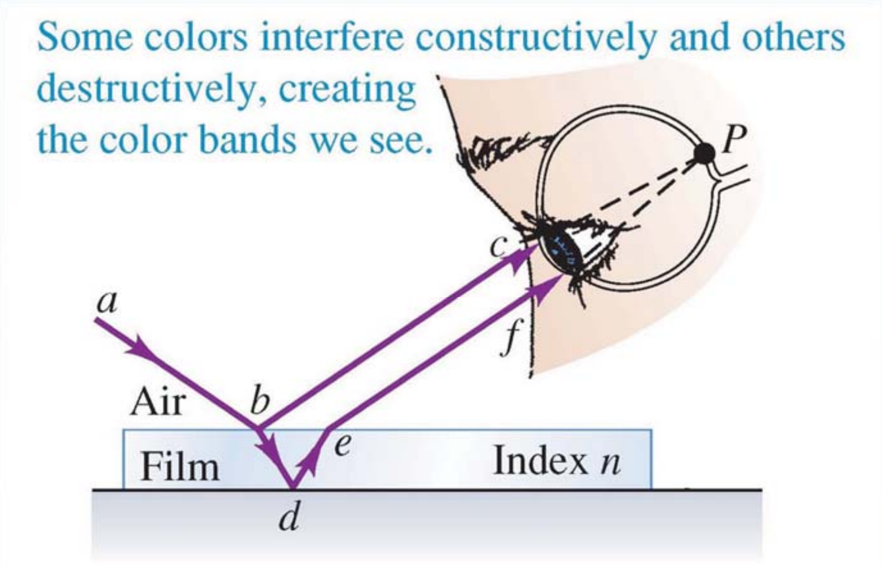
Interference

Thin-film interference

Interference



Interference



Different colours have different wavelengths so some will interfer constructively and other destructively.

Interference

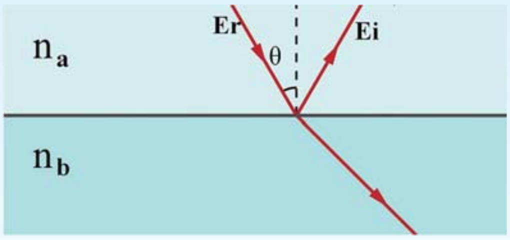
Reflections

Reflected Amplitude

Incoming Amplitude

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i$$

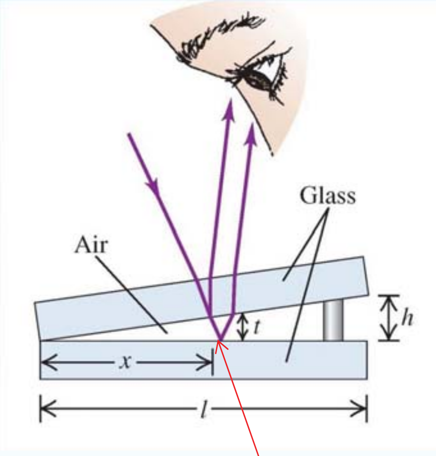
for $\theta = 0$



Positive if $n_a > n_b$ No phase shift

Negative if $n_b > n_a$ Phase shift with π

Interference



Phase shift with π

If we have one reflection with phase shift we get the following:

Constructive reflections:

$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots)$$

Destructive reflections:

$$2t = m\lambda \qquad (m = 0, 1, 2, \dots)$$

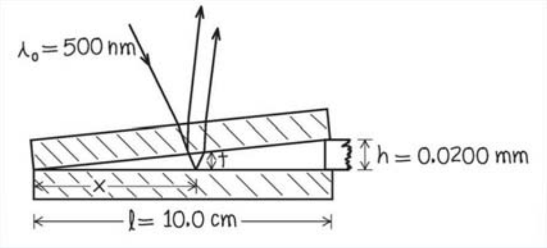
This is opposite to what we normally have without a phase shift.

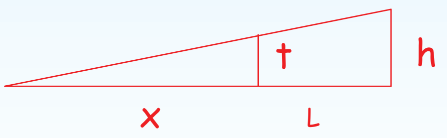
Interference

Problem solving

Interference

two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Assume monochromatic light with a wavelength in air of $\lambda = \lambda_0 = 500$ nm.





$$\frac{t}{x} = \frac{h}{l}$$

Destructive reflections: $2t = m\lambda \qquad (m = 0, 1, 2, \ldots)$

$$\frac{2xh}{l} = m\lambda_0$$

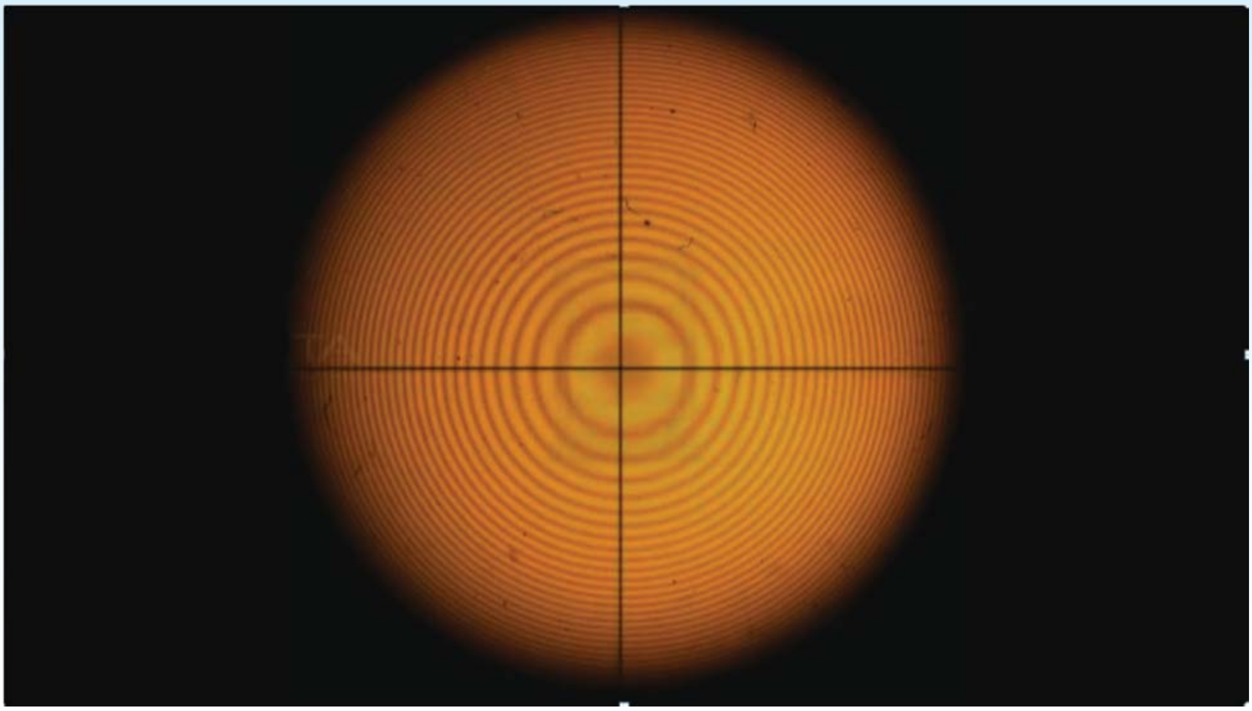
$$x = m\frac{l\lambda_0}{2h} = m\frac{(0.100\ \text{m})(500 \times 10^{-9}\ \text{m})}{(2)(0.0200 \times 10^{-3}\ \text{m})} = m(1.25\ \text{mm})$$

Successive dark fringes, corresponding to $m = 1, 2, 3, \ldots$, are spaced 1.25 mm apart.

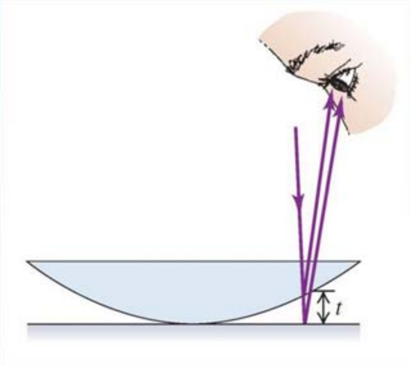
Interference

Newton's rings

Interference



Interference



Newton's rings can be used to study the surface of lenses to a very high precision.

Between each dark ring the distance t has changed with one half wavelength.

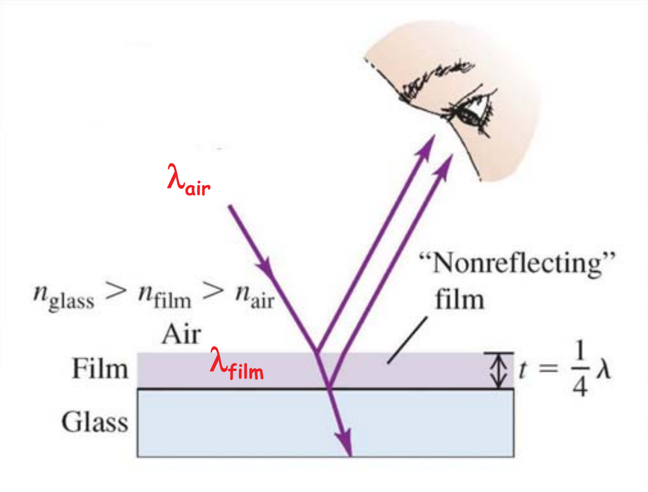
Destructive reflections: $2t = m\lambda \quad (m = 0, 1, 2, \ldots)$

Interference

Non-reflecting coating

Interference

Non-reflecting film



Film thickness: $\lambda_{film}/4$
Film refractive index: $n_{film} < n_{glass}$

↓

Destructive interference = no reflections

The wavelength in the film has to be a quarter of the film thickness.

This is not the same wavelength as that of the incoming light but it can be easily calculated with:

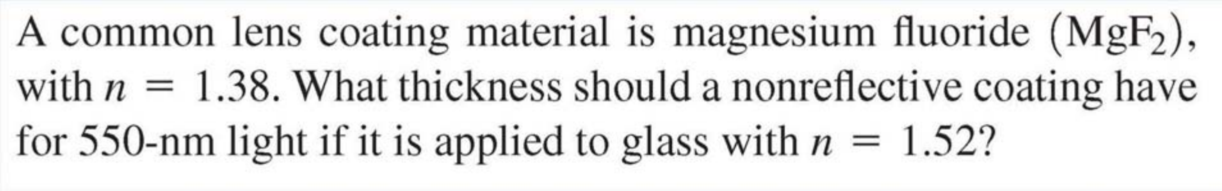
$$\lambda = v / f \qquad n > 1$$
$$\lambda_0 = c / f \qquad n = 1$$
$$n = c / v = \lambda_0 / \lambda$$

$$\lambda_{film} = \lambda_{air} / n_{film}$$

Interference

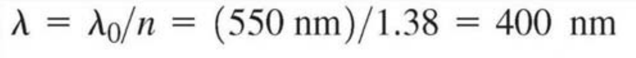
Problem solving

Interference



A common lens coating material is magnesium fluoride (MgF_2), with $n = 1.38$. What thickness should a nonreflective coating have for 550-nm light if it is applied to glass with $n = 1.52$?

$\lambda_{film} = \lambda_{air} / n_{film}$



$$\lambda = \lambda_0/n = (550\ \text{nm})/1.38 = 400\ \text{nm}$$

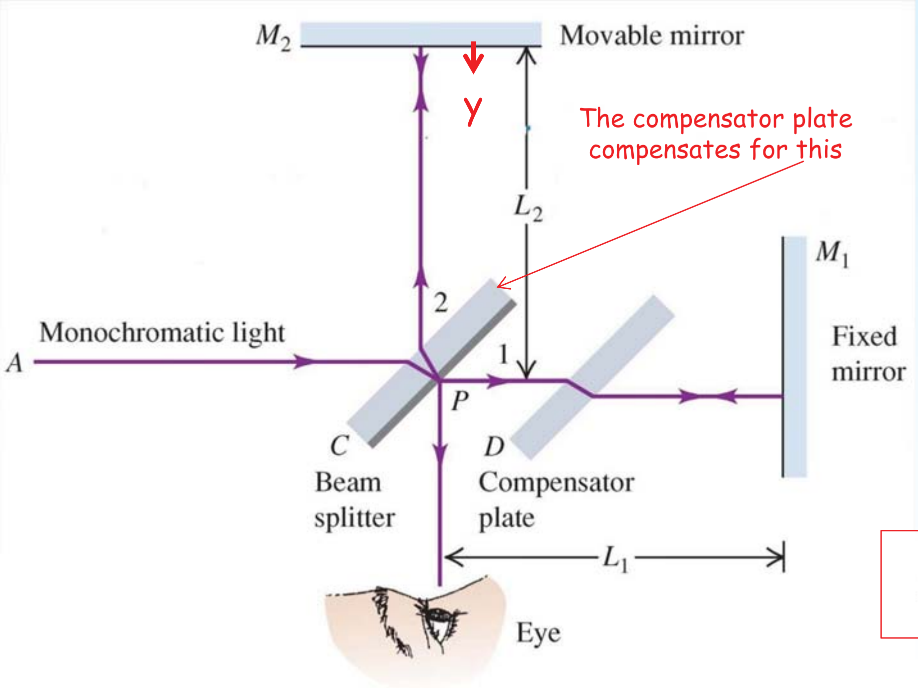
Film thickness = $\lambda/4$ = 400 / 100 = 100 nm

Interference

The Michelson Interferometer

Interference

The Michelson Interferometer



The compensator plate compensates for this

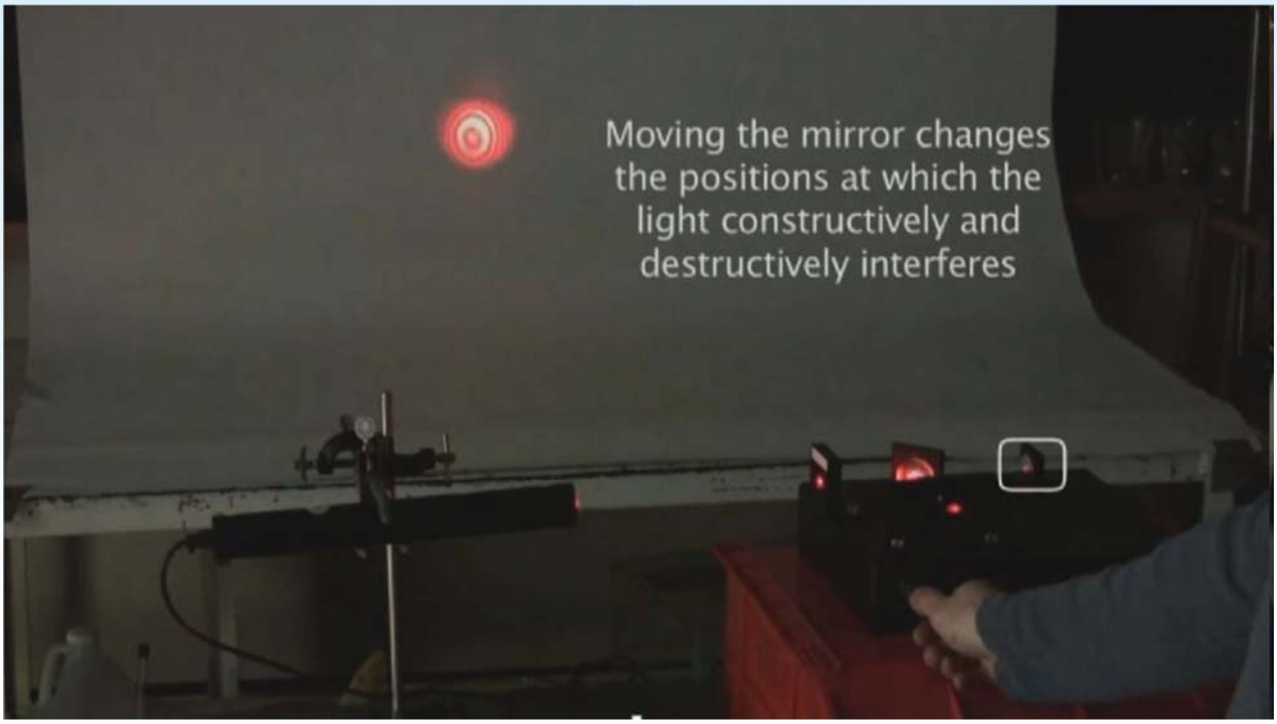
The observer will see an **interference pattern** with rings.

The **fringes** in the pattern will **move** when the mirror is moved.

The number of fringes (m) can be used to calculate y or λ

$$y = m\frac{\lambda}{2} \qquad \lambda = \frac{2y}{m}$$

Interference



Moving the mirror changes the positions at which the light constructively and destructively interferes