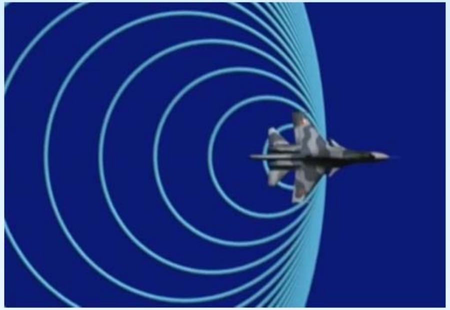
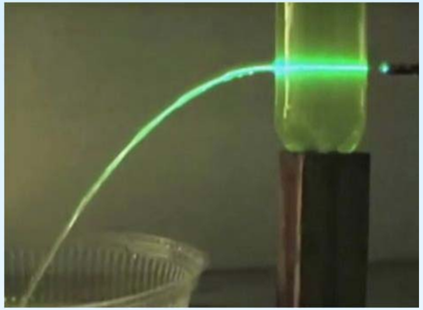
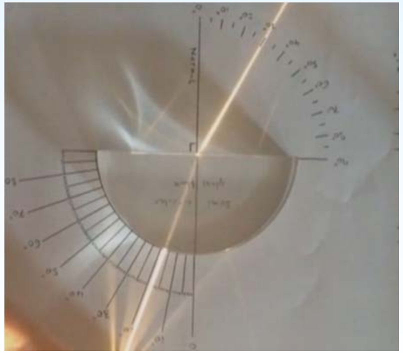
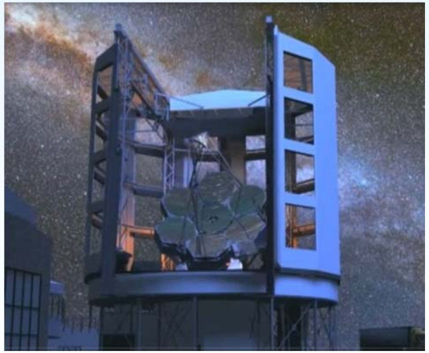
Wavemechanics and optics

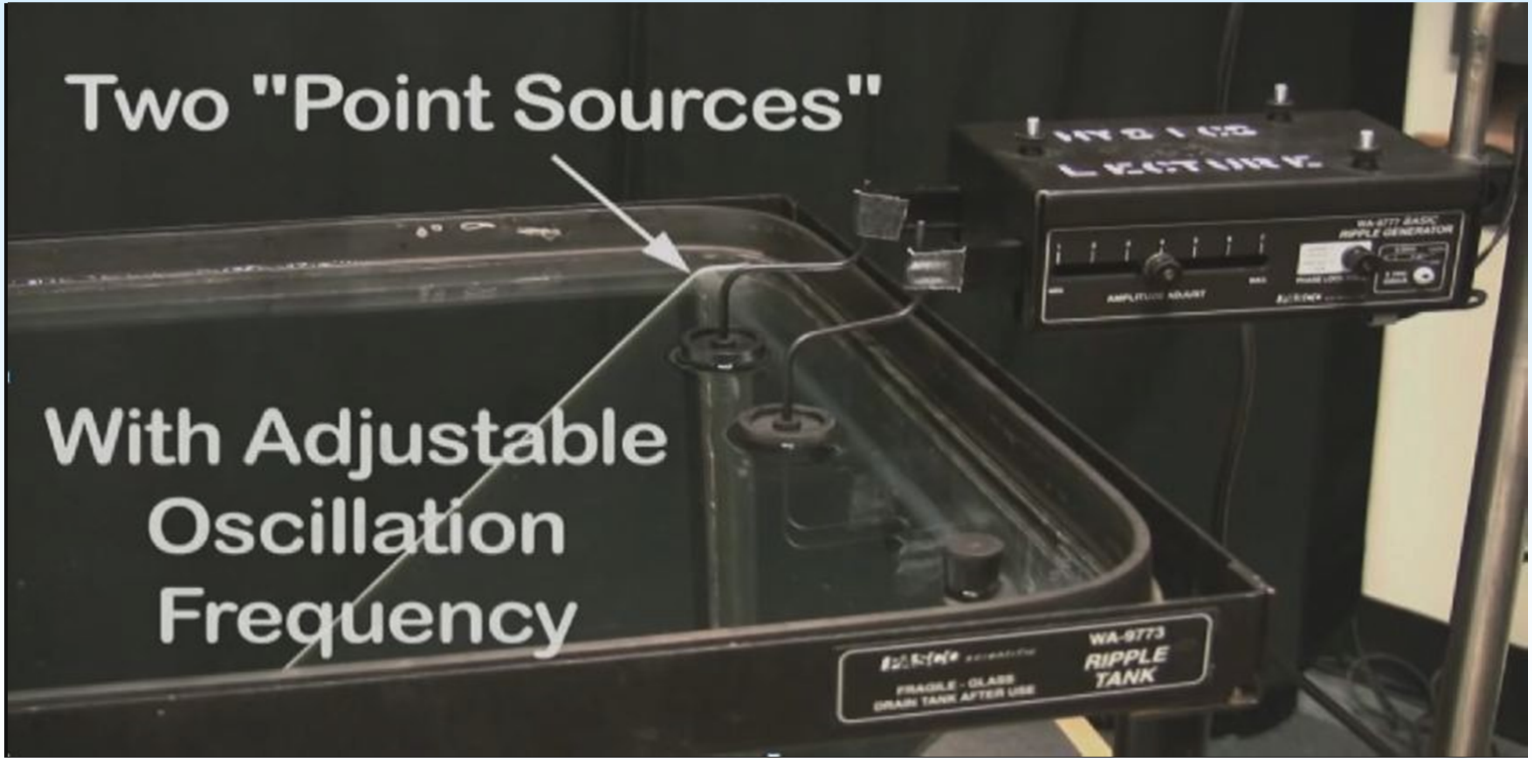




Chapter 35 - Interference

Vincent Hedberg - Lunds Universitet

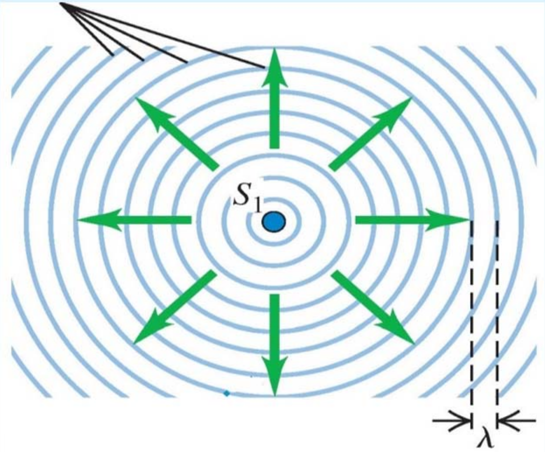
Interference



https://www.youtube.com/watch?v=UMkAXvWIRY4

Interference

Wave fronts: wave peaks in a wave separated by one λ



Interference:
Waves overlap in space

Coherent sources: Same frequency (or wavelength) and constant phase relationship (not necessarily in phase).

The Principle of Superposition

When two or more waves are superimposed, the instantaneous movement

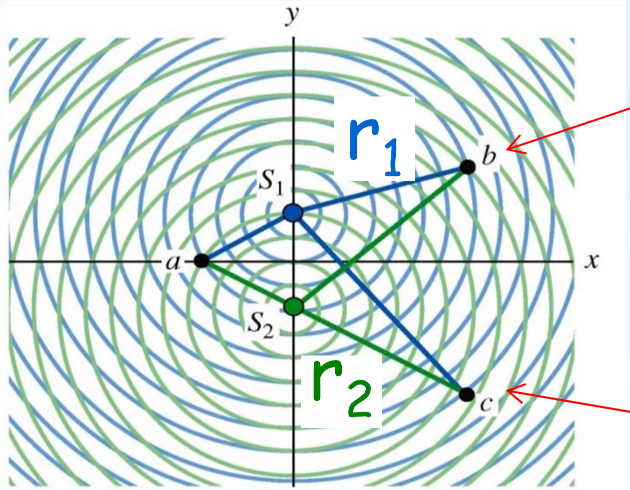
=

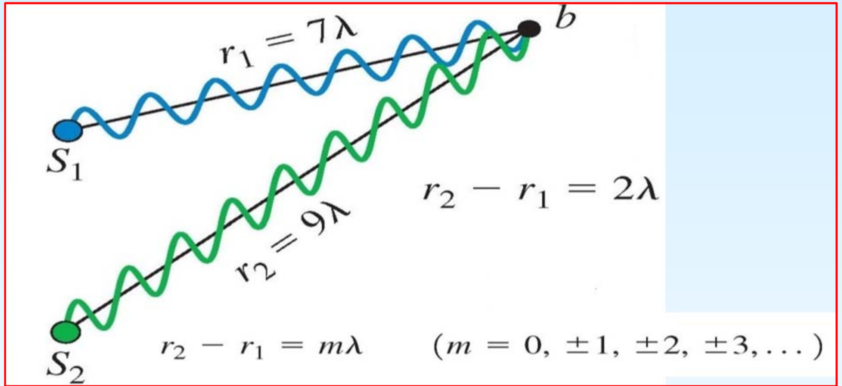
The sum of the displacement from the individual waves

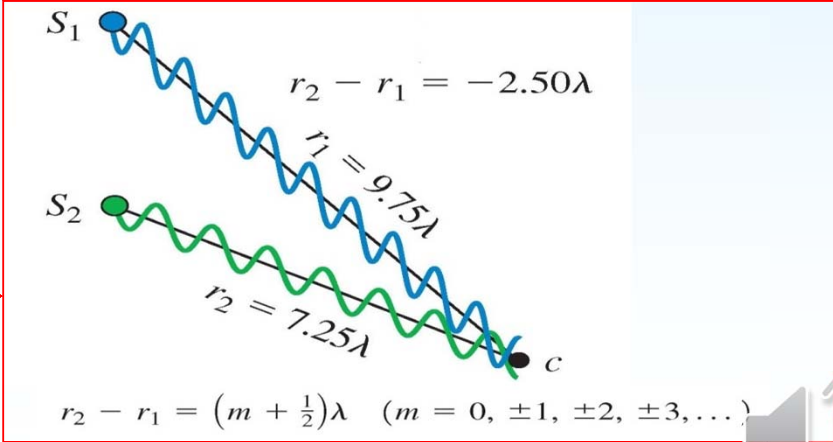
Interference

Contructive Interference

$$\delta = r_2 - r_1 = m\lambda$$







Destructive Interference

$$\delta = r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda$$

Interference

Suppose in one point:

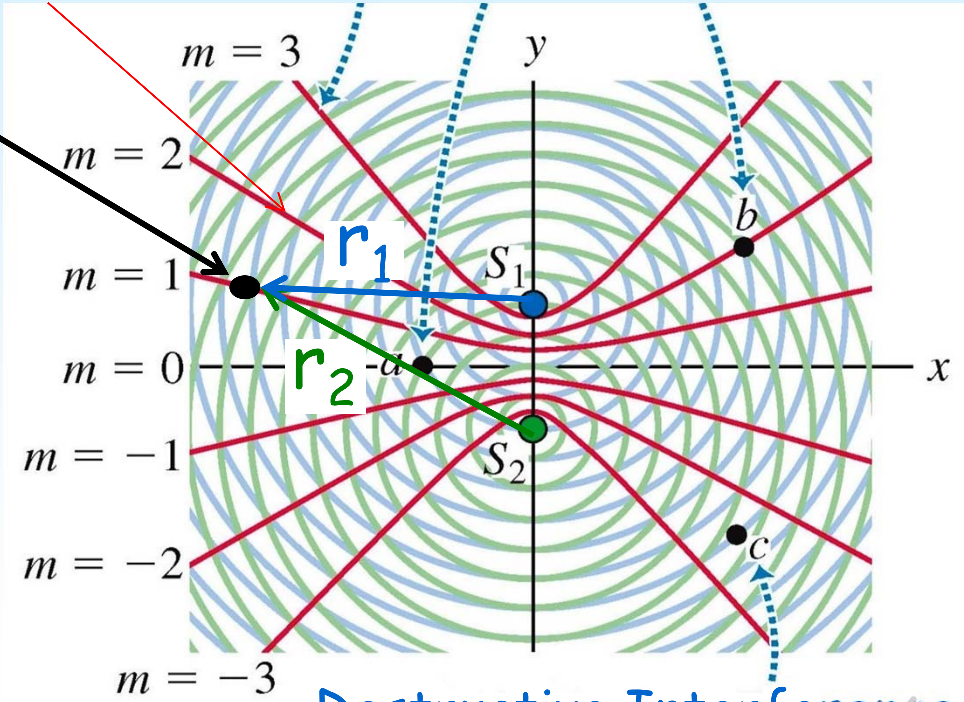
$$E_1 = E_{max}\cos\left(\frac{2\pi}{\lambda}r_1 - \omega t + \varphi\right)$$

$$E_2 = E_{max}\cos\left(\frac{2\pi}{\lambda}r_2 - \omega t\right)$$

$$E = E_1 + E_2$$

$r_1 = r_1 + \lambda$

Antinodal curves = Constructive Interference

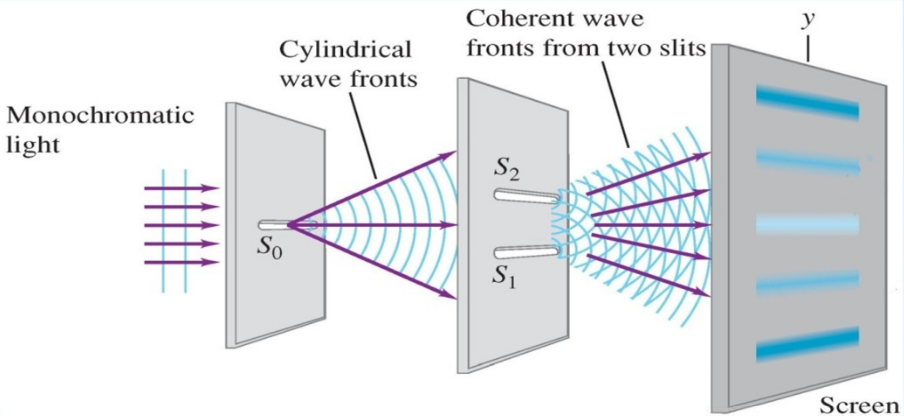


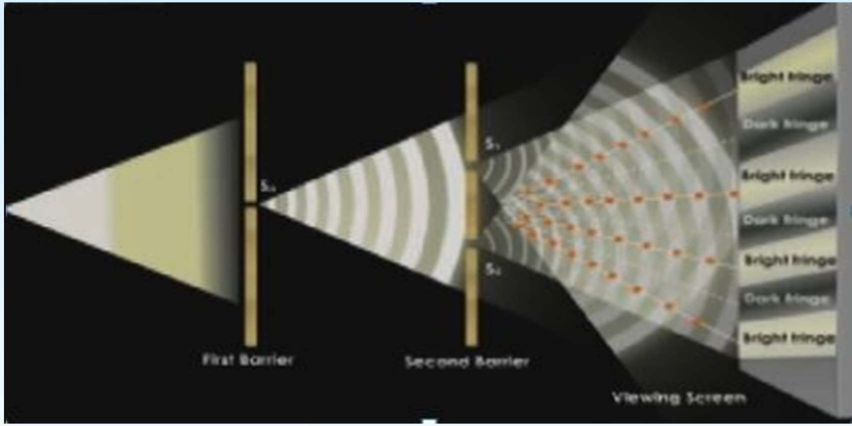
Destructive Interference

Increasing r_1 with one wavelength gives the same change of E as increasing the phase angle ϕ by 2π:

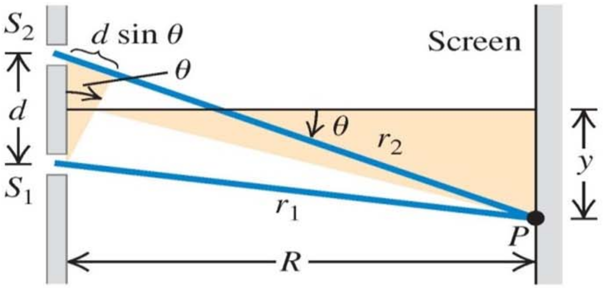
$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

Interference

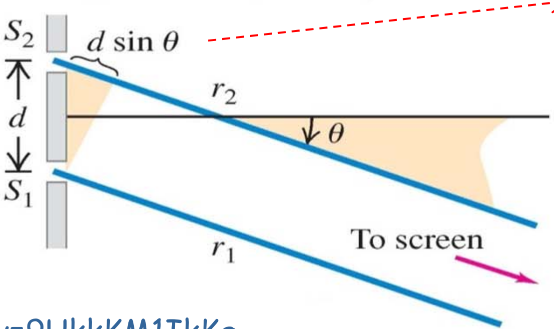




Actual geometry (seen from the side)



Approximate geometry



$$\delta = r_2 - r_1 = d \sin\theta$$

Constructive

$$d \sin\theta = m\lambda$$

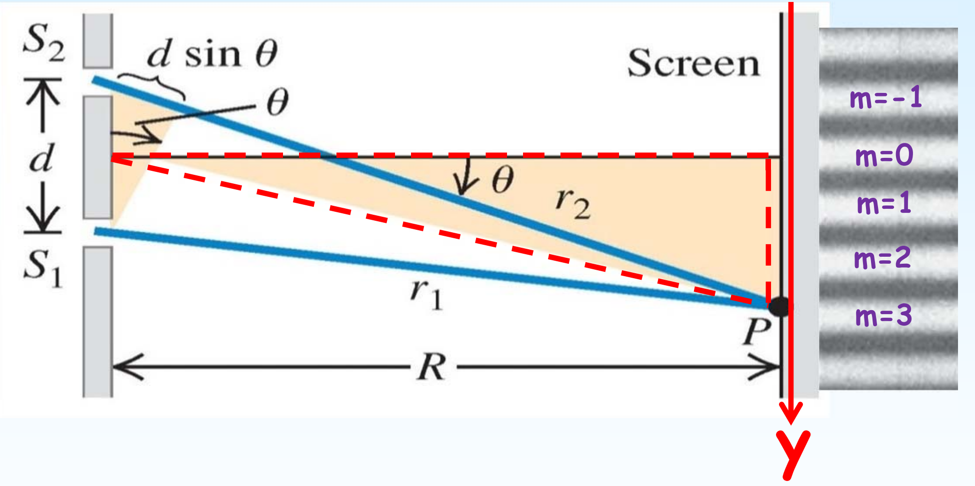
Destructive

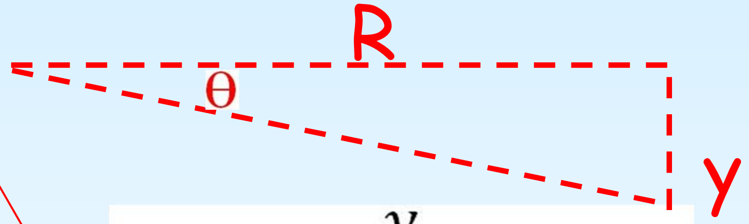
$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

https://www.youtube.com/watch?v=9UkkKM1IkKg

Interference

Geometry: $r_2 - r_1 = d\sin(\theta)$





$$\tan(\theta) = \frac{y}{R} \approx \sin(\theta)$$

$$r_2 - r_1 = d\frac{y}{R}$$

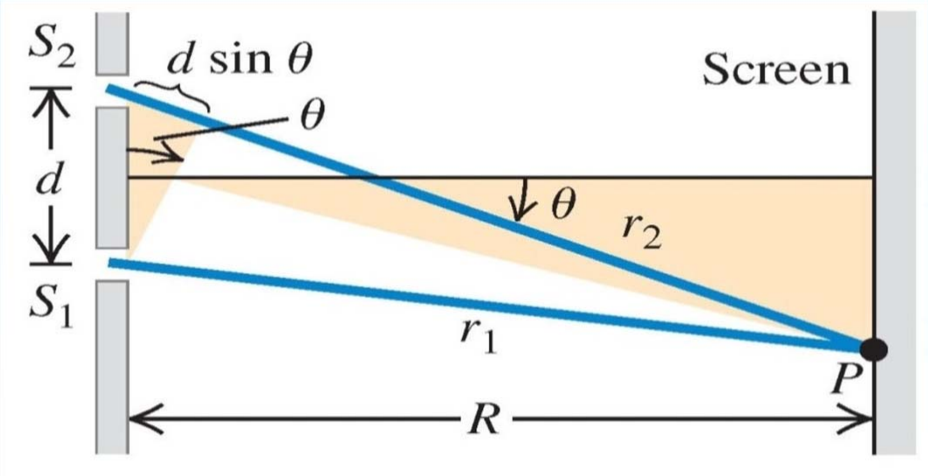
Constructive Interference:

$$r_2 - r_1 = m\lambda$$

$$y_m = R\frac{m\lambda}{d}$$

$$m = 0, \pm 1, \pm 2, \pm 3 \ldots.$$

Interference: Intensity



A path difference of one wavelength corresponds to a phase difference of 2π

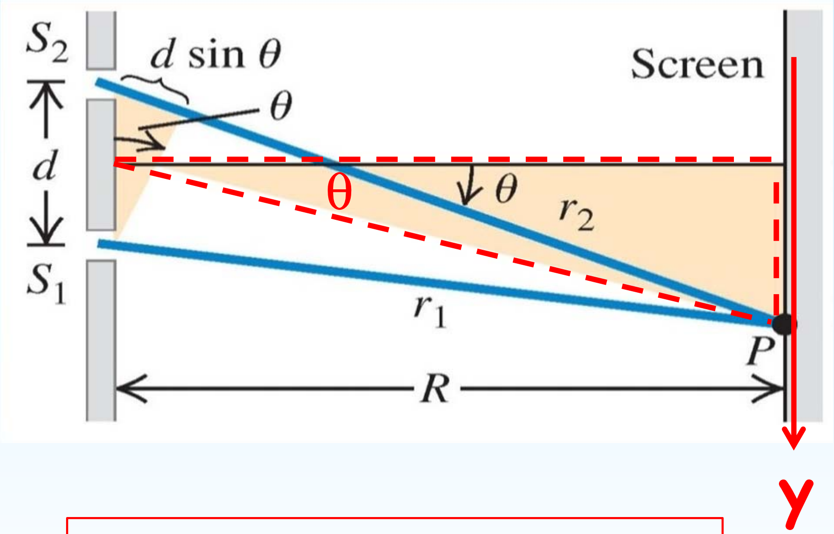
$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda} = \frac{\delta}{\lambda}$$

Path difference

$$\delta = r_2 - r_1 = d \sin\theta$$

$$\phi = \frac{2\pi\delta}{\lambda} = \frac{2\pi d}{\lambda}\sin\theta$$

Interference: Intensity



Introduce y in the formula

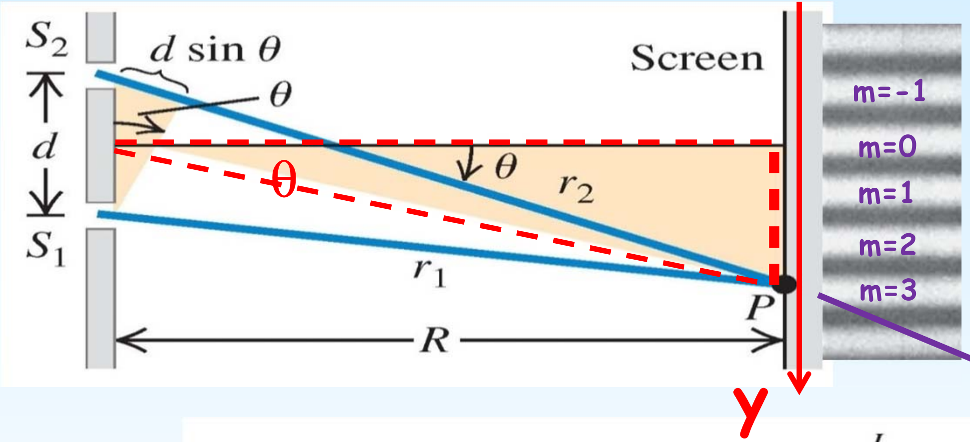
$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

$$\tan(\theta) = y / R \approx \sin(\theta)$$

small θ

$$\phi = \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi dy}{\lambda R}$$

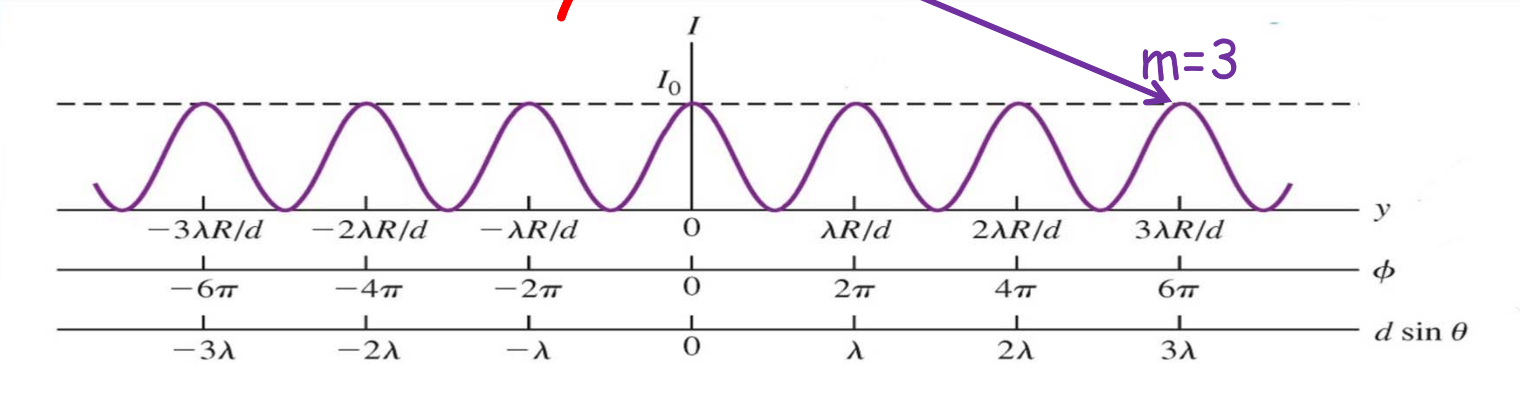
Interference: Intensity



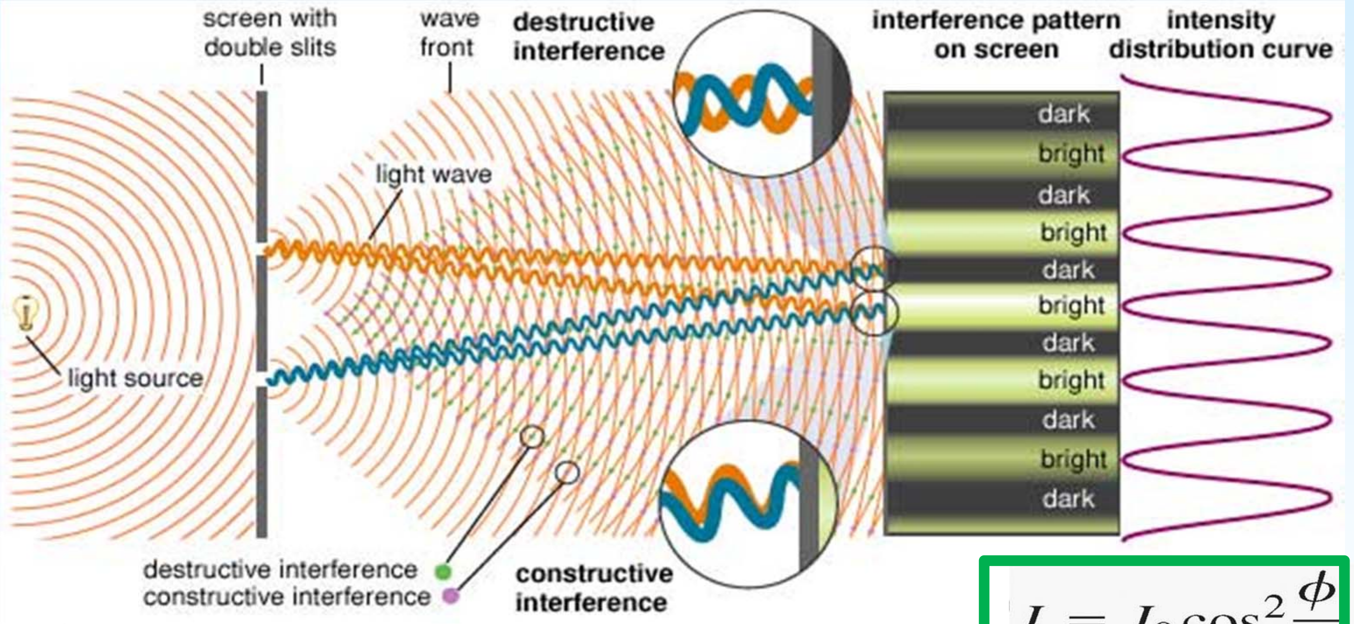
$$\phi \approx \frac{2\pi d y}{\lambda R}$$

Intensity:

$$I = I_0 \cos^2 \frac{\phi}{2} = I_0 \cos^2\left(\frac{\pi d y}{\lambda R}\right)$$



Interference: Intensity



Constructive interference:

$$r_2 - r_1 = d\sin(\theta) = m\lambda$$

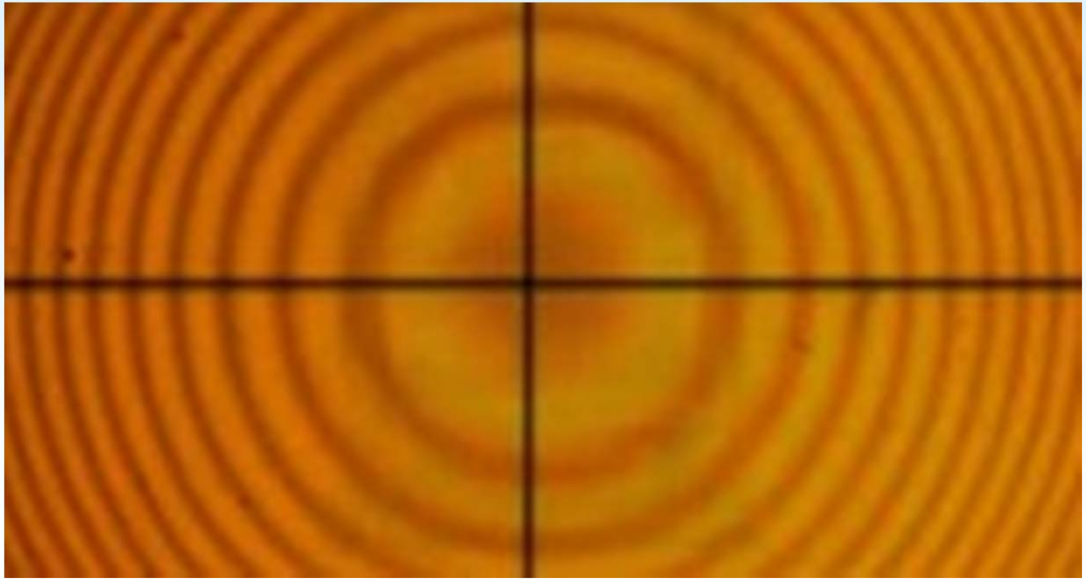
$$y_m \approx m \cdot (R\lambda / d)$$

Intensity:

$$I = I_0 \cos^2\frac{\phi}{2}$$

$$\phi = \frac{2\pi\delta}{\lambda} \approx \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d y}{\lambda R}$$

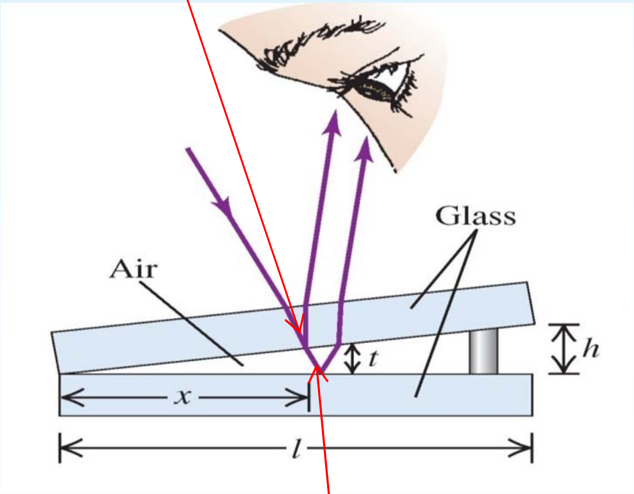
Thin film interference



Interference: Thin Film

$n_b < n_a$

Phase shift = 0



$n_b > n_a$

Phase shift = π

After a reflection with one phase shift ($n_b > n_a$) the following is true:

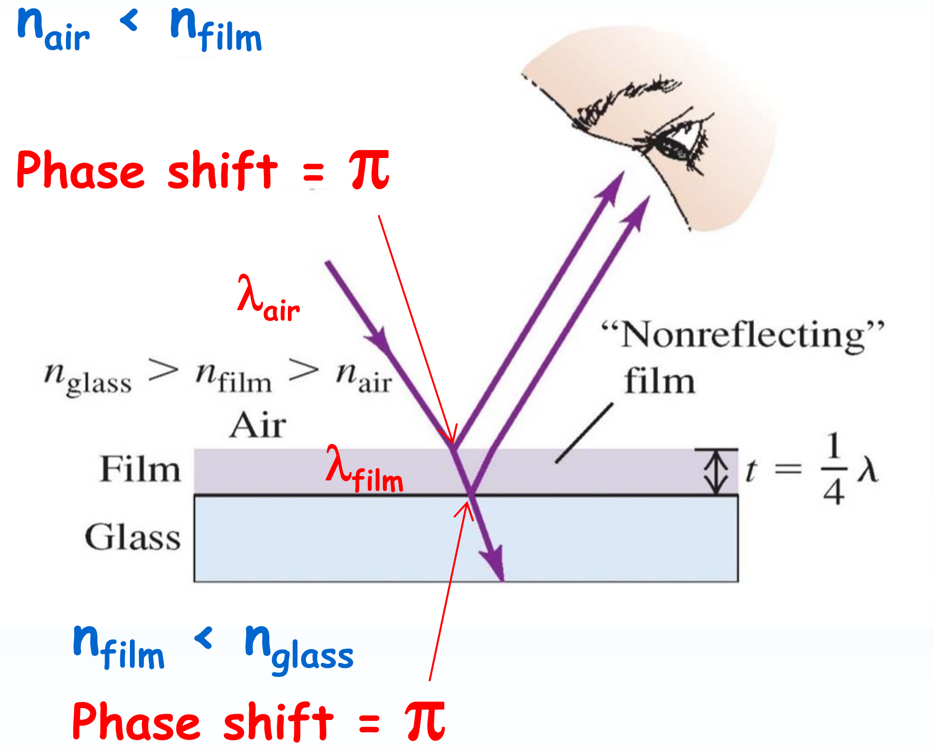
Constructive reflections:

$$2t = \left(m + \frac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \dots)$$

Destructive reflections:

$$2t = m\lambda \qquad (m = 0, 1, 2, \dots)$$

This is the opposite of what we normally have without a phase shift (or after two phase shifts).



Non-reflecting coating

Destructive interference:

$$2t = \left(m + \frac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \ldots)$$

Film thickness: $t = \lambda_{film} / 4$

Film refractive index: $n_{film} < n_{glass}$

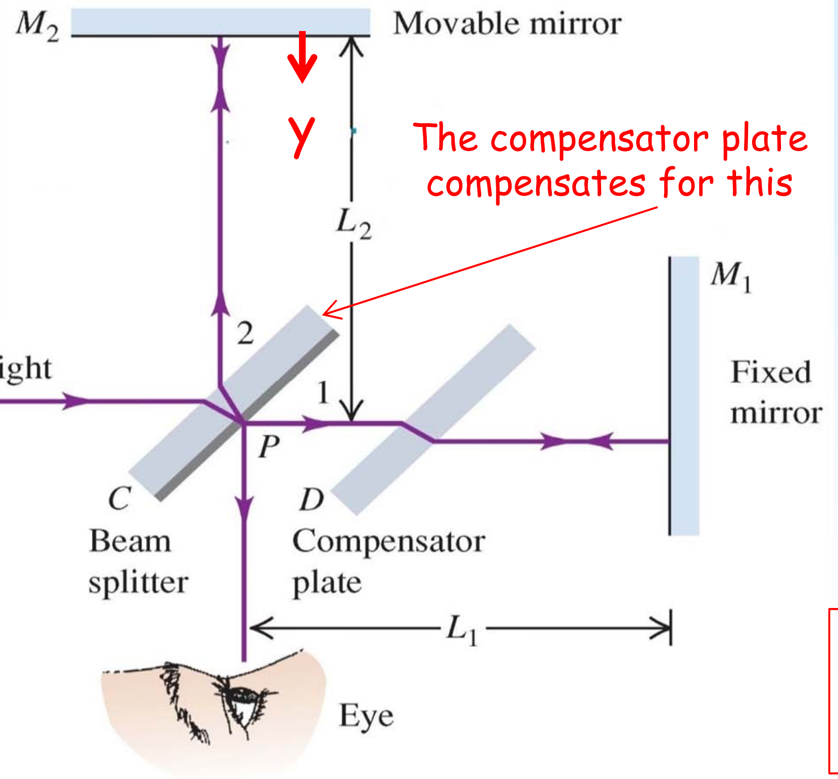
⇩

Destructive interference = No reflections

Interference: The Michelson interferometer

The Michelson interferometer

Interference: The Michelson interferometer



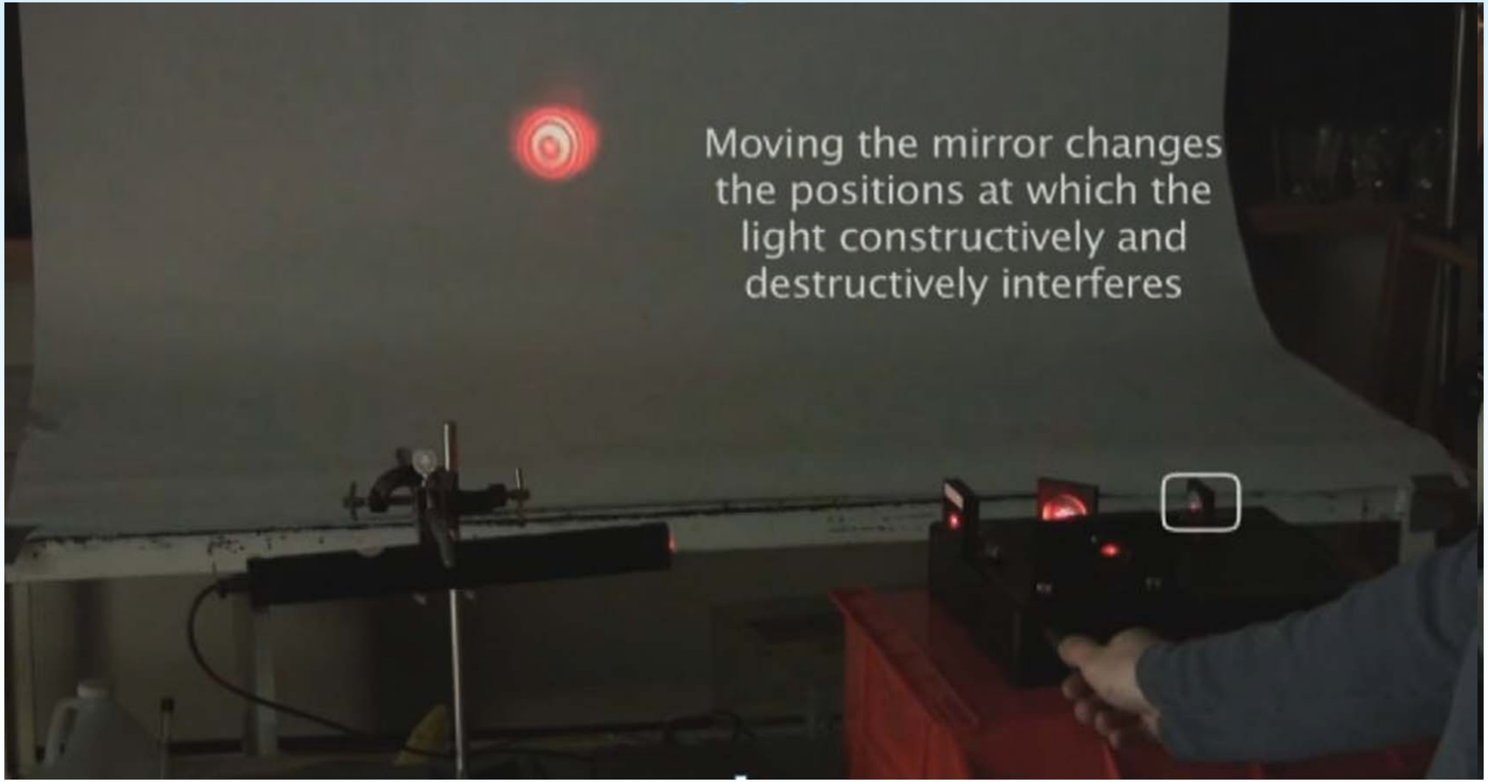
The observer will see an **interference pattern** with rings.

The **fringes** in the pattern will **move** when the mirror is moved.

The number of fringes (m) can be used to **calculate y or λ**:

$$y = m\frac{\lambda}{2} \qquad \lambda = \frac{2y}{m}$$

Interference: The Michelson interferometer



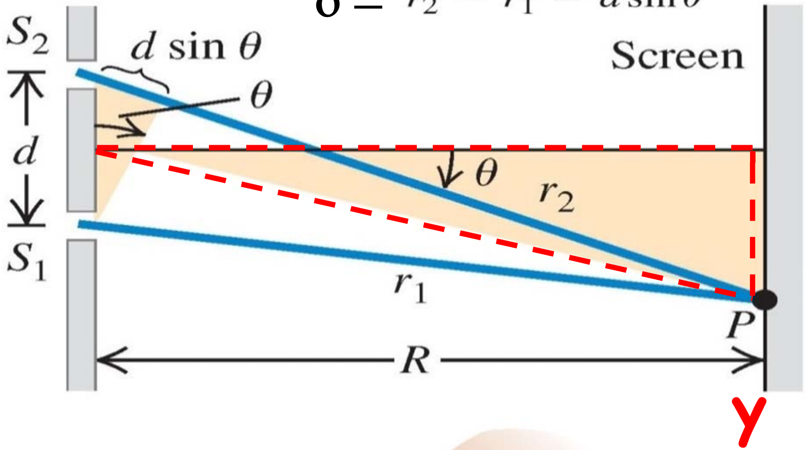
https://www.youtube.com/watch?v=j-u3IEgcTiQ

SUMMARY

Interference

Interference: Summary

Young's double-slit experiment



Constructive interference:

$$d\sin\theta = m\lambda \qquad y_m = R\frac{m\lambda}{d} \qquad m = 0, \pm 1 \ldots$$

Destructive interference:

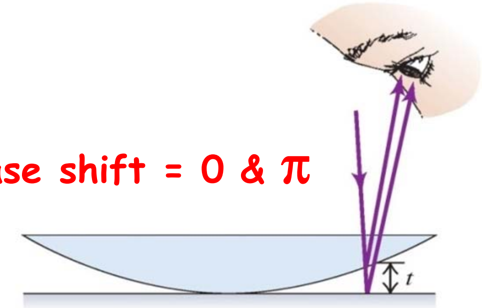
$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = 0, \pm 1 \ldots.$$

$$I = I_0 \cos^2\frac{\phi}{2}$$

$$\phi = \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d y}{\lambda R}$$

Thin film & Newton's rings

Phase shift = 0 & π



Constructive reflections:

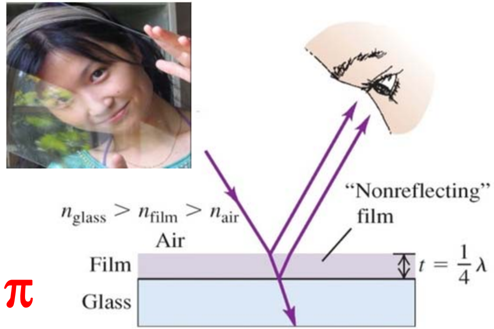
$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \ldots)$$

Destructive reflections:

$$2t = m\lambda \qquad (m = 0, 1, 2, \ldots)$$

Interference: Summary

Non-reflecting coating

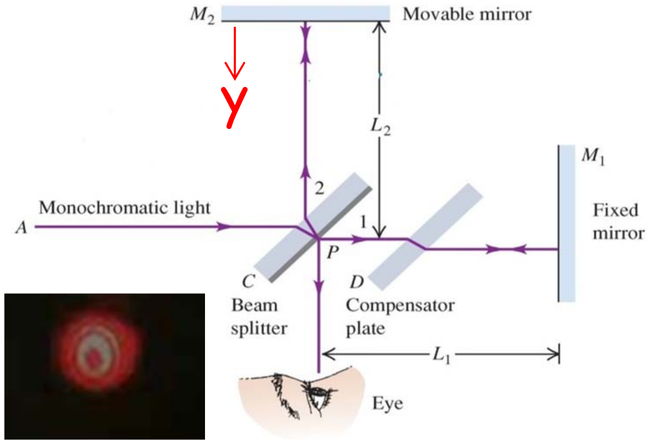


$t = \lambda_{film} / 4$

$\lambda_{air} = \lambda_{film} n_{film}$

Phase shift = π & π

The Michelson Interferometer



$$y = m\frac{\lambda}{2}$$

$$\lambda = \frac{2y}{m}$$