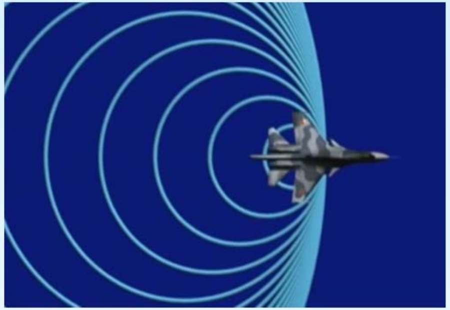
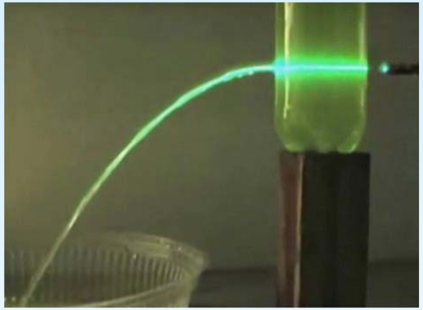
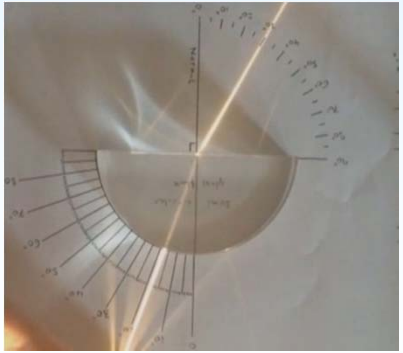
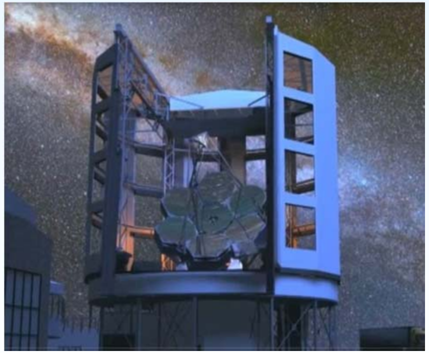
Wavemechanics and optics





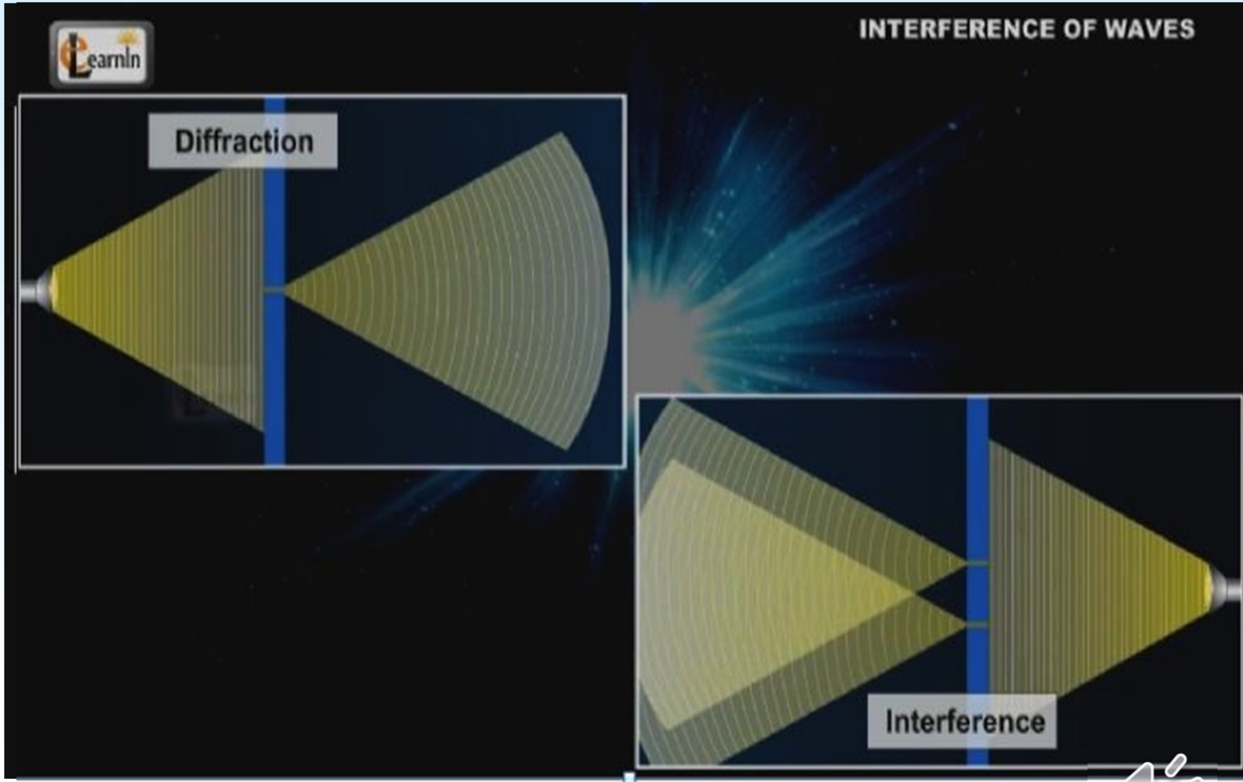
Chapter 36 - Diffraction

Vincent Hedberg - Lunds Universitet

Innehåll

Diffraction

Part 1.
Diffraction



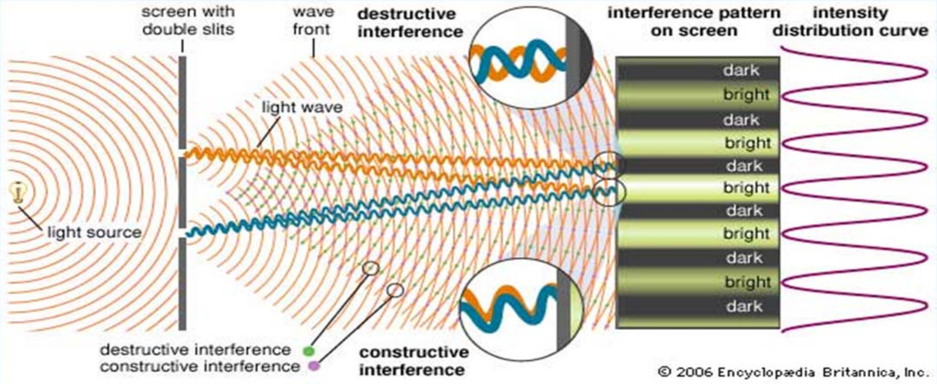
https://www.youtube.com/watch?v=CAe3lkYNKt8

Diffraction

https://www.youtube.com/watch?v=egRFqSKFmWQ

Diffraction

Interference:
Double slit
experiment



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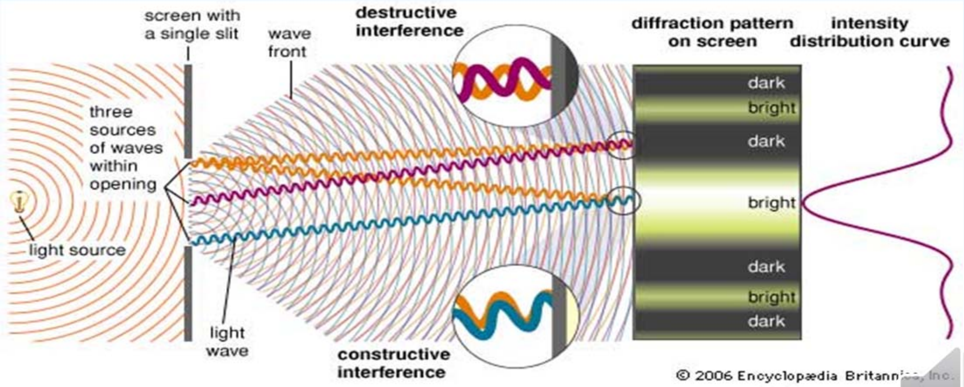
Diffraction:
single slit
experiment



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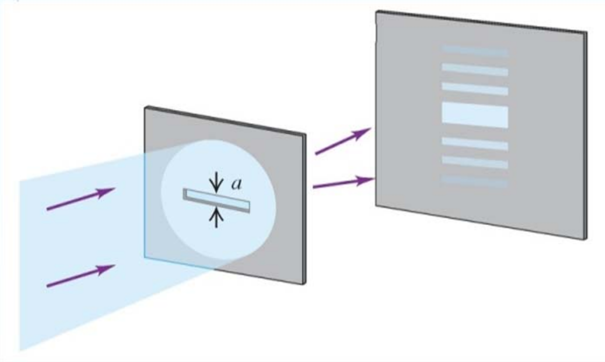
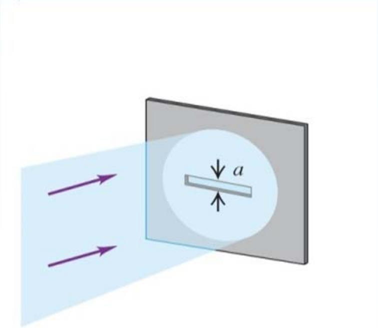
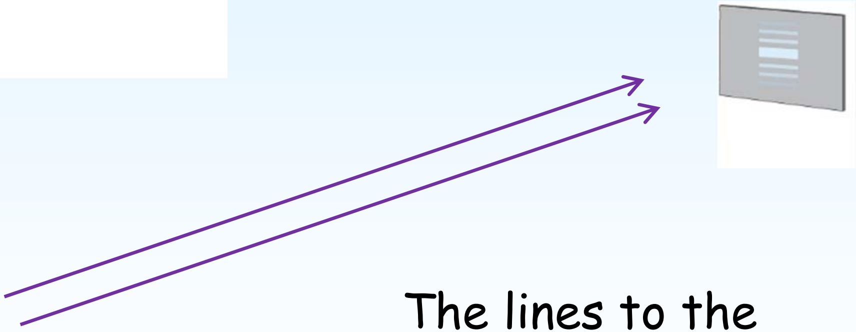
Diffraction

Fresnel diffraction or near-field diffraction.



Frauenhofer diffraction or far-field diffraction.



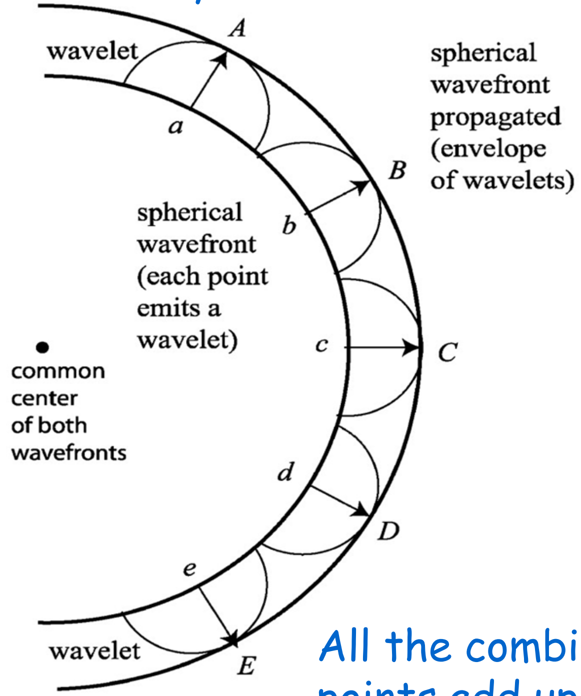


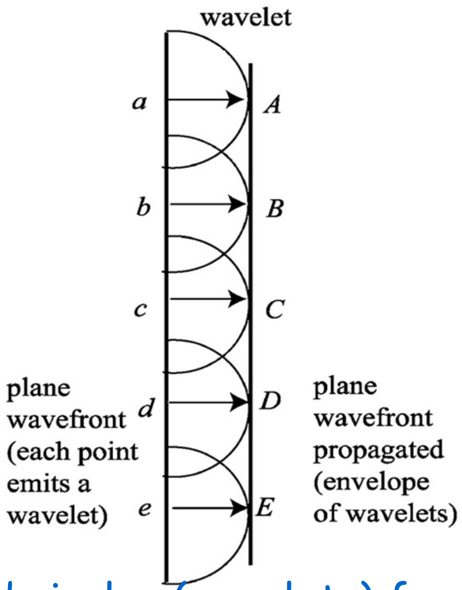
The lines to the screen are assumed to be parallel

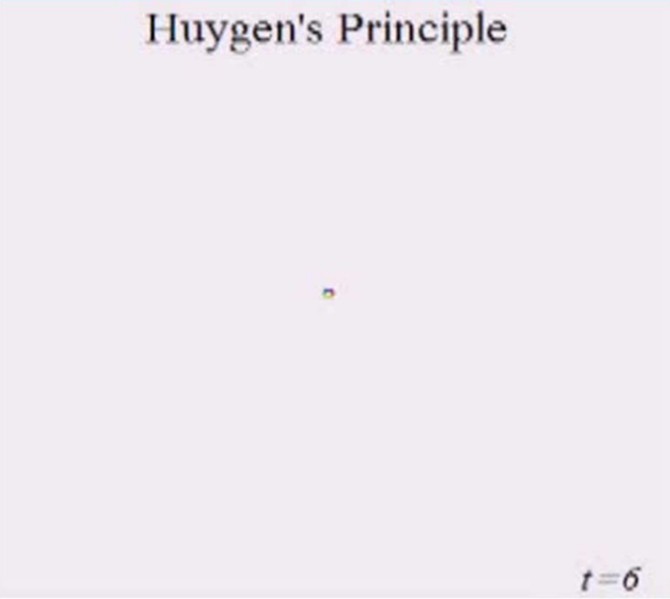
Diffraction

Huygen's principle

Each point in a wavefront is regarded as a new source of secondary wavelets.

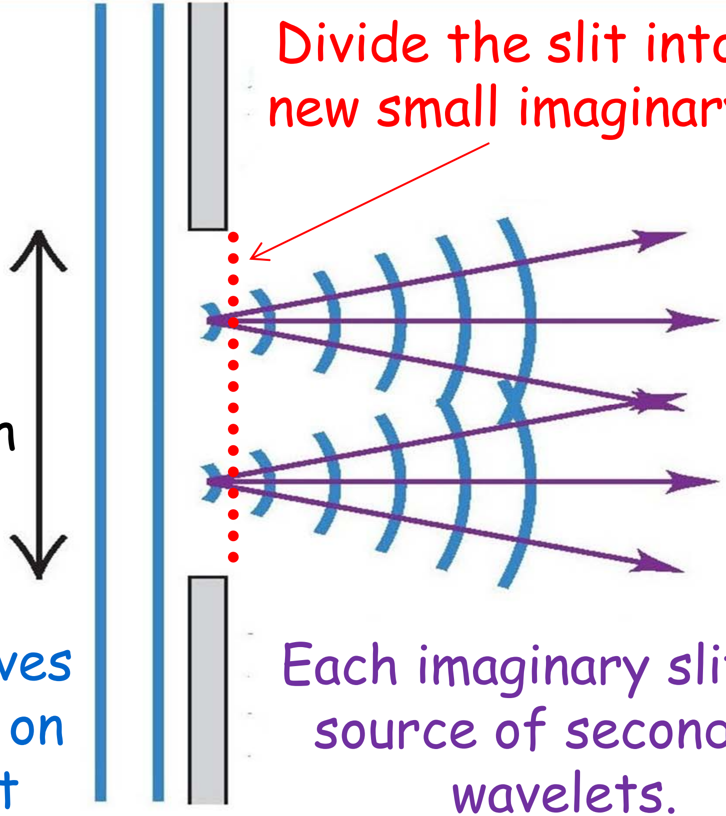






All the combined circles (wavelets) from all the points add up to create the new wavefronts.

Diffraction

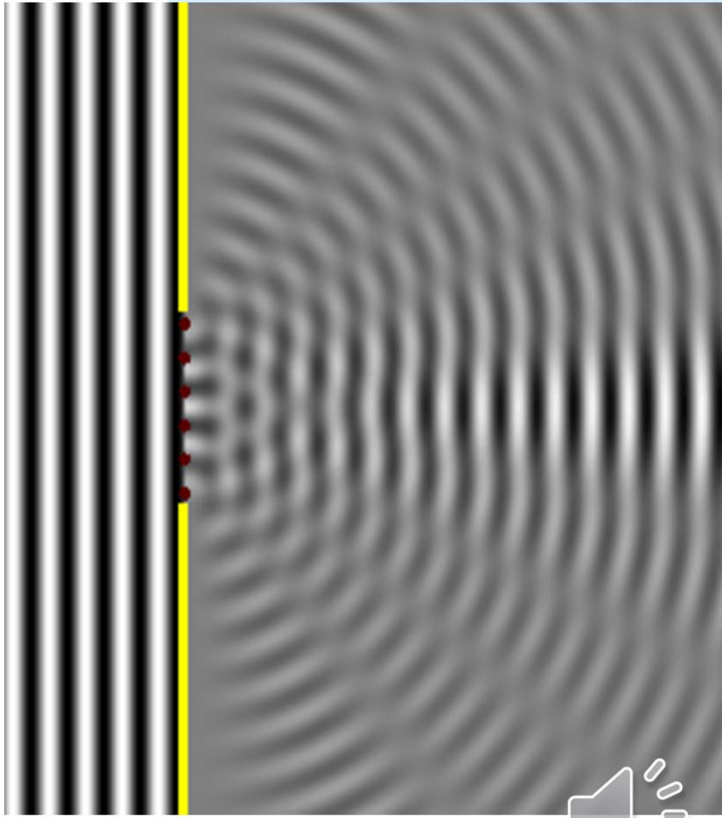


Divide the slit into many new small imaginary slits.

a = slit width

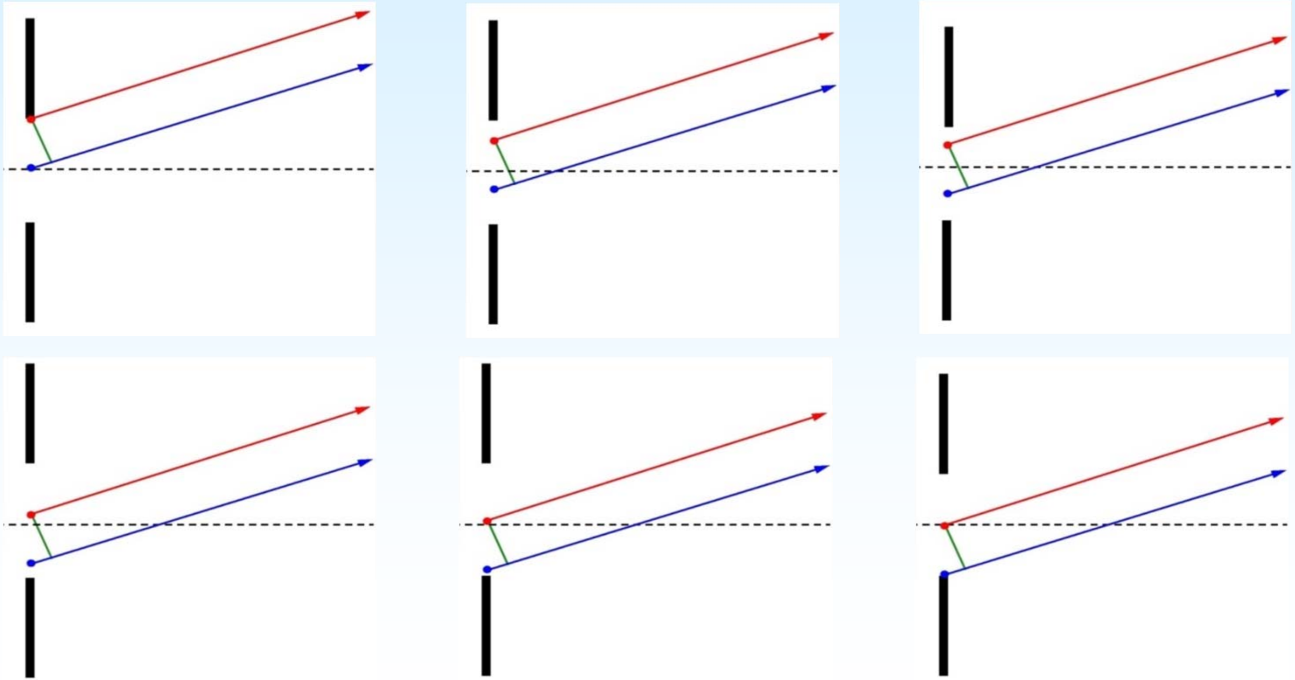
Plane waves incident on the slit

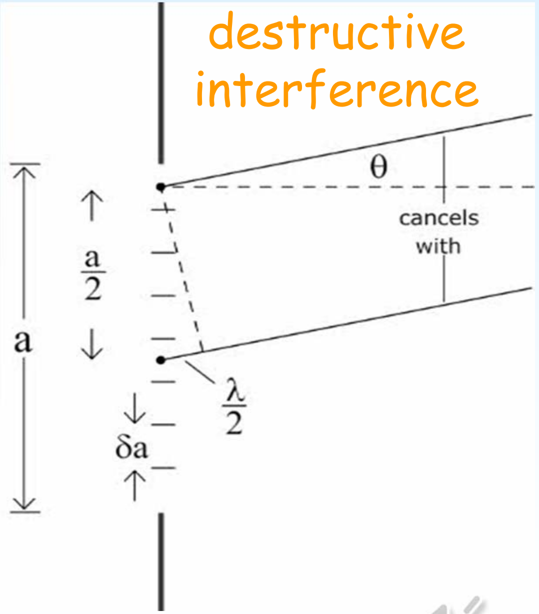
Each imaginary slit is a source of secondary wavelets.



Diffraction

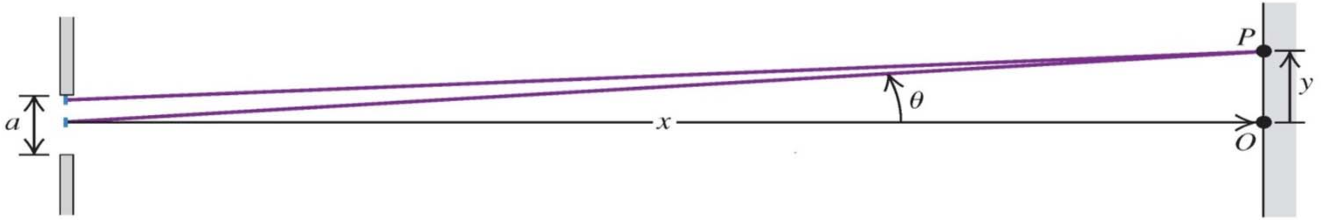
For every point in the top half of the slit there is a corresponding point in the bottom half.





destructive interference

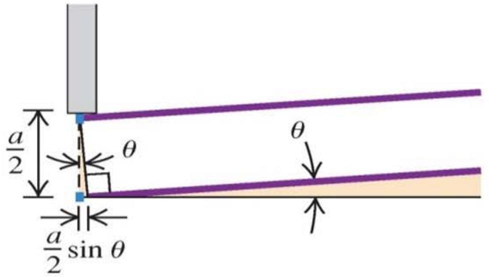
Diffraction



Geometry:

$$\tan(\theta) = y / x$$

Enlarged view of the top half of the slit



Destructive interference:

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}\,\mathbf{m}$$

$$\sin(\theta) = m\lambda/a \qquad m = \pm 1, \ \pm 2,$$

Small angles:

$$\tan(\theta) \approx \theta$$

$$\sin(\theta) \approx \theta$$

$$y_m = x\frac{m\lambda}{a} \qquad (\text{for } y_m \ll x)$$

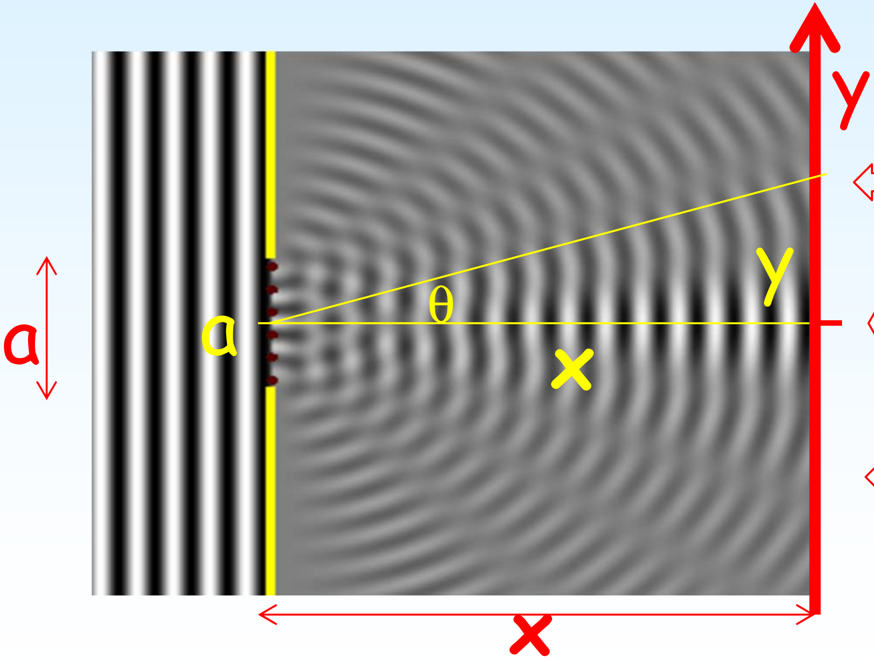
$$m = \pm 1, \ \pm 2,$$

Diffraction

$$tan(\theta) = \frac{y}{x} \approx sin(\theta)$$

Destructive Interference:

$$y_m = x\frac{m\lambda}{a} \qquad (\text{for } y_m \ll x)$$



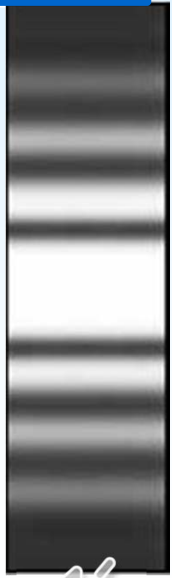
Minimum

$$\frac{a}{2}\sin\theta = +\frac{\lambda}{2} \Rightarrow y \approx x\frac{\lambda}{a}$$

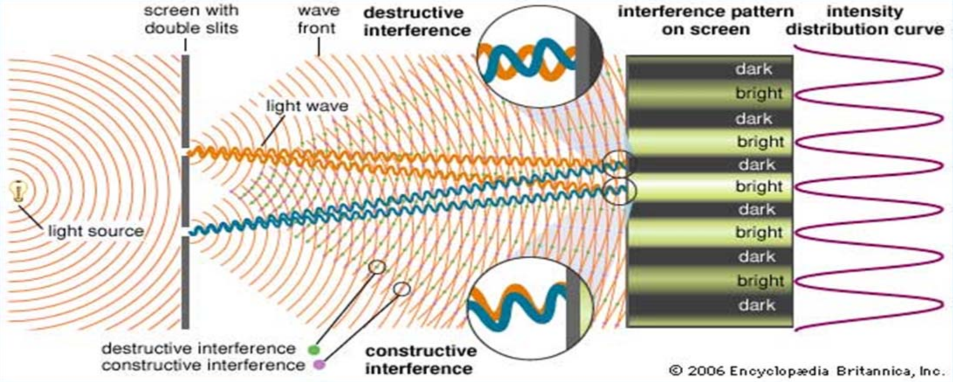
Broad central maximum

Minimum

$$\frac{a}{2}\sin\theta = -\frac{\lambda}{2} \Rightarrow y \approx -x\frac{\lambda}{a}$$



Diffraction



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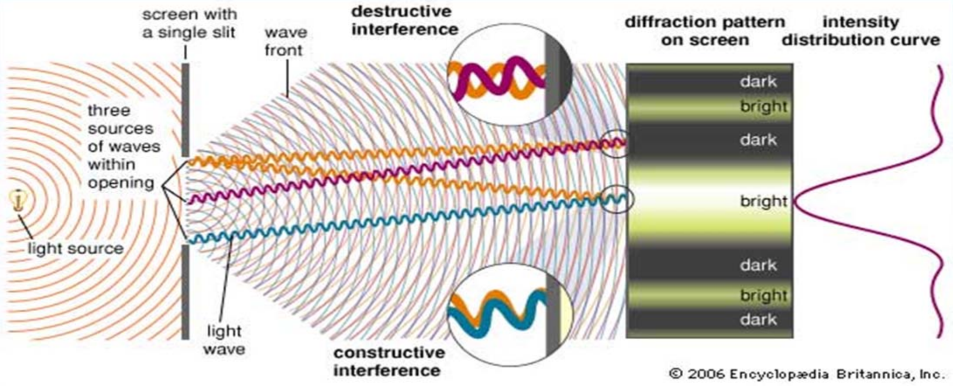
Bright bands:

Distance to screen
Wavelength
Distance between slits

$$y_m = R\frac{m\lambda}{d}$$

$$m = \mathbf{0},\quad \pm 1,\quad \pm 2,$$



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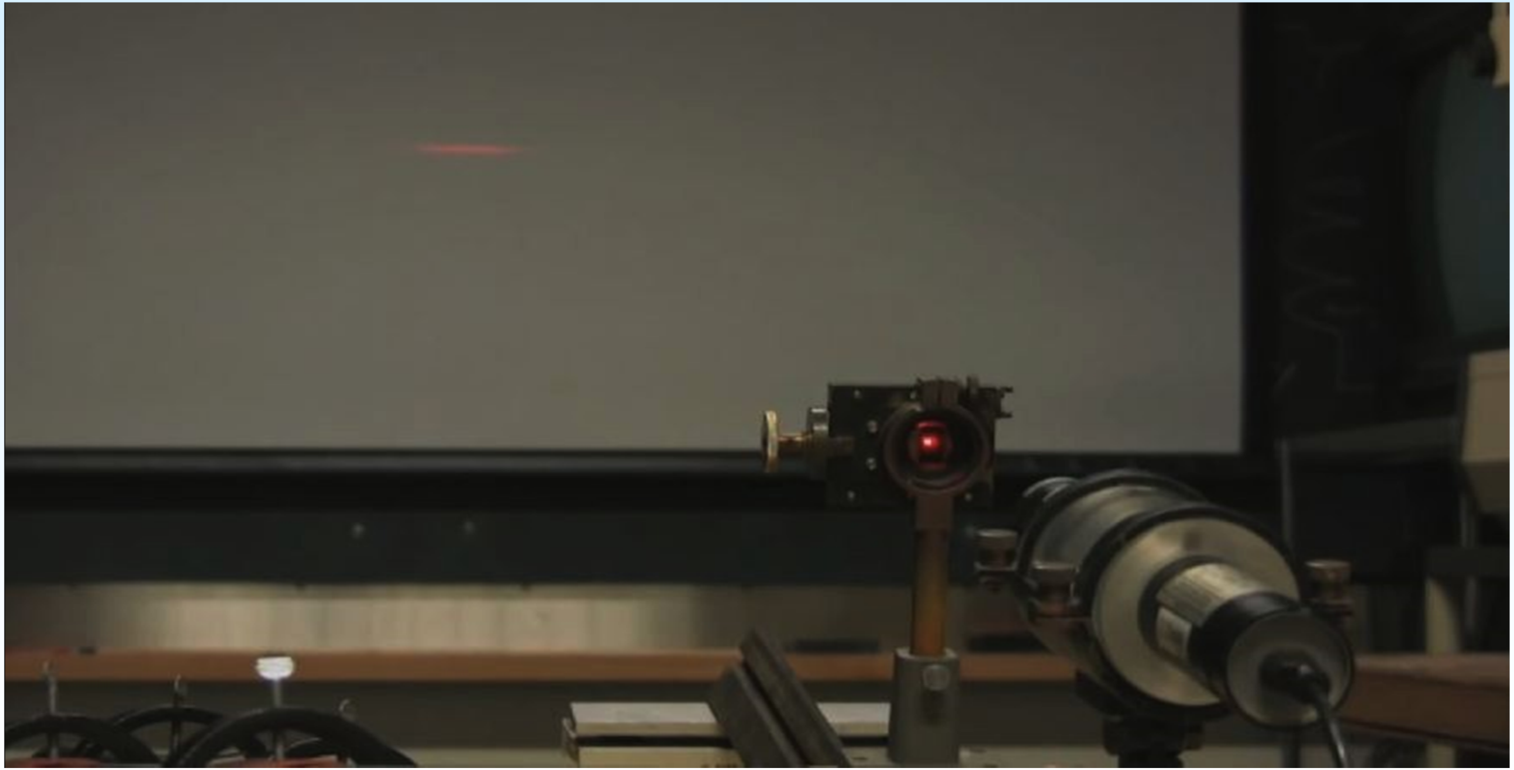
Dark bands:

Distance to screen
Wavelength
Slit width

$$y_m = x\frac{m\lambda}{a}$$

$$m = \pm 1,\quad \pm 2,$$

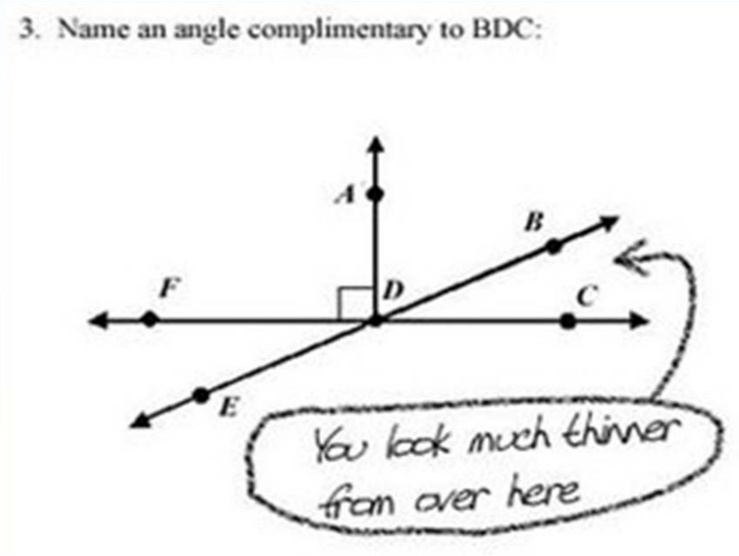
Diffraction



https://www.youtube.com/watch?v=9D8cPrEAGyc

Diffraction: Problem

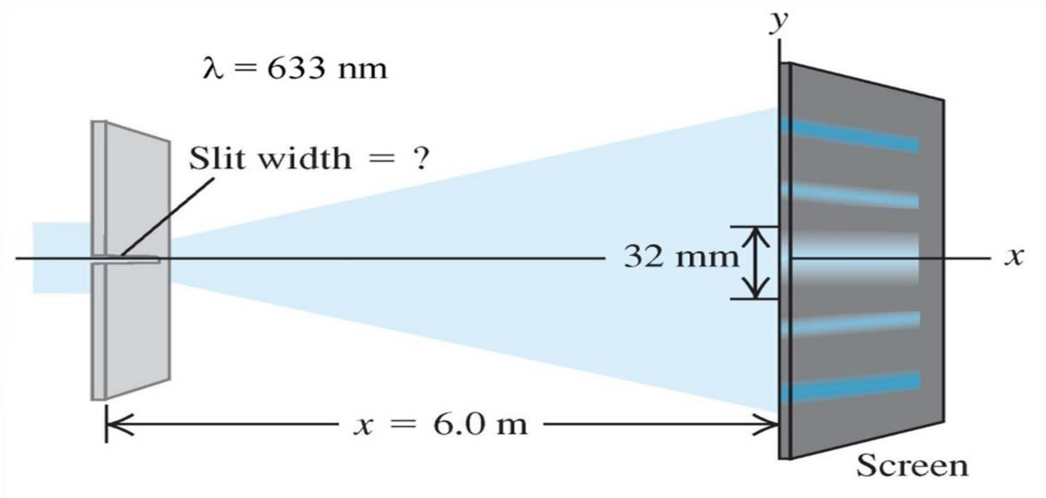
Part 2. Problems



3. Name an angle complimentary to BDC:

Diffraction: Problem

What is the slit width ?

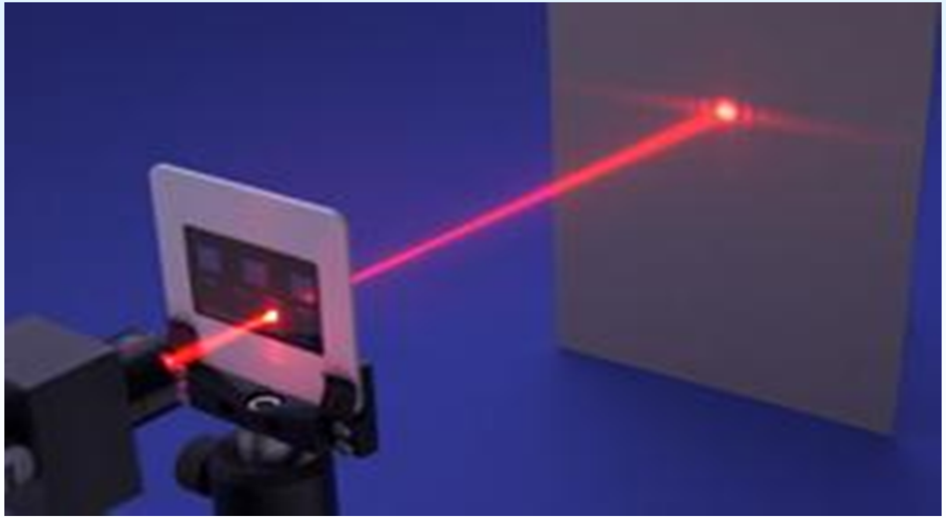


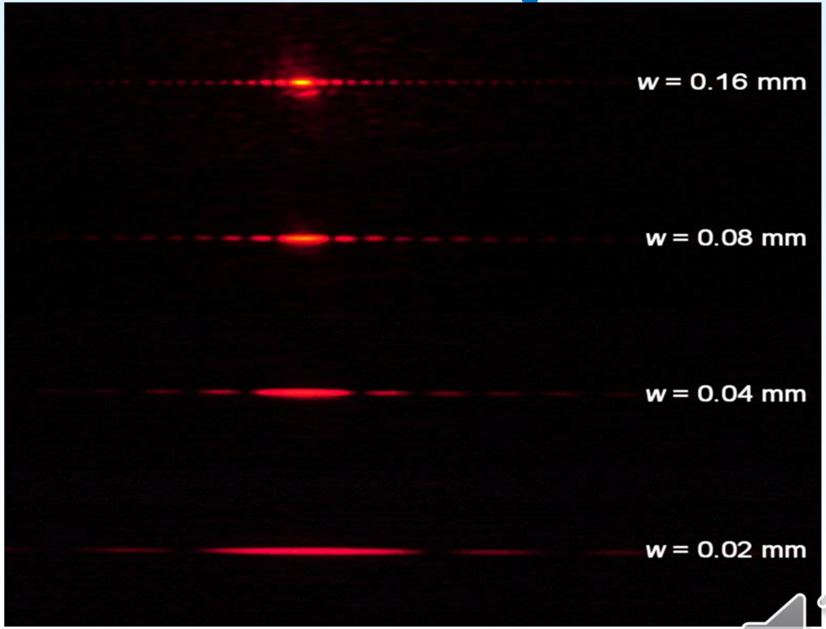
$$y_m = x\frac{m\lambda}{a}$$

$$y = (32\text{ mm})/2 = 16\text{ mm}$$

$$a = \frac{x\lambda}{y} = \frac{(6.0\text{ m})(633 \times 10^{-9}\text{ m})}{16 \times 10^{-3}\text{ m}} = 2.4 \times 10^{-4}\text{ m} = 0.24\text{ mm}$$

Part 3. Intensity





Diffraction: Intensity

The intensity of light (I) is proportional to the square of the amplitude of the total electric field (E_p)

$$I \sim {E_p}^2$$

So what is E_p ?

Diffraction: Intensity

Strategy for the intensity calculation

Task 1:

Use phase vectors to calculate the total amplitude E_p of the electric field after a superposition of all interfering waves.

Task 2:

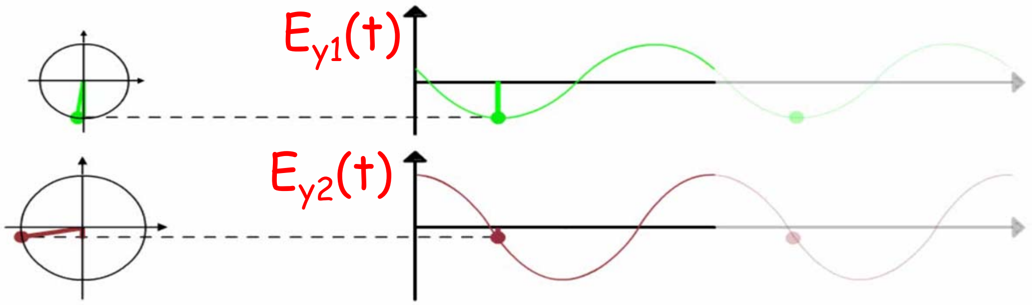
Put the new E_p into the formula: $I \sim E_p^2$

Task 3:

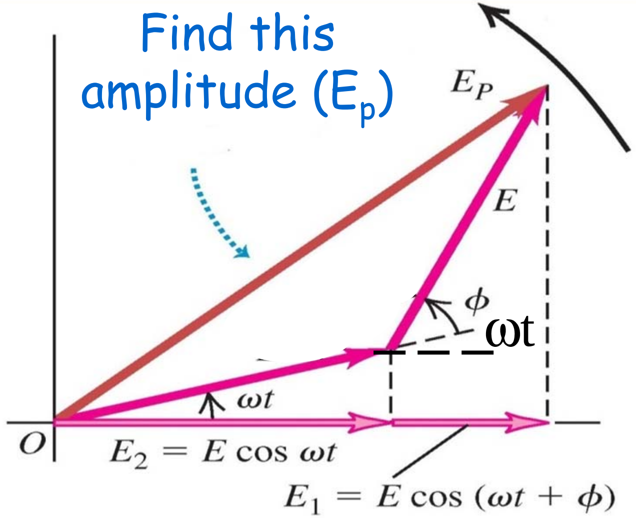
Derive a relationship between intensity and a, y, λ and x.

Interference: Fas vektorer

Interference of two waves



$E_y(t) = E_{y1}(t) + E_{y2}(t)$



Find this amplitude (E_p)

The total wave can be described by the projection on the x-axis:

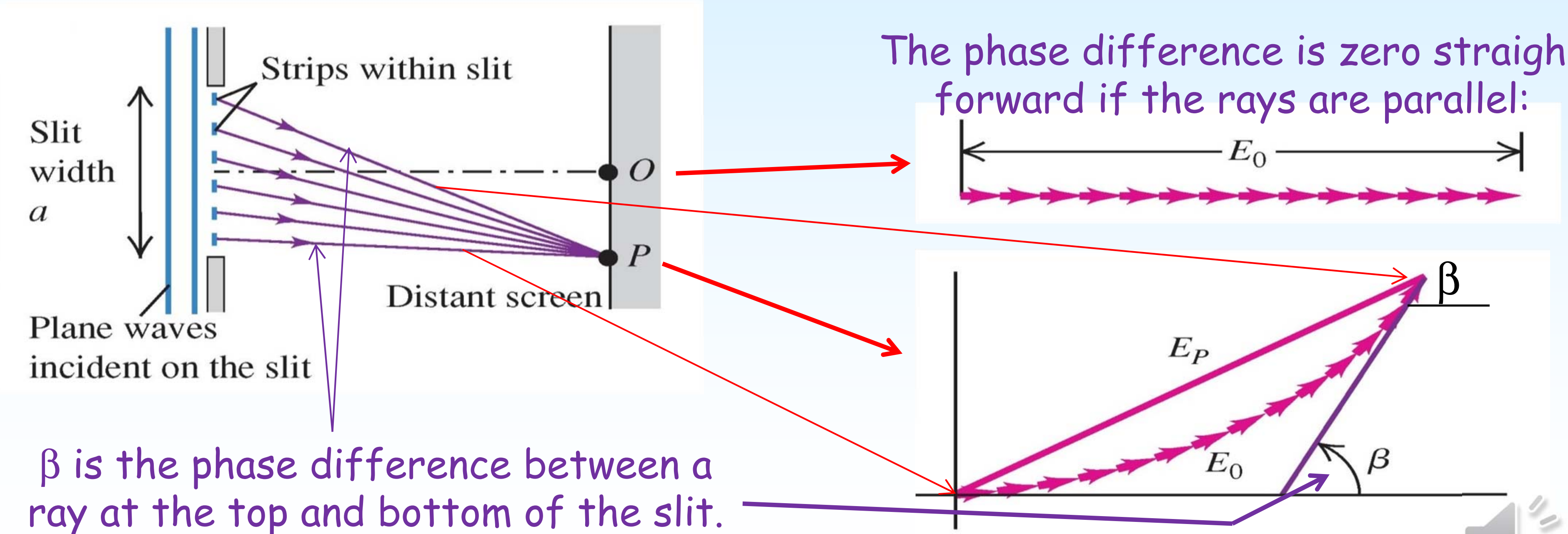
$$E_1(t) = E\cos(\omega t + \phi)$$

$$E_2(t) = E\cos\omega t$$

$$E(t) = E_1(t) + E_2(t)$$

Diffraction: Intensity

Assume many small phase vectors with an amplitude E_0 are giving the total electric field strength (E_p) in a point P:

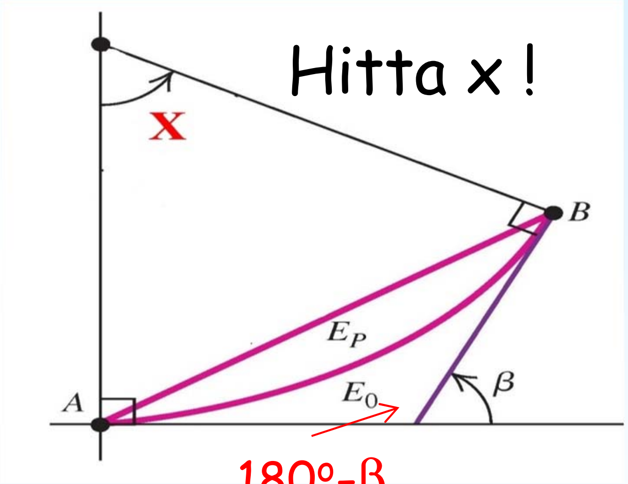


The phase difference is zero straight forward if the rays are parallel:

β is the phase difference between a ray at the top and bottom of the slit.

Diffraction: Intensity

Step 1



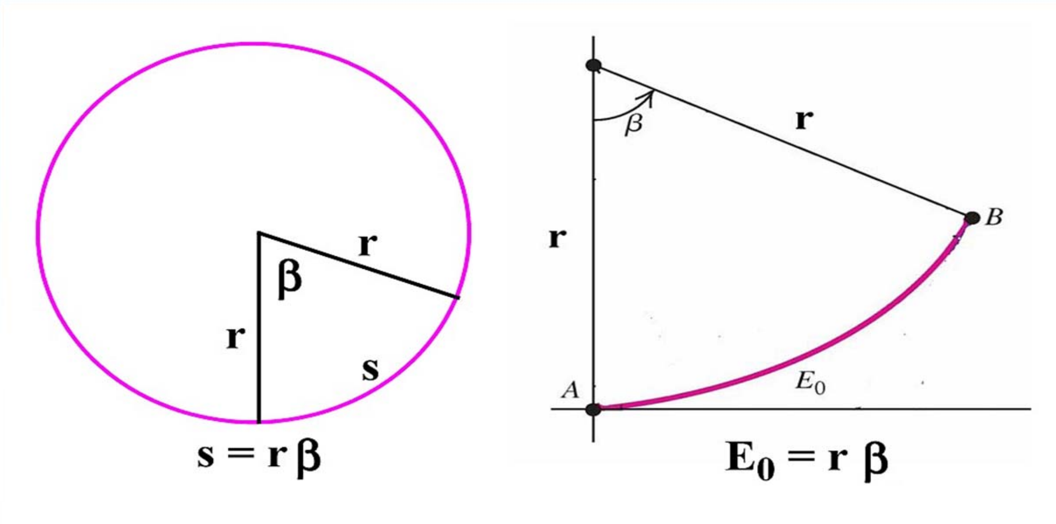
180°-β

Quadrangle = 360°

$180° - \beta + 90° + X + 90° = 360°$

$X = \beta$

Step 2 Find r from the perimeter of a circle segment !



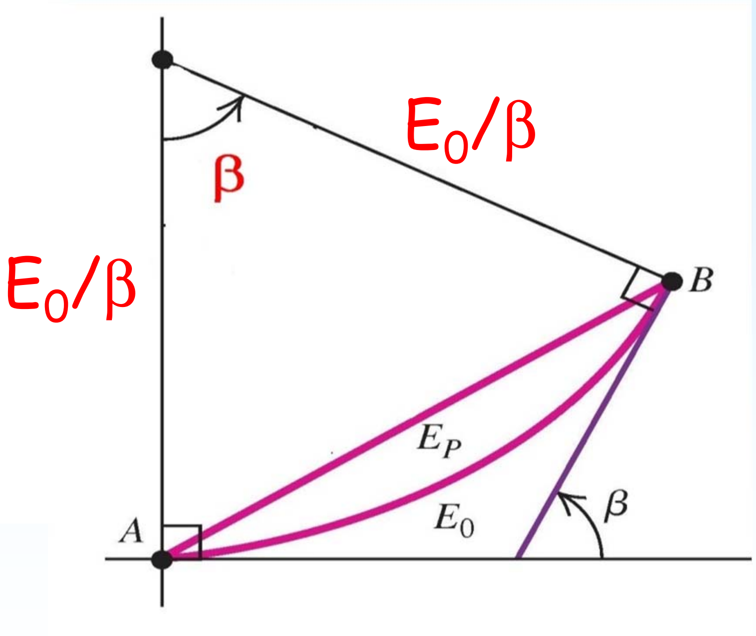
$s = r\beta$

$E_0 = r\beta$

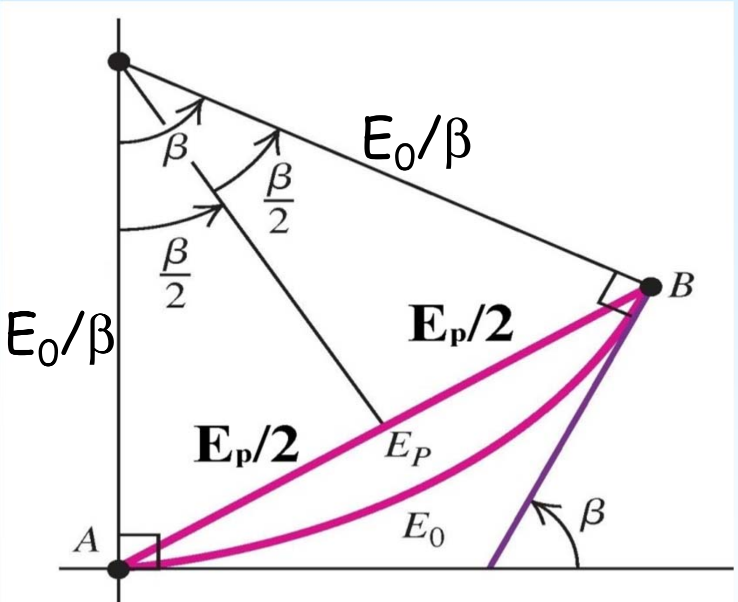
$r = E_0/\beta$

Diffraction: Intensity

Step 1 & 2



Step 3



Divide the triangle into two halves and take the sine:

$$\sin(\beta/2) = (E_p/2) \,/\, (E_0/\beta)$$

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

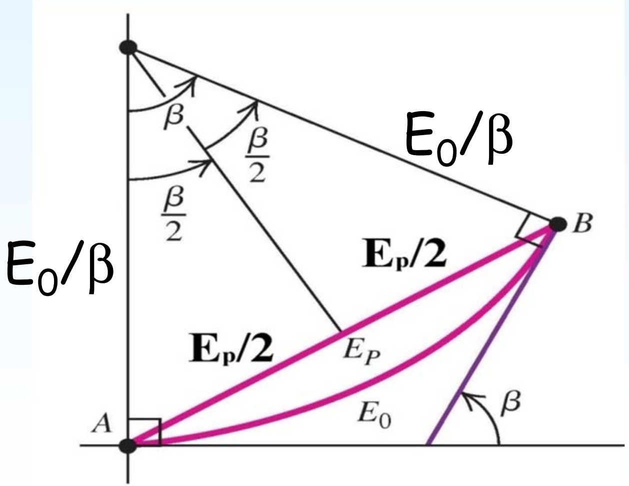
Diffraction: Intensity

$$I \sim E_p^2$$

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

Intensity

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$



But what is β ?

(the phase shift between a ray from the top and bottom of the slit)

Diffraction: Intensity

Task 1:

Use phase vectors to calculate the total amplitude E_p of the electric field after a superposition of all interfering waves.

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

Task 2:

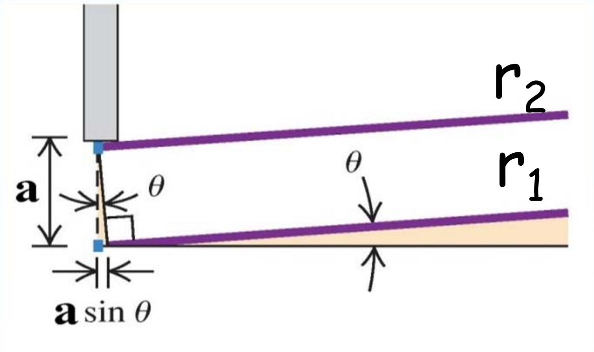
Put the new E_p into the formula: $I \sim E_p^2$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Task 3:

Derive a relationship between intensity and a, y, λ and x.

Diffraction: Intensity



Path difference:

$r_2 - r_1 = a \sin(\theta)$

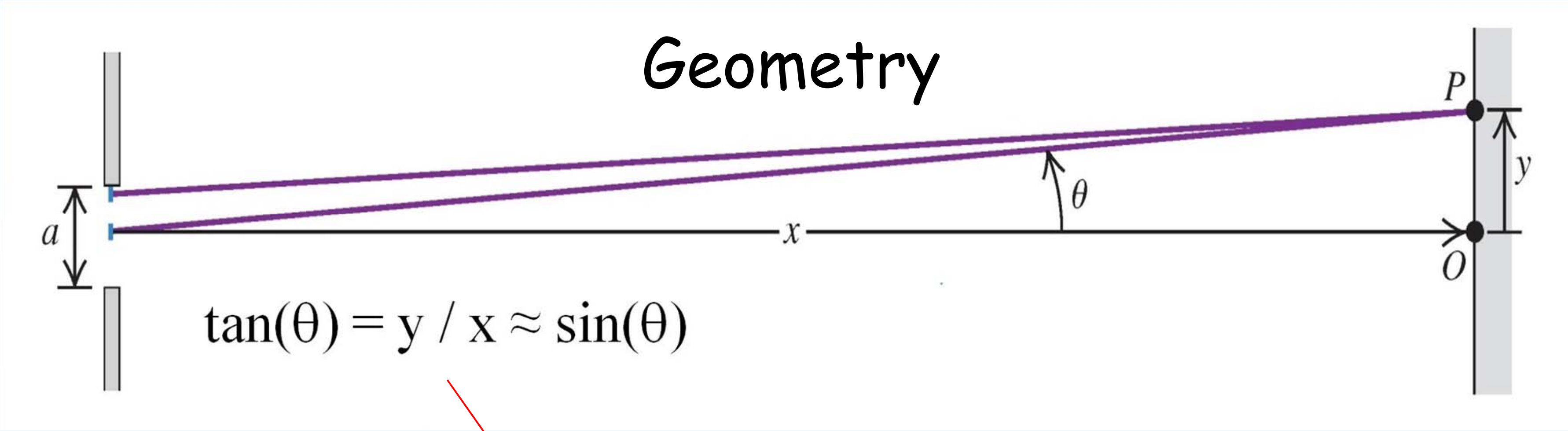
A path difference of one wavelength corresponds to a phase difference of 2π

$$\frac{\beta}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

$$\boxed{\beta = \frac{2\pi}{\lambda} a \sin\theta}$$

r_2-r_1 is the path difference between a ray at the top and bottom of the slit.

Diffraction: Intensity

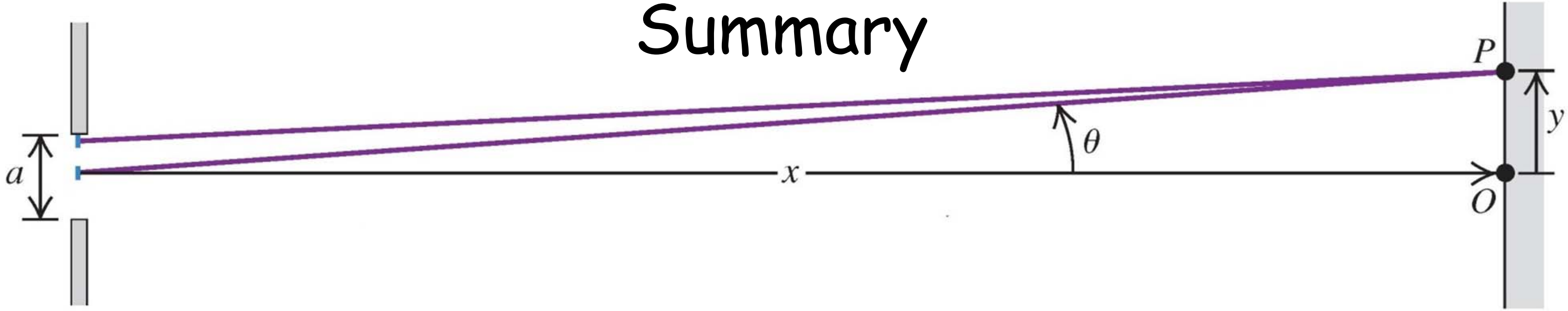


$\tan(\theta) = y / x \approx \sin(\theta)$

$$\boxed{\beta = \frac{2\pi}{\lambda} a \sin\theta} \Rightarrow \boxed{\beta = \frac{2\pi}{\lambda} a \sin(\theta) \approx \frac{2\pi}{\lambda} a \frac{y}{x}}$$

Diffraction: Intensity

Summary



$$I \sim E_p^2$$

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\beta = \frac{2\pi}{\lambda} a \sin(\theta) \approx \frac{2\pi}{\lambda} a \frac{y}{x}$$

Diffraction: Intensity

Task 1:

Use phase vectors to calculate the total amplitude E_p of the electric field after a superposition of all interfering waves.

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

Task 2:

Put the new E_p into the formula: $I \sim E_p^2$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

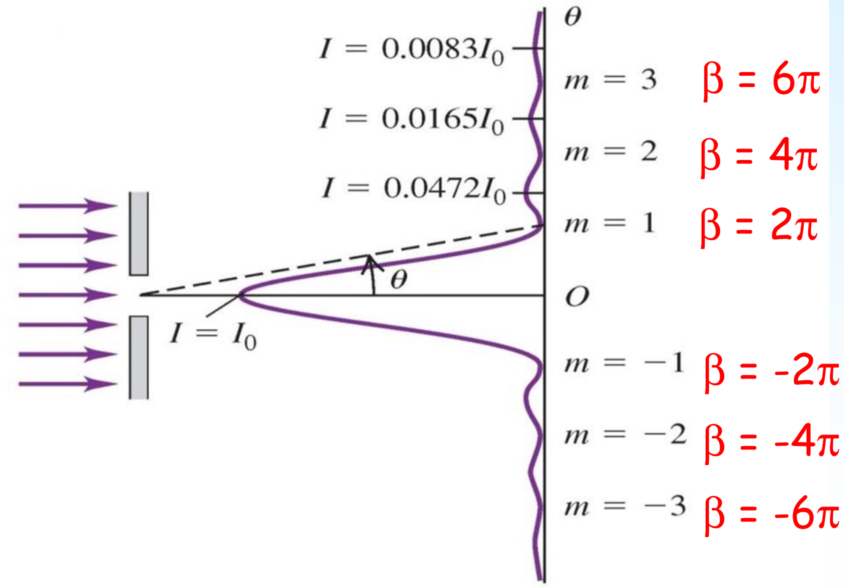
Task 3:

Derive a relationship between intensity and a, y, λ and x.

$$\beta = \frac{2\pi}{\lambda} a \sin(\theta) \approx \frac{2\pi}{\lambda} a \frac{y}{x}$$

Diffraction: Intensity

$$\boxed{I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2}$$



Destructive interference:

The intensity has a minimum for

$$0 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$$0 = \sin^2(\beta/2)$$

$$0 = \sin(\beta/2)$$

Not possible

$$\beta = \cancel{0}, 2\pi, 4\pi, 6\pi\ldots\ldots = \pm 2\pi\ m$$

This gives again:

$$y_m = x\frac{m\lambda}{a}$$

Constructive interference:

The intensity has a maximum for

$$\frac{\mathbf{dI}}{\mathbf{d\beta}} = \mathbf{0}$$

But the resulting equation has no analytical solution.

The peaks are close but not exactly at $\beta = 0, 3\pi, 5\pi, 7\pi\ldots.$

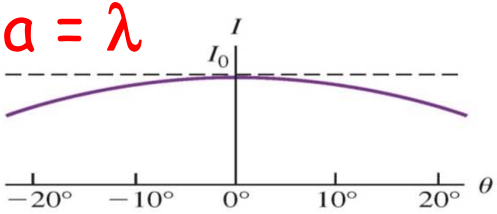
Diffraction: Intensity

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$ where $$\beta = \frac{2\pi}{\lambda}a\sin\theta$$

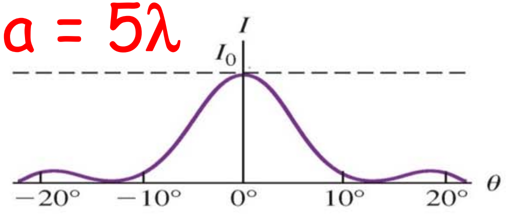
Dark bands: $\sin(\theta) = m\lambda/a$

$m = \pm 1, \ \pm 2,$

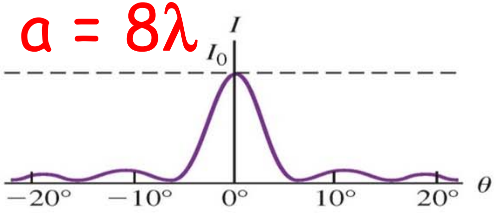
a = λ



a = 5λ



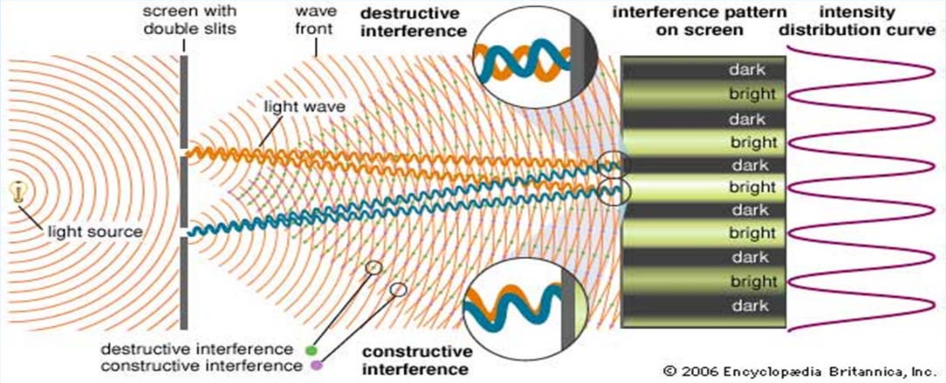
a = 8λ



If the width of the slit is equal or smaller than λ then only one broad maximum is observed.

A broader slit makes a narrower central peak.

Diffraction: Intensity



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First dark band:

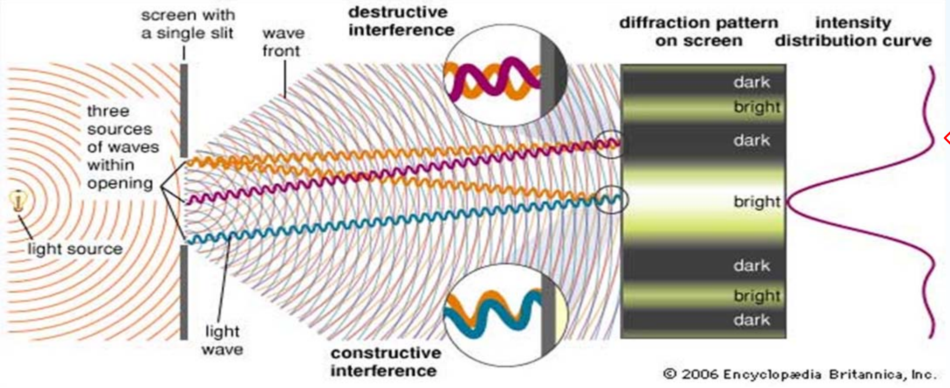
$\delta = \lambda/2$

$\phi = \pi$

$$I = I_0 \cos^2 \frac{\phi}{2}$$

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\tan(\theta) = y / R \approx \sin(\theta)$$



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$\delta = \lambda$

$\beta = 2\pi$

(top and bottom ray)

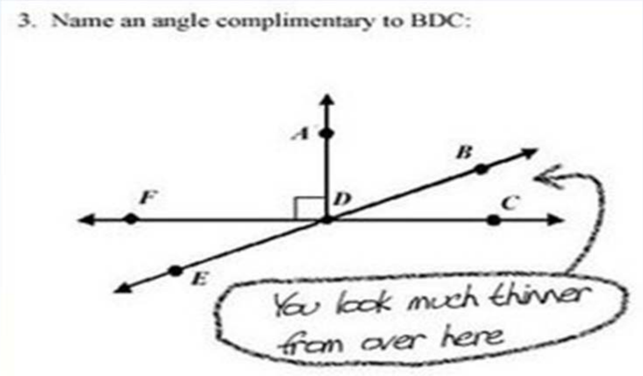
$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$\tan(\theta) = y / x \approx \sin(\theta)$$

Diffraction: Problem

Part 4. Problems



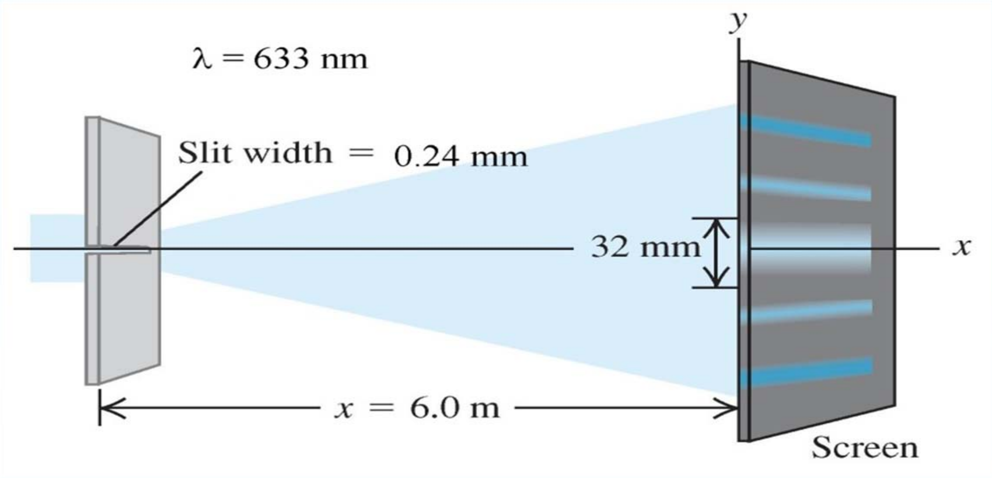
3. Name an angle complimentary to BDC:

You look much thinner from over here

Diffraction: Problem

The intensity in the central peak is I_0.

What is the intensity 3.0 mm away from this peak ?



λ = 633 nm
x = 6.00 m
a = 0.24 mm
y = 3.0 mm

$$\tan\theta = y/x = (3.0 \times 10^{-3}\ \mathrm{m})/(6.0\ \mathrm{m}) = 5 \times 10^{-4} = \sin(\theta)$$

$$\boxed{\beta = \frac{2\pi}{\lambda} a \sin\theta} = \frac{2\pi(2.4 \times 10^{-4}\ \mathrm{m})(5.0 \times 10^{-4})}{6.33 \times 10^{-7}\ \mathrm{m}} = 1.20\ \mathrm{rad}$$

$$\boxed{I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2} = I_0\left(\frac{\sin 0.60}{0.60}\right)^2 = 0.89 I_0$$

Diffraction: Problem

The intensity in the central peak in a single slit spectrum is I_0.

What is the intensity at a point where the phase difference between waves from the top and bottom of the gap is 66 radians ?

If this point is 7.0° from the central peak, how many wavelengths wide is the gap ?

$\beta = 66$ rad

$$\boxed{I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2}$$

$$I = I_0\left[\frac{\sin(33\ \text{rad})}{33\ \text{rad}}\right]^2 = (9.2 \times 10^{-4})I_0$$

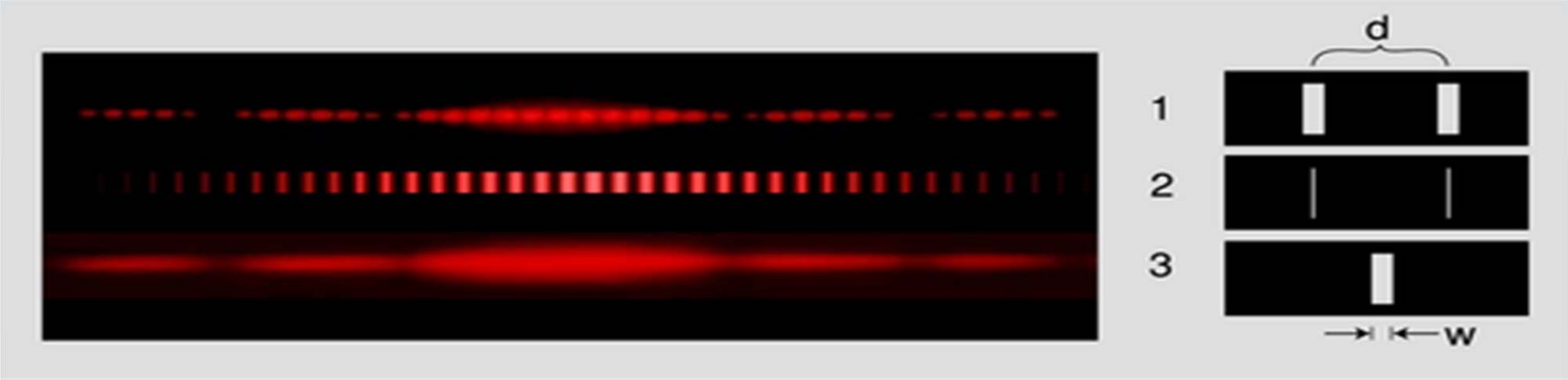
$\theta = 7.0^\circ$

$$\boxed{\beta = \frac{2\pi}{\lambda}a\sin\theta}$$

$$\frac{a}{\lambda} = \frac{\beta}{2\pi\sin\theta} = \frac{66\ \text{rad}}{(2\pi\ \text{rad})\sin 7.0^\circ} = 86$$

Diffraction: Two broad slits

Part 5. Two broad slits



Diffraction: Two broad slits

In the analysis of interference from two slits it was assumed that they were very narrow. What if they are broad ?

Two narrow slits:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

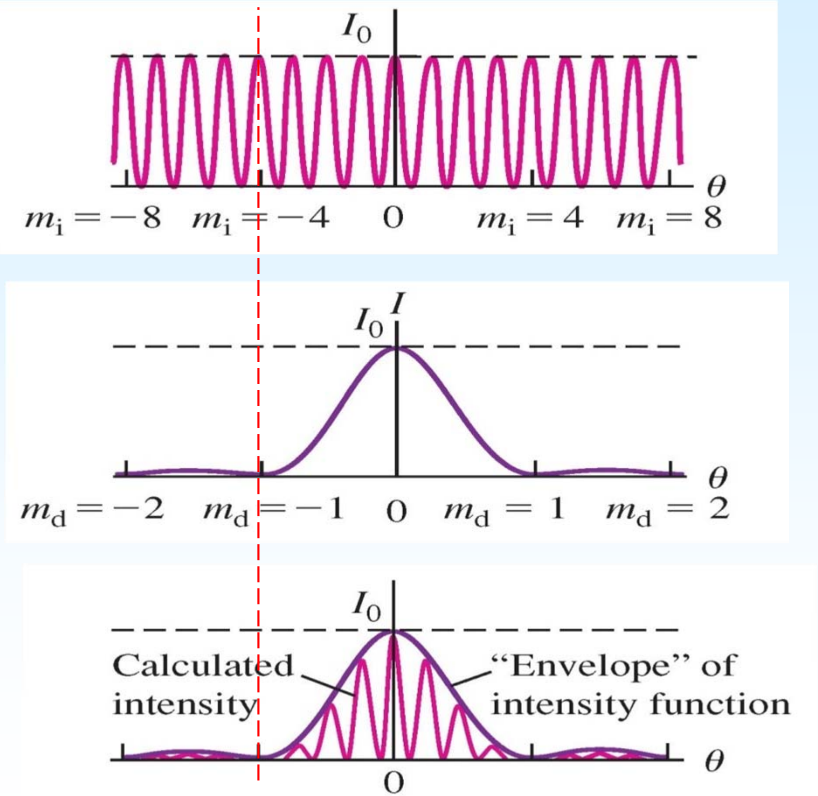
$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

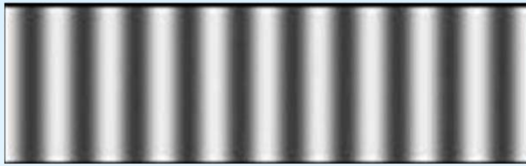
$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$

Diffraction: Two broad slits



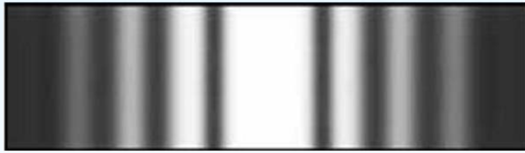
Two narrow slits:

$$I = I_0 \cos^2 \frac{\phi}{2}$$



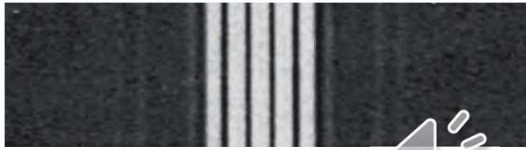
One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$



Two broad slits:

$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$



Diffraction: Two broad slits

Two narrow slits:

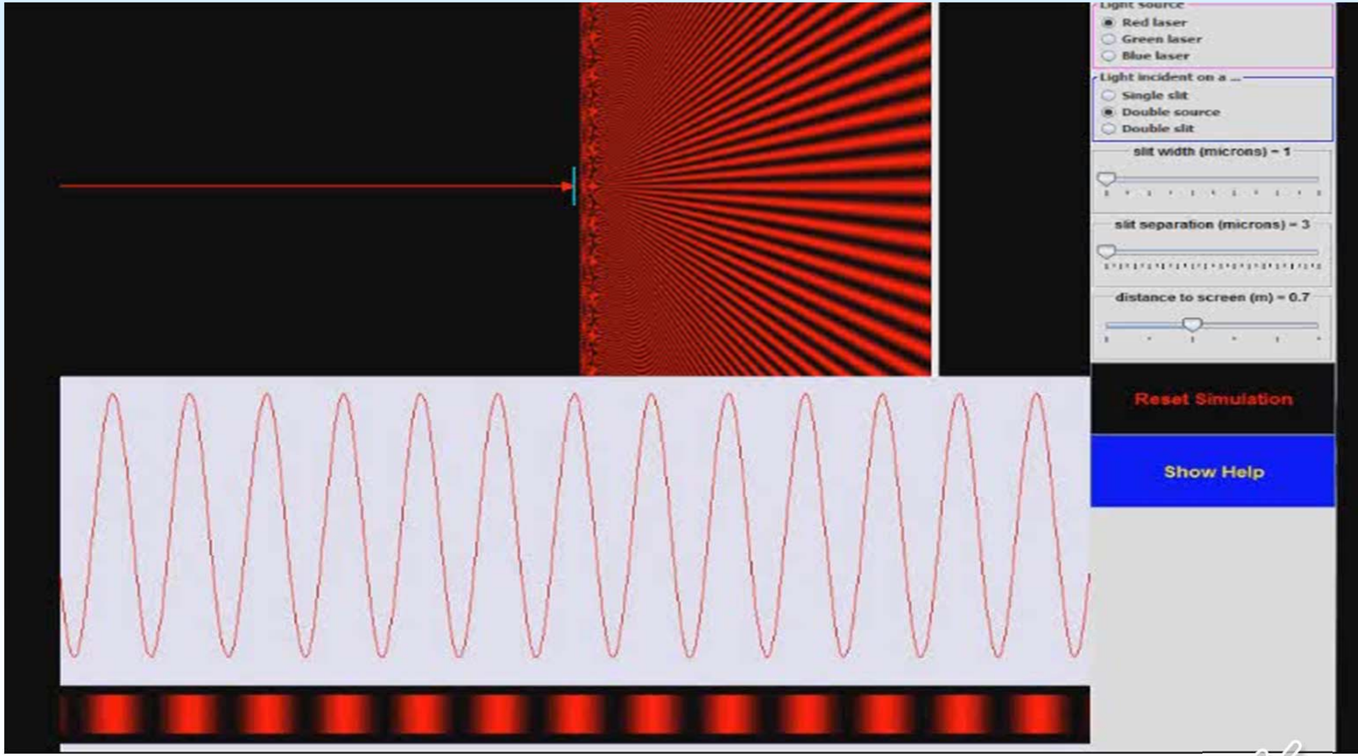
$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$



Light source
● Red laser
○ Green laser
○ Blue laser

Light incident on a ...
○ Single slit
● Double source
○ Double slit

slit width (microns) = 1

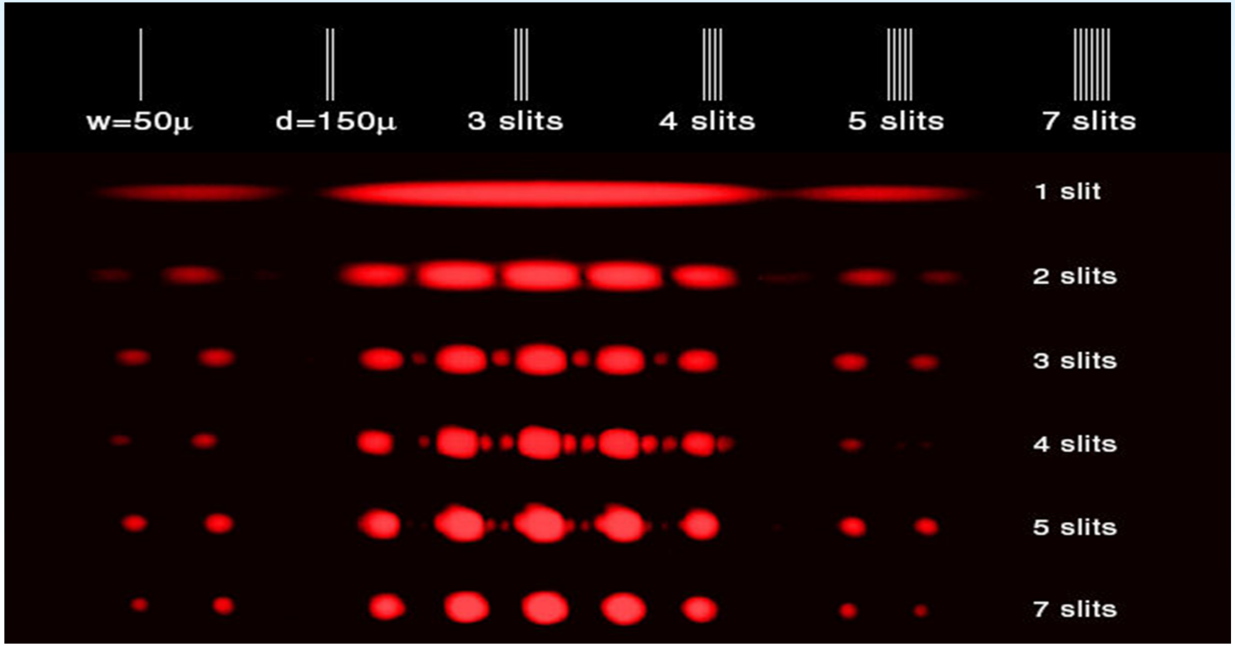
slit separation (microns) = 3

distance to screen (m) = 0.7

http://www.opensourcephysics.org/items/detail.cfm?ID=9988

Diffraction: Multiple slits

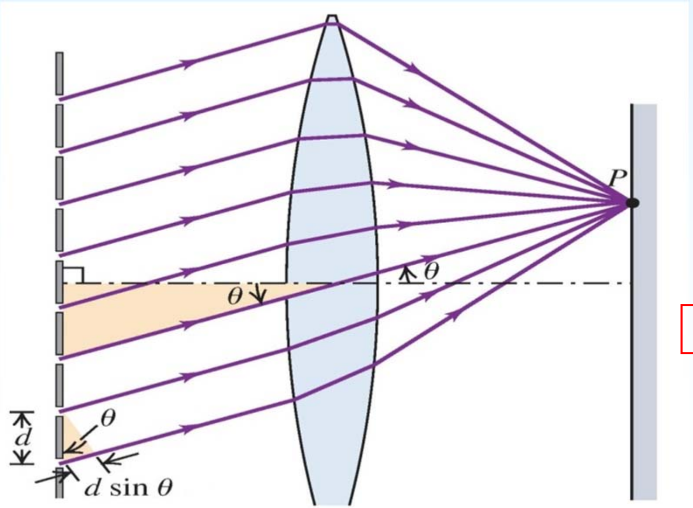
Part 6. Multiple slits

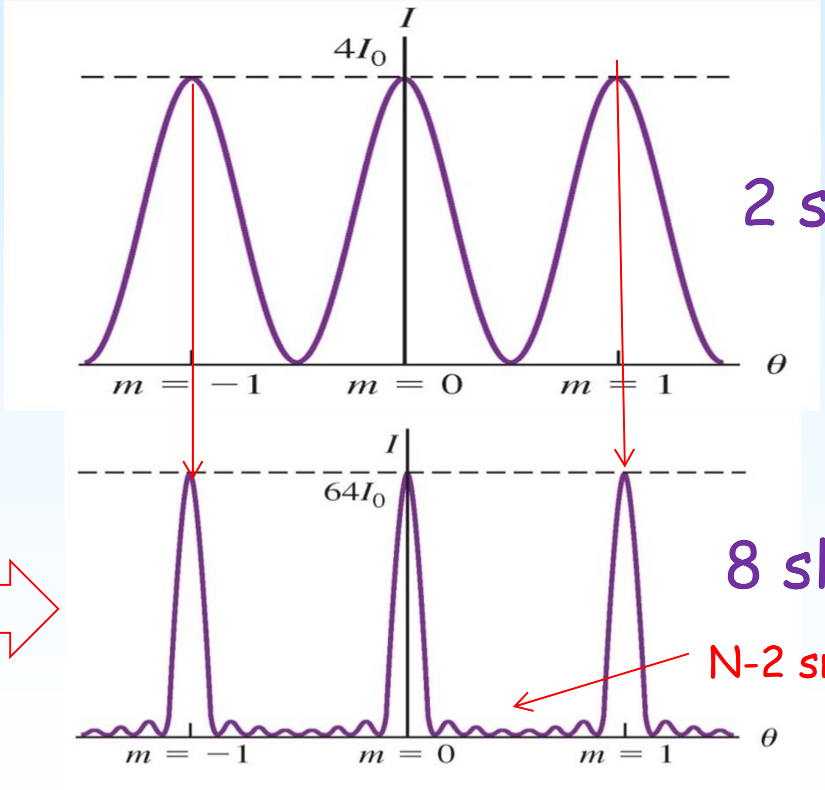


Diffraction: Multiple slits

The path difference between adjacent slits gives the principal peak intensity and is always:

$$d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \dots)$$





2 slits

8 slits

N-2 small peaks

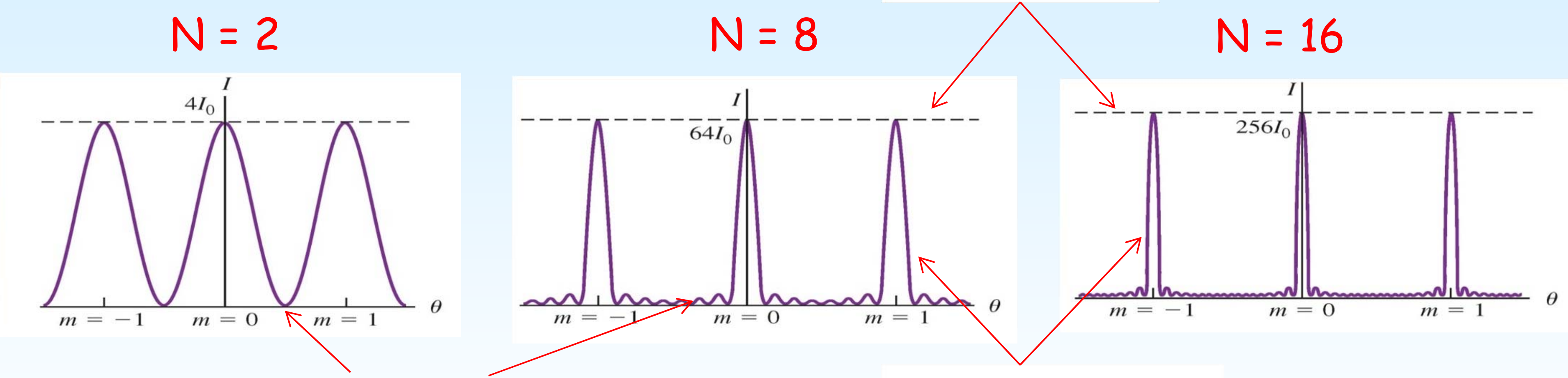
Diffraction: Multiple slits

$I_{max} \sim N^2$

N = 2

N = 8

N = 16



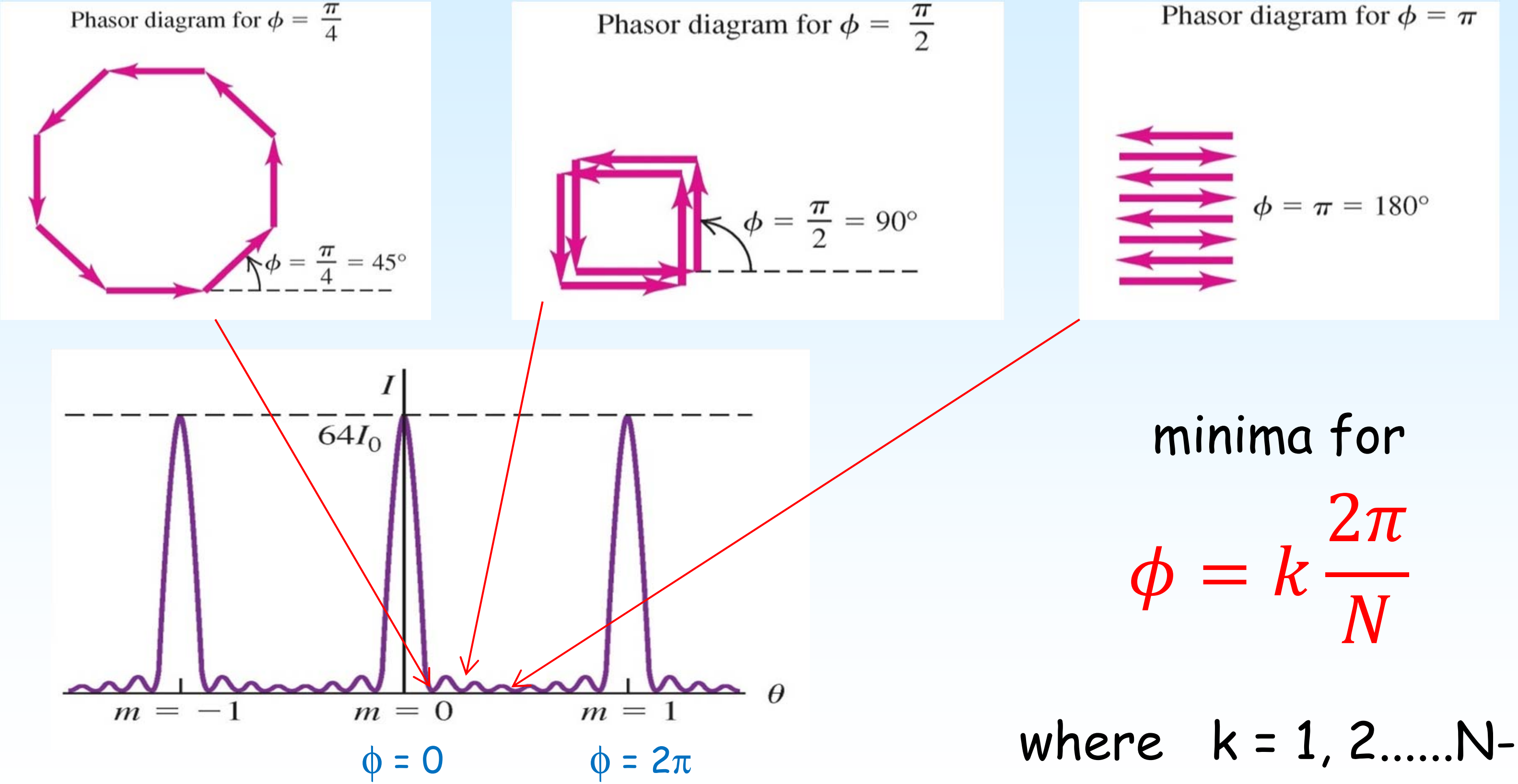
N-1 minima

$I_{width} \sim 1 / N$

Principal maxima: $d \sin\theta = m\lambda \qquad (m = 0, \pm1, \pm2, \pm3, \ldots)$

Diffraction: Multiple slits

N = 8

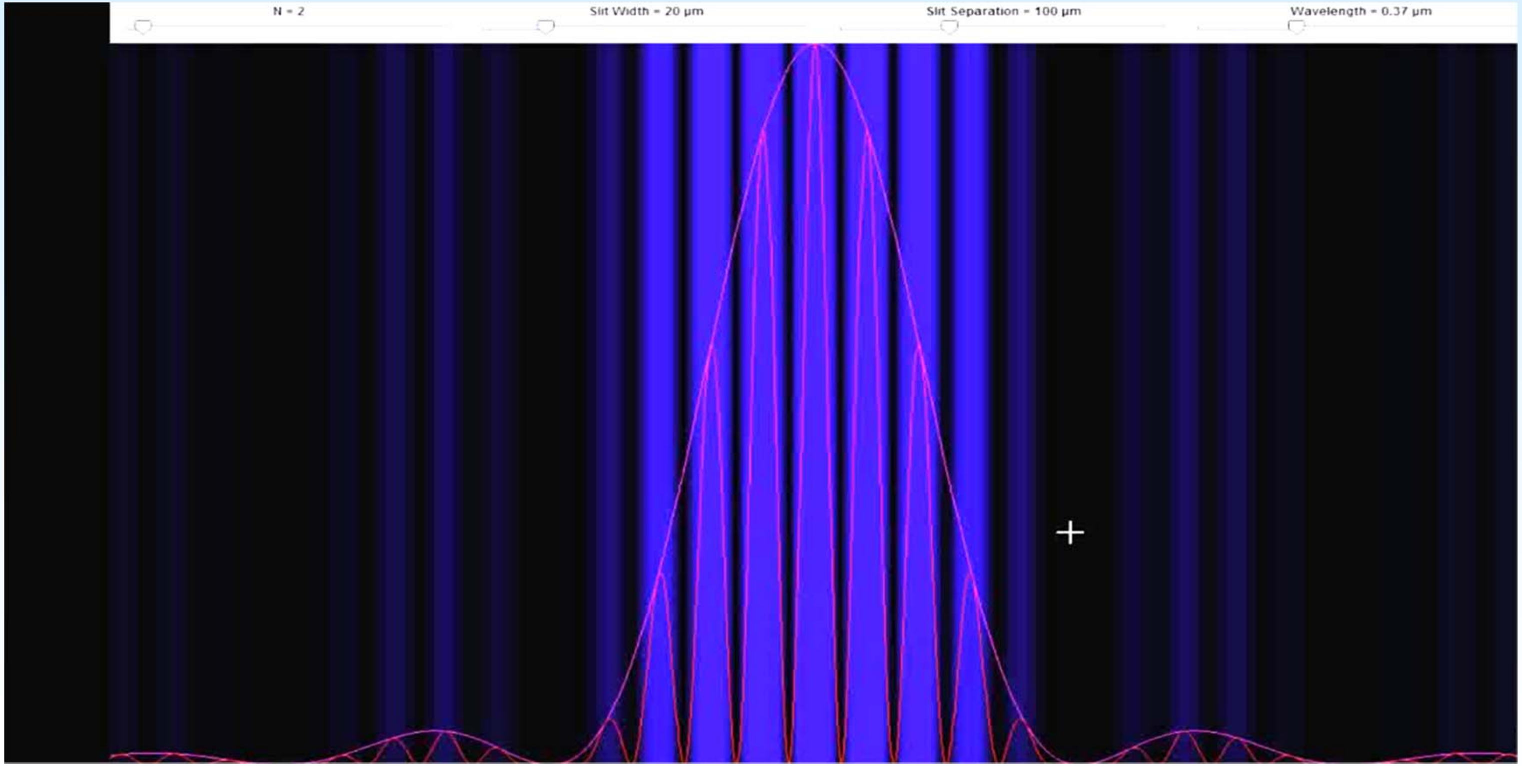


minima for

$$\phi = k\frac{2\pi}{N}$$

where k = 1, 2.....N-1

Diffraction: Multiple slits



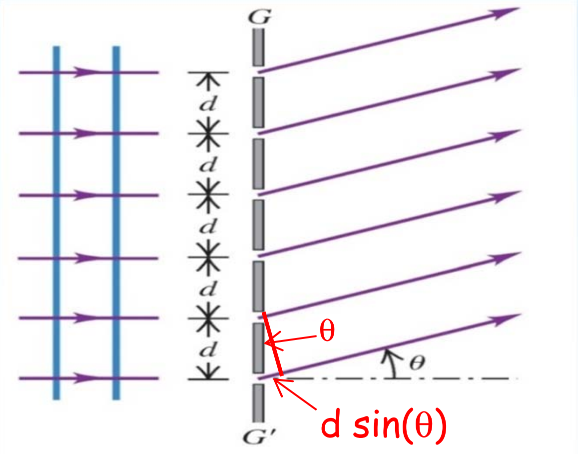
N = 2
Slit Width = 20 μm
Slit Separation = 100 μm
Wavelength = 0.37 μm

http://www.opensourcephysics.org/items/detail.cfm?ID=8331

Diffraction: Multiple slits

In diffraction grating one uses devices with thousands of slits or reflecting surfaces. This gives very narrow principal maximum that can be used to determine the wavelength of light.

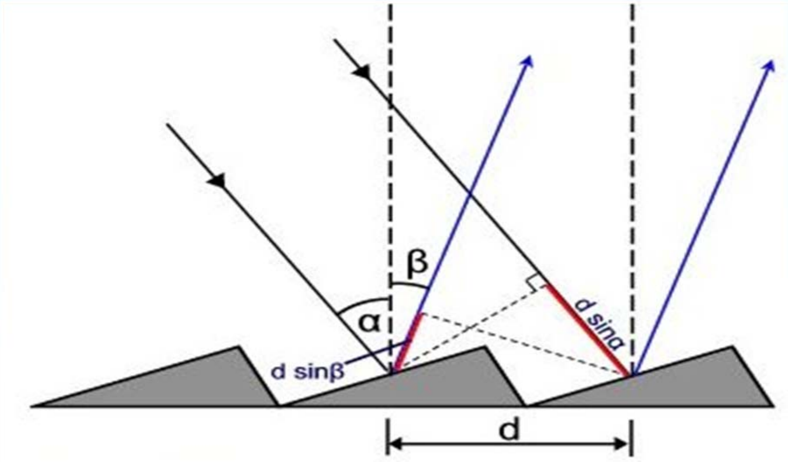
Transmission grating



Path difference for maxima:

$$\boxed{\delta = d\sin(\theta) = m\lambda}$$

Reflection grating



Path difference for maxima:

$$\delta = d\sin(\alpha) - d\sin(\beta) = m\lambda$$

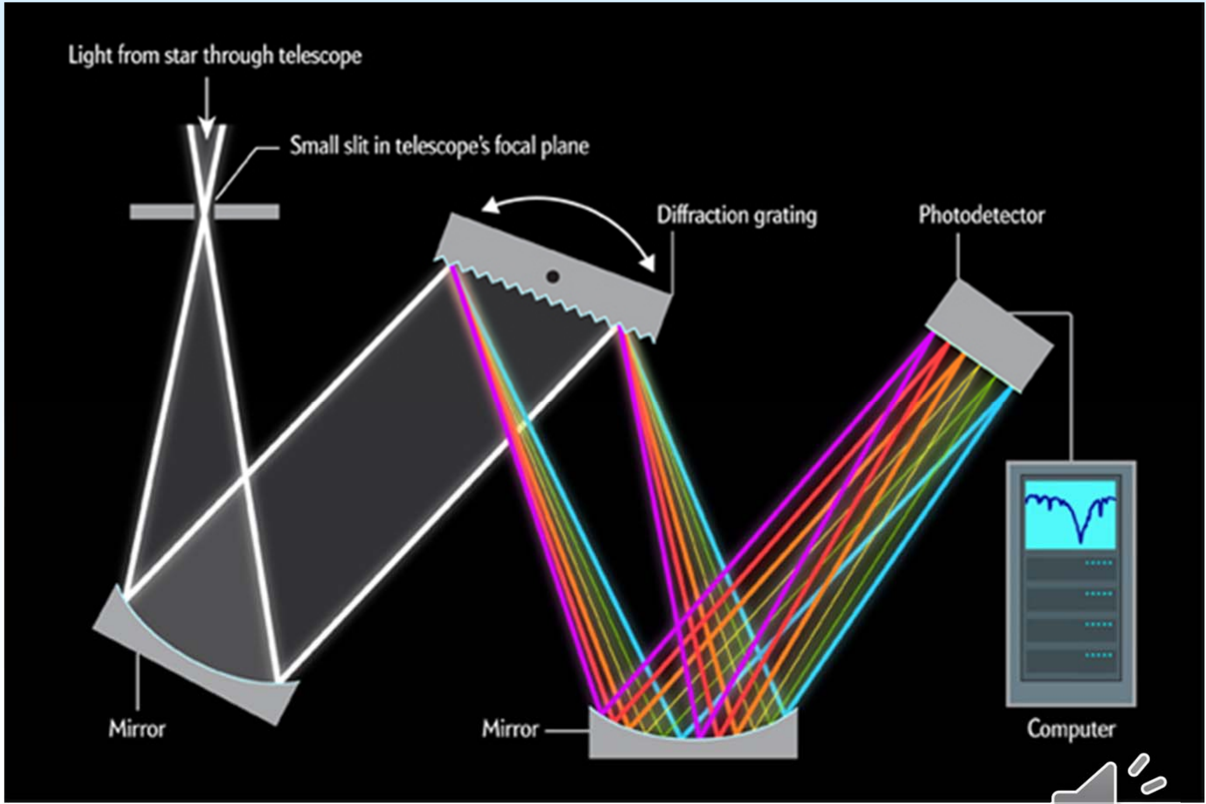
Part 7. The spectrometer

Diffraction: The spectrometer

A spectrometer for astronomy:

Light incident on a grating is dispursed into a spectrum.

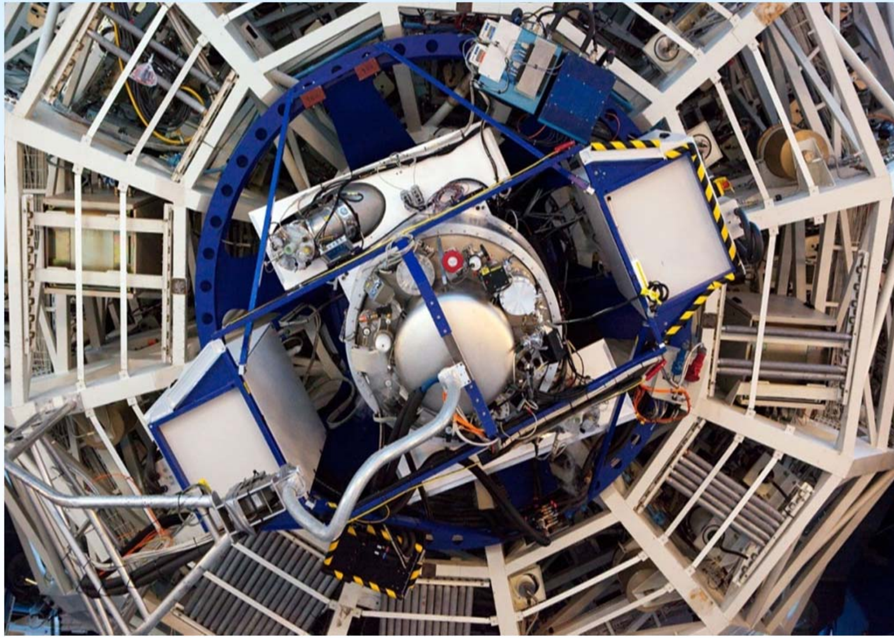
The angles of deviations of the maxima are measured to calculate the wave length.



Diffraction: The spectrometer

The ESO Very Large Telescope (VLT) in Chile

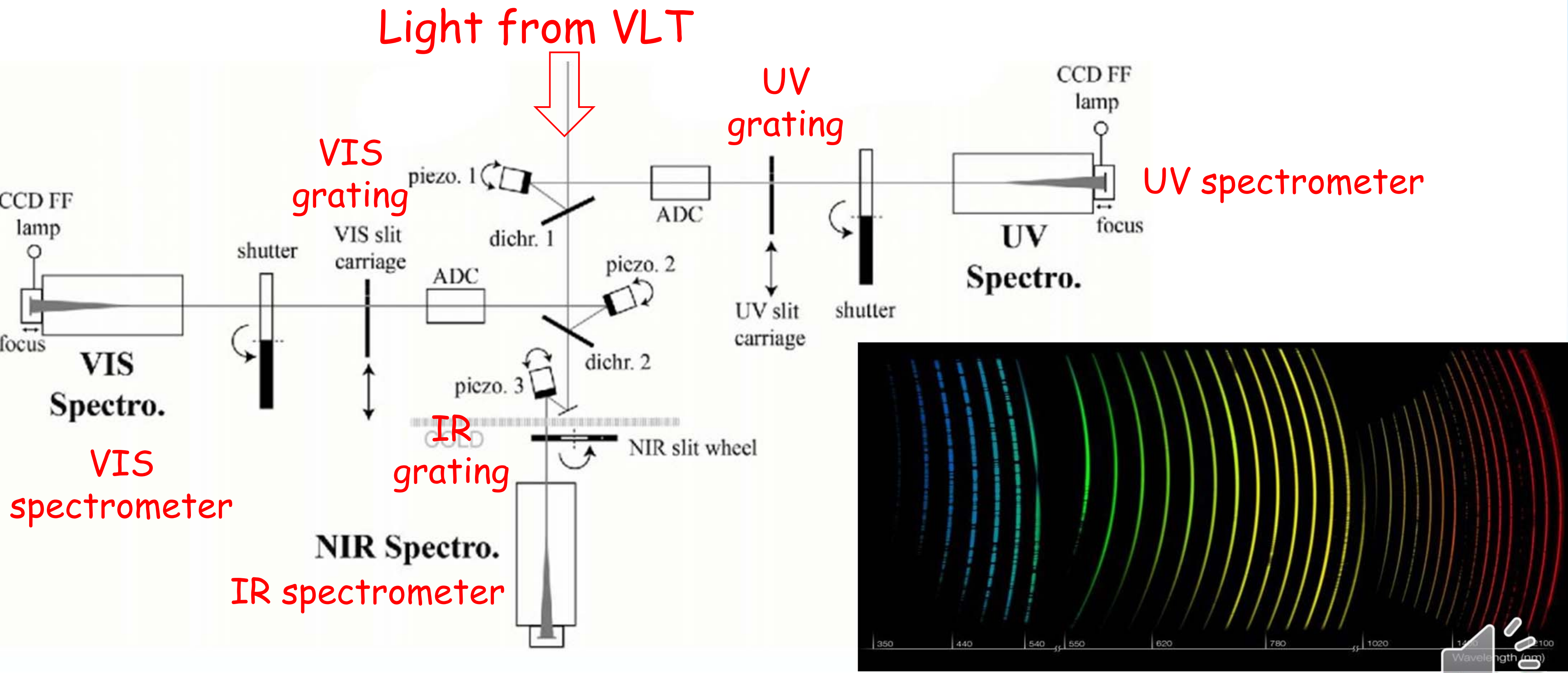
The XSHOOTER spectrometer in the VLT



ESO: European Southern Observatory

https://www.eso.org/public/

Diffraction: The spectrometer



Diffraction: The spectrometer

Chromatic resolving power (R):

The minimum wavelength difference ($\Delta\lambda$) that can be distinguished by a spectrograph:

$$R = \frac{\lambda}{\Delta\lambda} \qquad \text{(chromatic resolving power)}$$

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

Number of slits in the grating

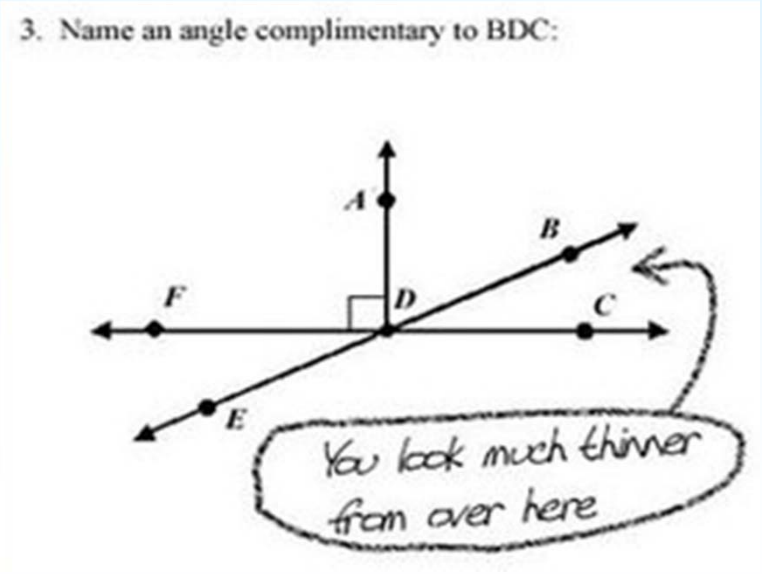
The order of the peak in the diffraction spectrum

R is higher for many slits and higher orders !

Example: XSHOOTER has R = 4000-7000 depending on wavelength.

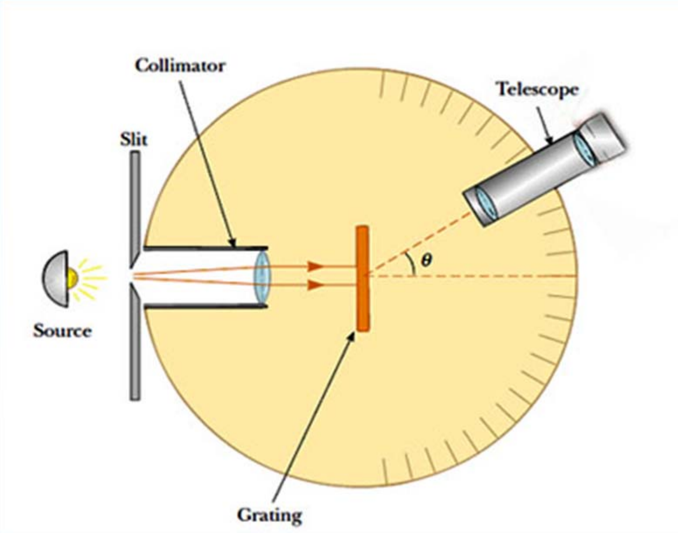
Diffraction: Problem

Part 8. Problems



3. Name an angle complimentary to BDC:

Diffraction: Problem



https://www.youtube.com/watch?v=b85paV77dS8

Grating: 1000 slits per mm   1st order maximum at 24°   What is λ ?

$d \sin\theta = m\lambda$ with $d = 1\ \text{mm} / 1000\ \text{slits} = 10^{-6}\ \text{m}$

$\theta = 24^\circ$

$\lambda = d \sin(\theta) = 10^{-6} \sin(24^\circ) = 0.407 \times 10^{-6} = 407\ \text{nm}$

Vincent Hedberg - Lunds Universitet

Diffraction: Summary

Part 9. Summary

Diffraction: Summary

One broad slit:

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$$\beta = \frac{2\pi}{\lambda}a\sin\theta$$

$$\tan(\theta) = y / x \approx \sin(\theta)$$

Two broad slits:

$$I = I_0\cos^2\frac{\phi}{2}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda}\sin\theta$$

$$\beta = \frac{2\pi a}{\lambda}\sin\theta$$

Multiple slits:

Path difference for principal maxima: $\delta = d\sin(\theta) = m\lambda$

Chromatic resolving power: $R = \frac{\lambda}{\Delta\lambda} = Nm$