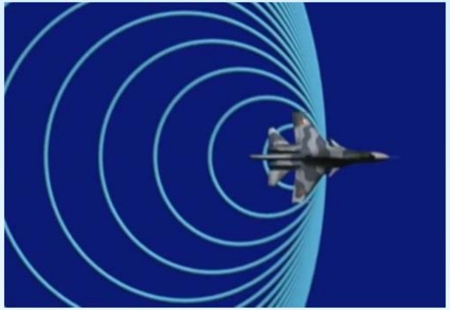
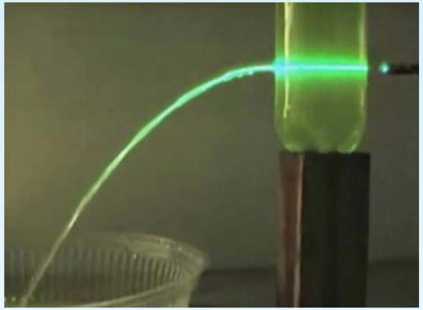
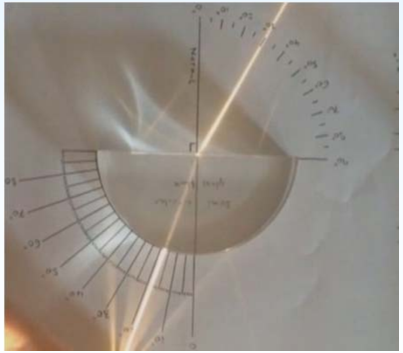
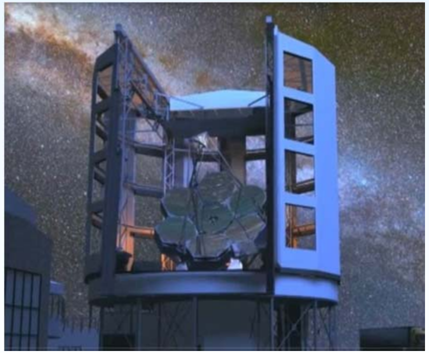
Wavemechanics and optics

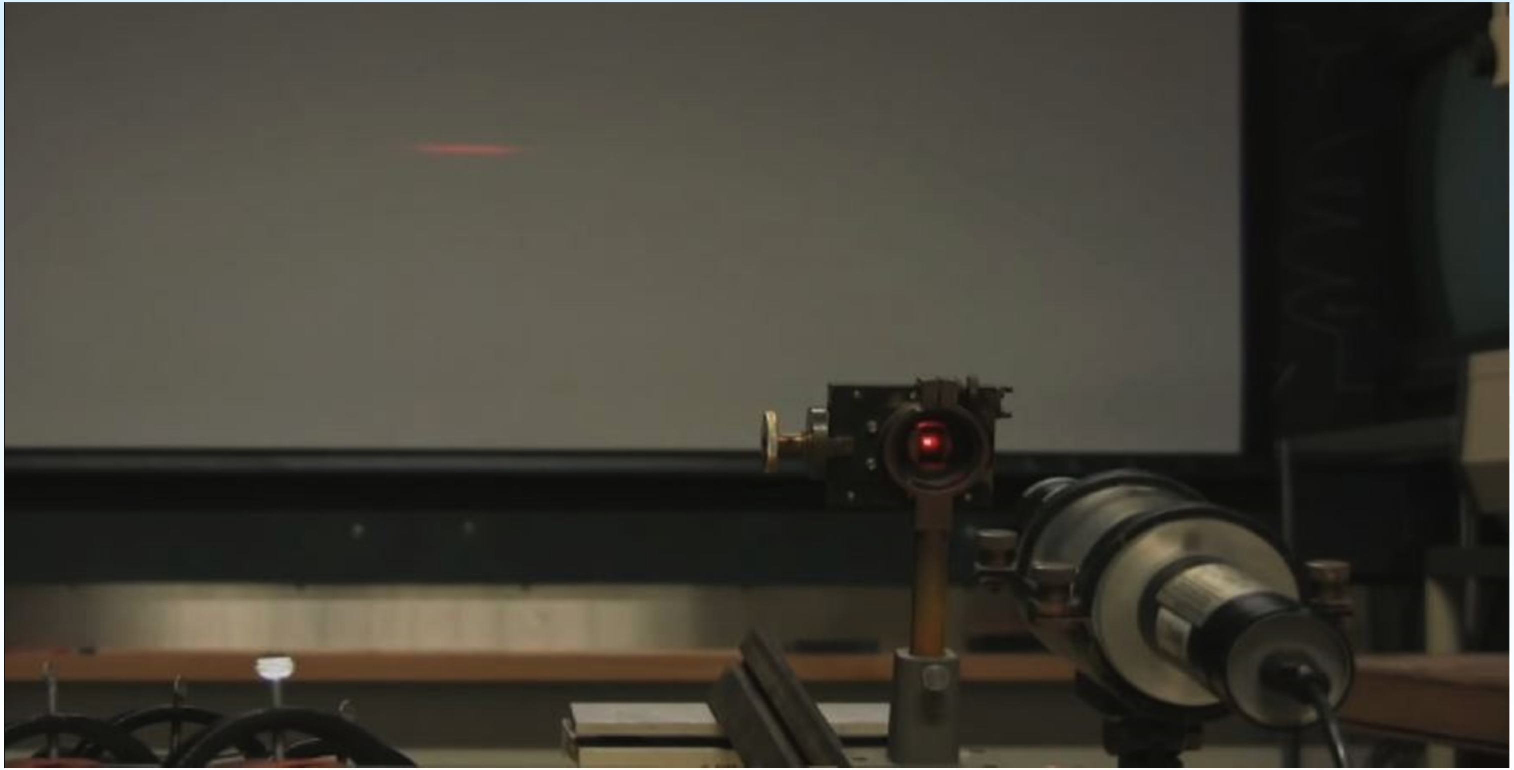




Chapter 36 - Diffraction

Vincent Hedberg - Lunds Universitet

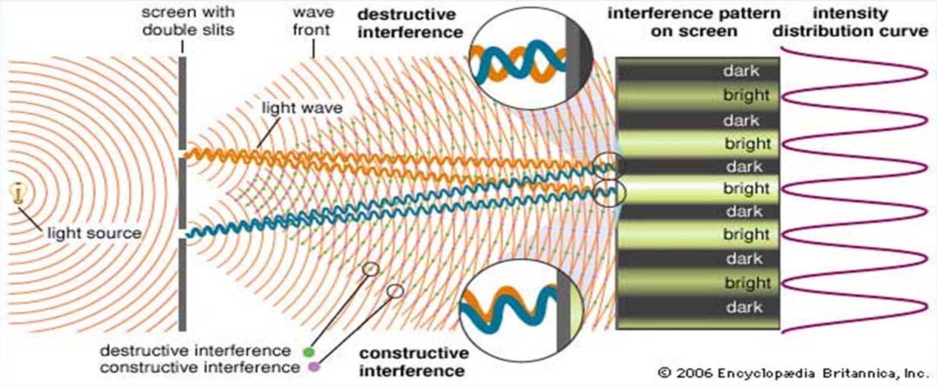
Diffraction



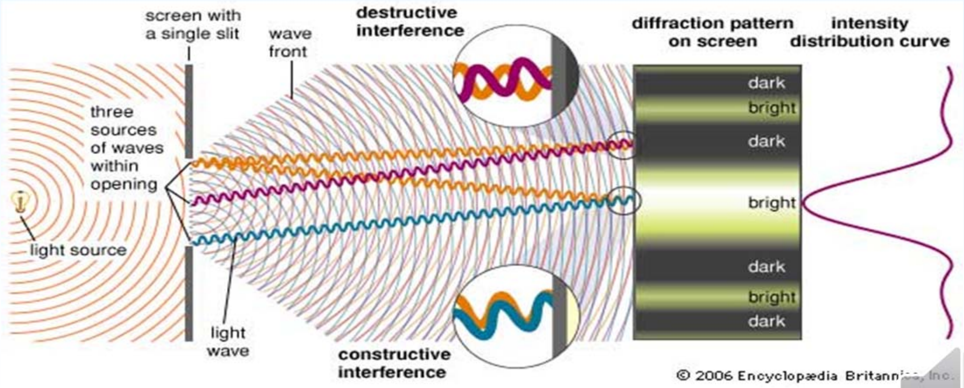
https://www.youtube.com/watch?v=9D8cPrEAGyc

Diffraction

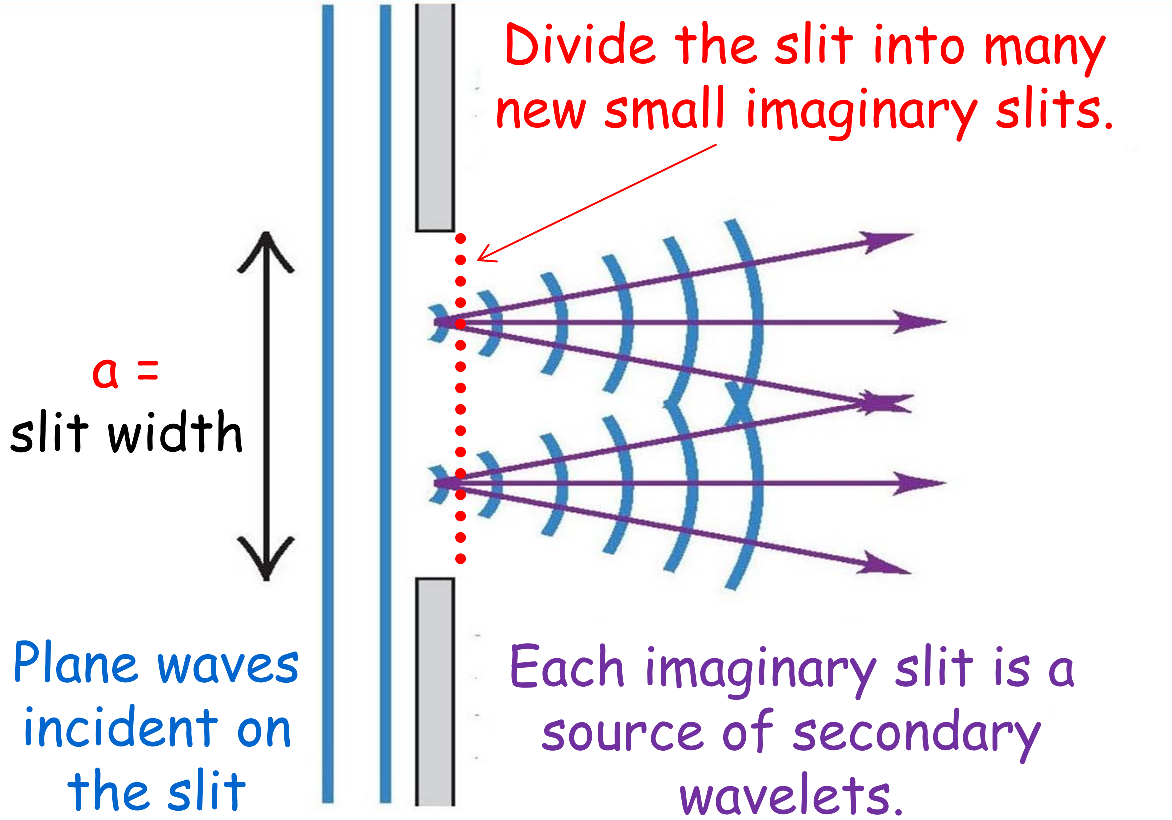
Interference:
Double slit
experiment

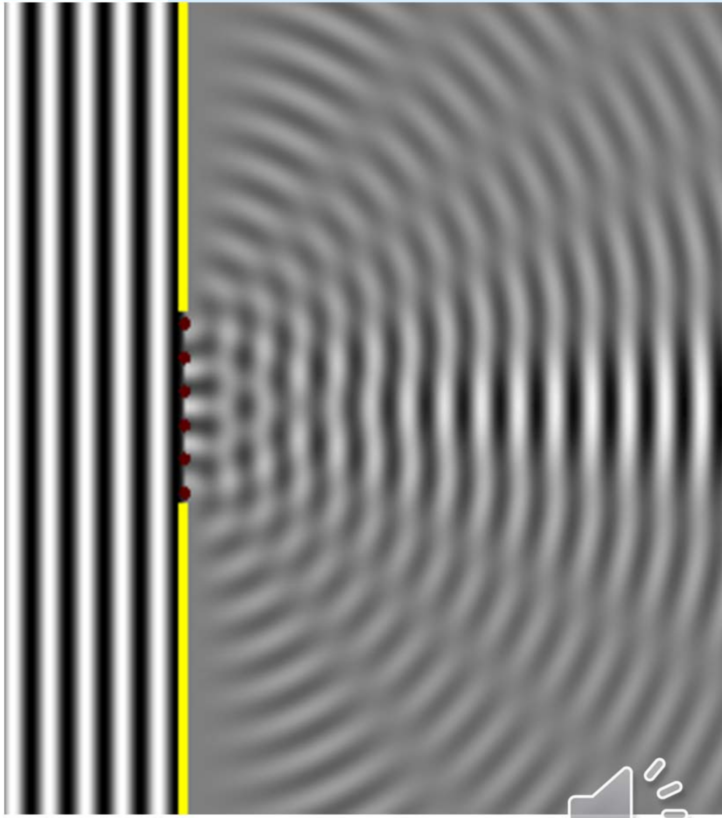


Diffraction:
single slit
experiment



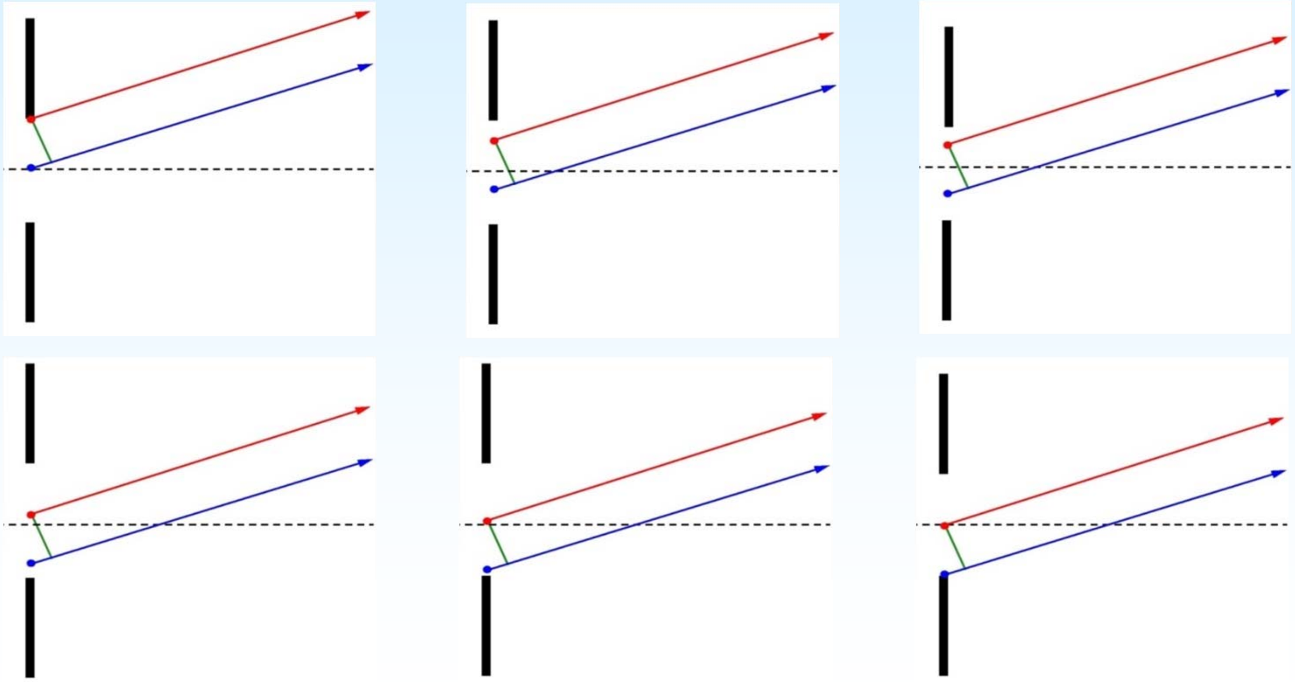
Diffraction

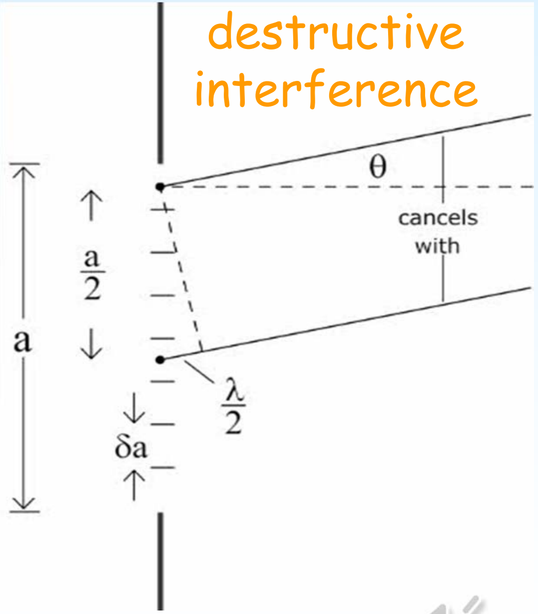




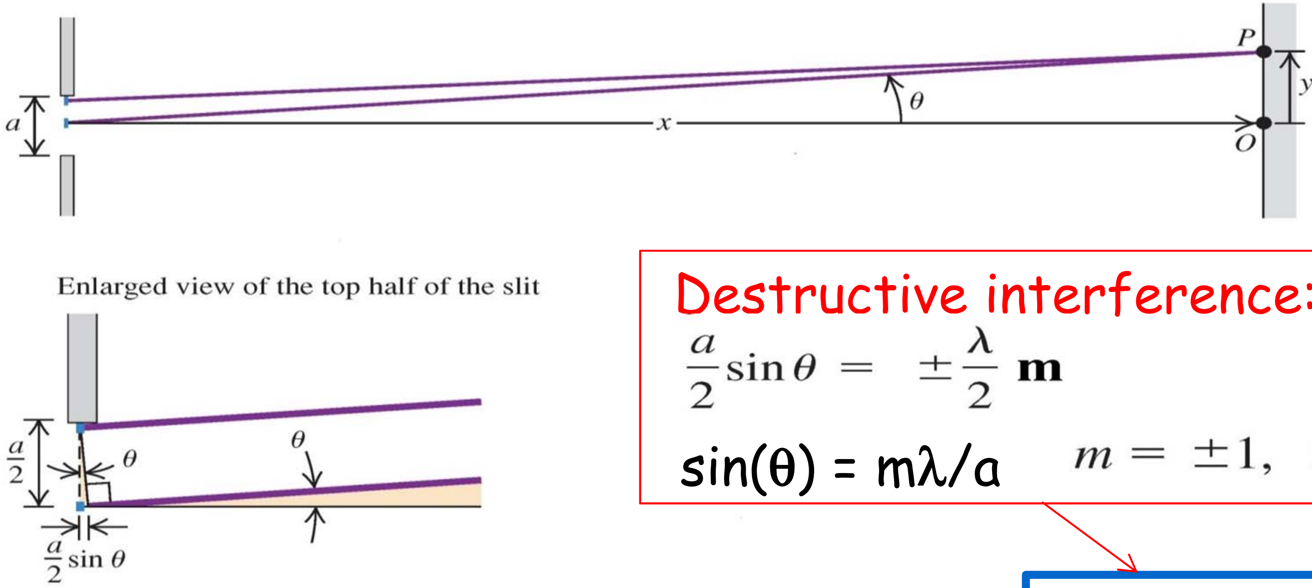
Diffraction

For every point in the top half of the slit there is a corresponding point in the bottom half.





Diffraction



Geometry:

tan(θ) = y / x

Destructive interference:

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}\,\mathbf{m}$$

sin(θ) = mλ/a $\quad m = \pm 1, \ \pm 2,$

Small angles:

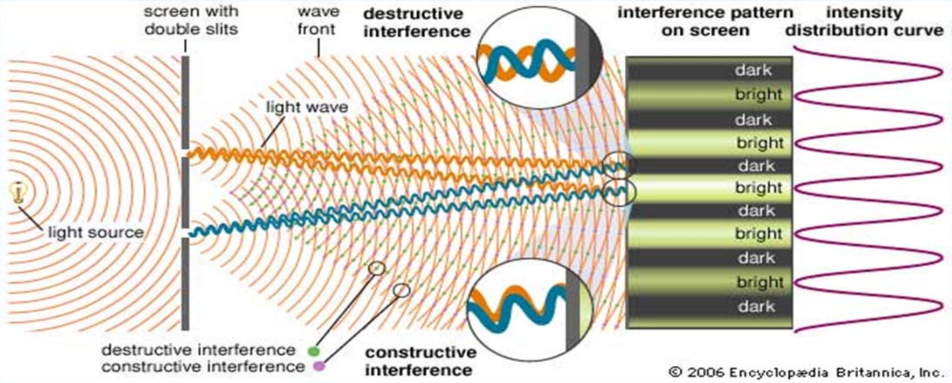
$\tan(\theta) \approx \theta$

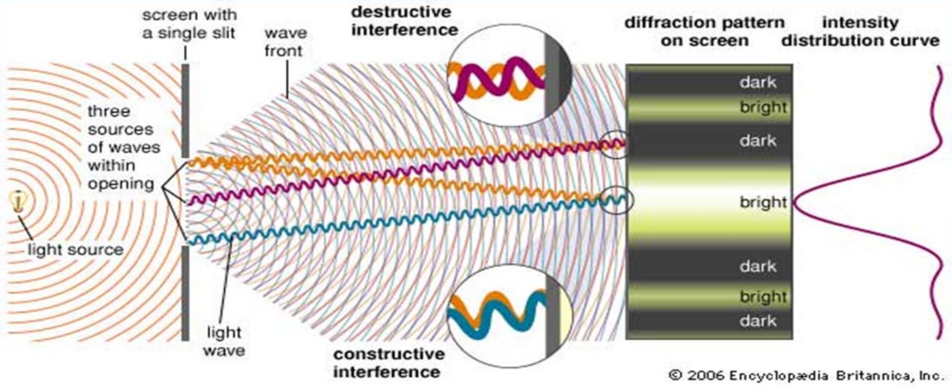
$\sin(\theta) \approx \theta$

$$y_m = x\frac{m\lambda}{a} \qquad (\text{for } y_m \ll x)$$

$m = \pm 1, \ \pm 2,$

Diffraction





Bright bands:

$$y_m = R\frac{m\lambda}{d}$$

- R: Distance to screen
- λ: Wavelength
- d: Distance between slits

$$m = 0, \quad \pm 1, \quad \pm 2,$$

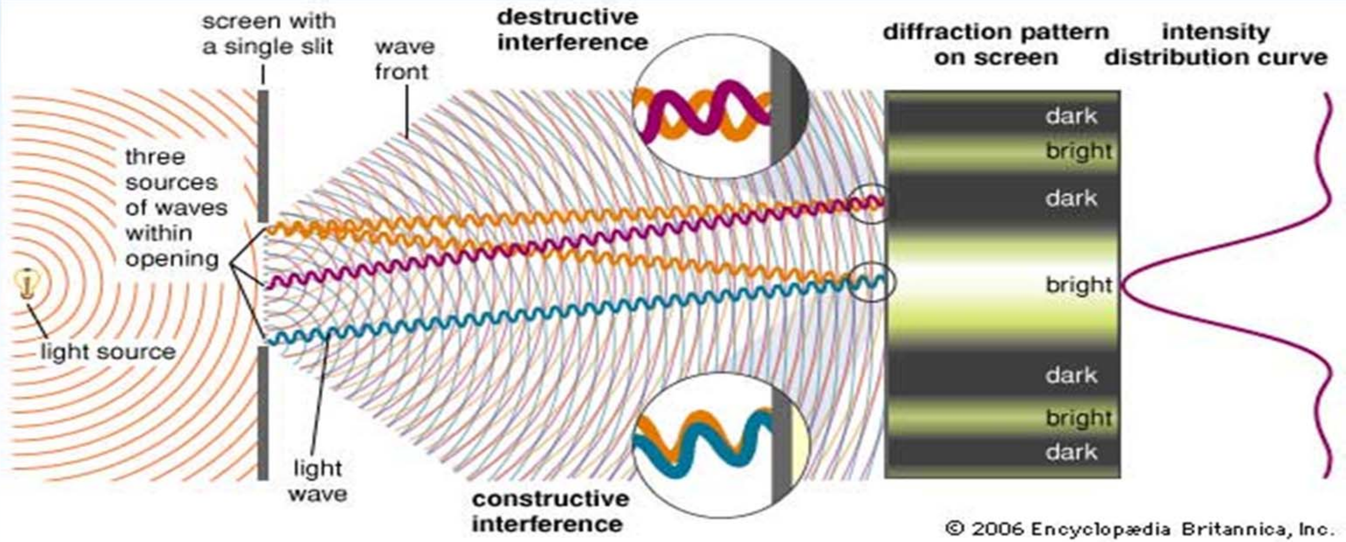
Dark bands:

$$y_m = x\frac{m\lambda}{a}$$

- x: Distance to screen
- λ: Wavelength
- a: Slit width

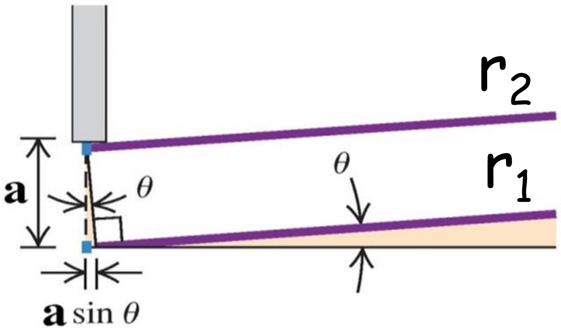
$$m = \pm 1, \quad \pm 2,$$

The intensity function



$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Diffraction: The phase angle β



Path difference:

$r_2 - r_1 = a \sin(\theta)$

The phase angle β

A path difference of one wavelength corresponds to a phase difference of 2π

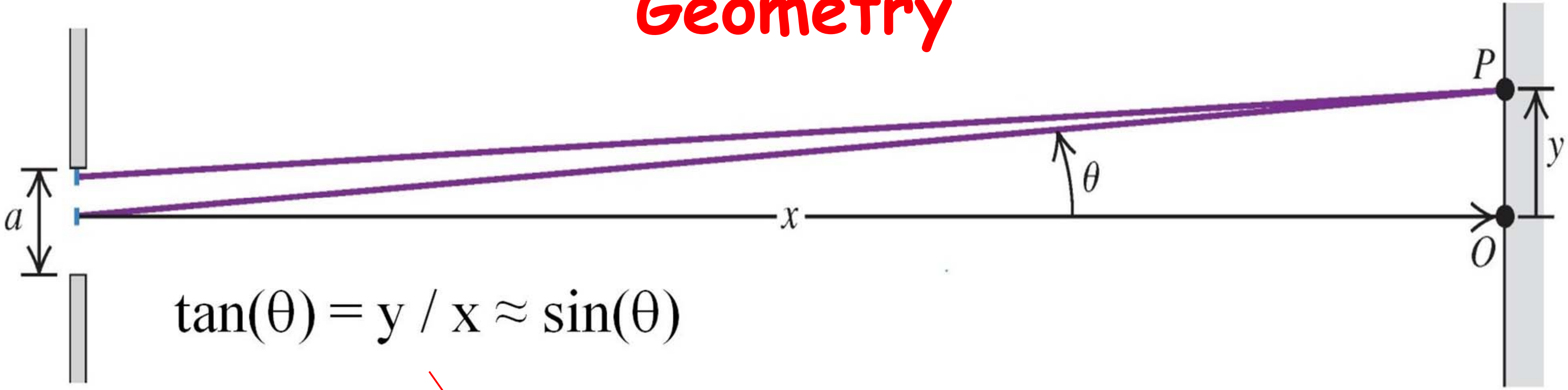
$$\frac{\beta}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

r_2-r_1 is the path difference between a ray at the top and bottom of the slit.

Diffraction: The phase angle β

Geometry

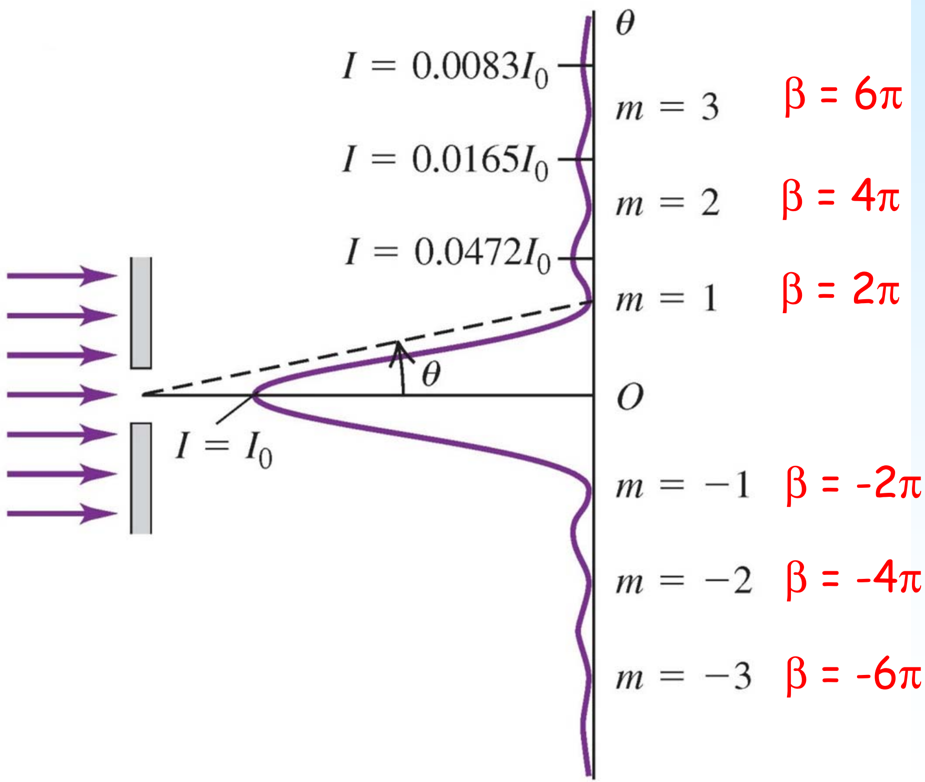


$$\tan(\theta) = y / x \approx \sin(\theta)$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$\beta = \frac{2\pi}{\lambda} a \sin(\theta) \approx \frac{2\pi}{\lambda} a \frac{y}{x}$$

Diffraction: Intensity

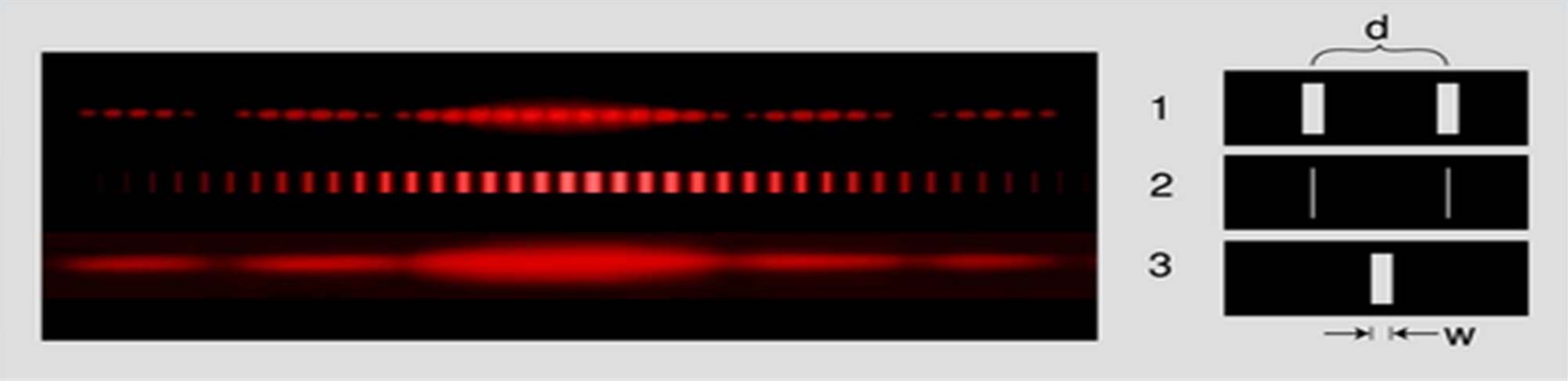


$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

where

$$\beta = \frac{2\pi}{\lambda} a \sin(\theta) \approx \frac{2\pi}{\lambda} a \frac{y}{x}$$

Two broad slits



Diffraction: Two broad slits

In the analysis of interference from two slits it was assumed that they were very narrow. What if they are broad ?

Two narrow slits:

$$I = I_0 \cos^2 \frac{\phi}{2}$$

One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

Two broad slits:

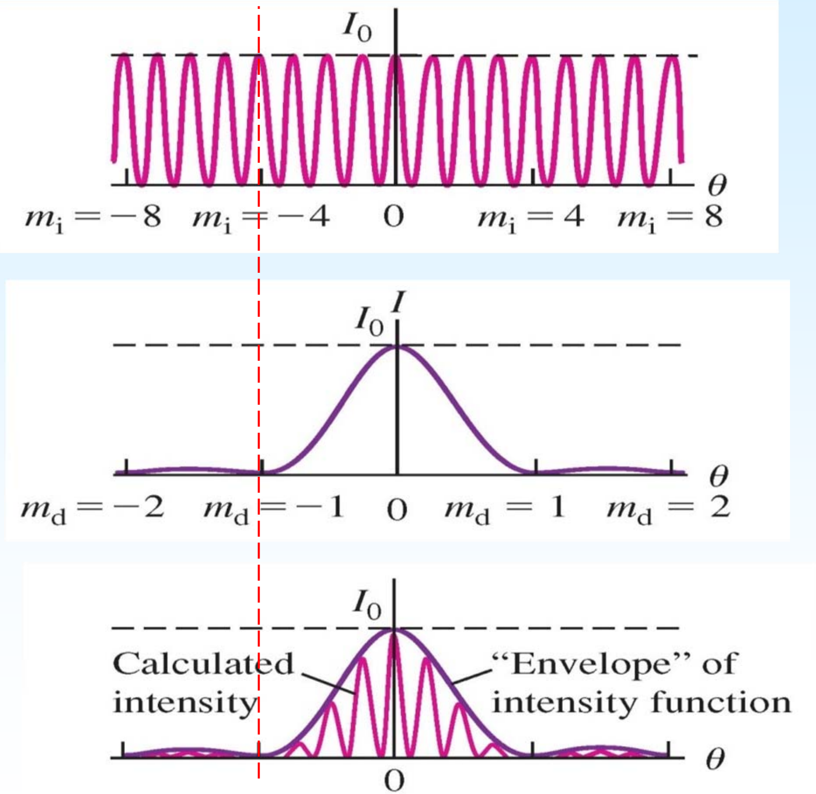
$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta = \frac{2\pi\delta}{\lambda}$$

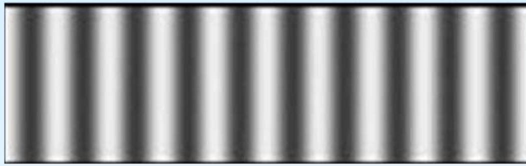
$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$

Diffraction: Two broad slits



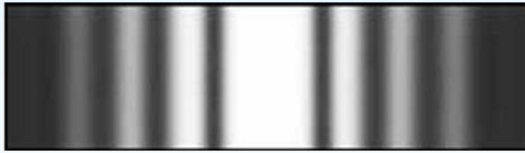
Two narrow slits:

$$I = I_0 \cos^2 \frac{\phi}{2}$$



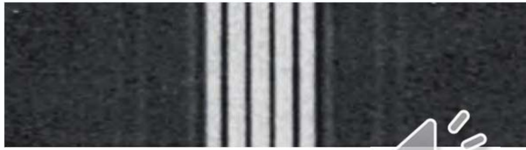
One broad slit:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

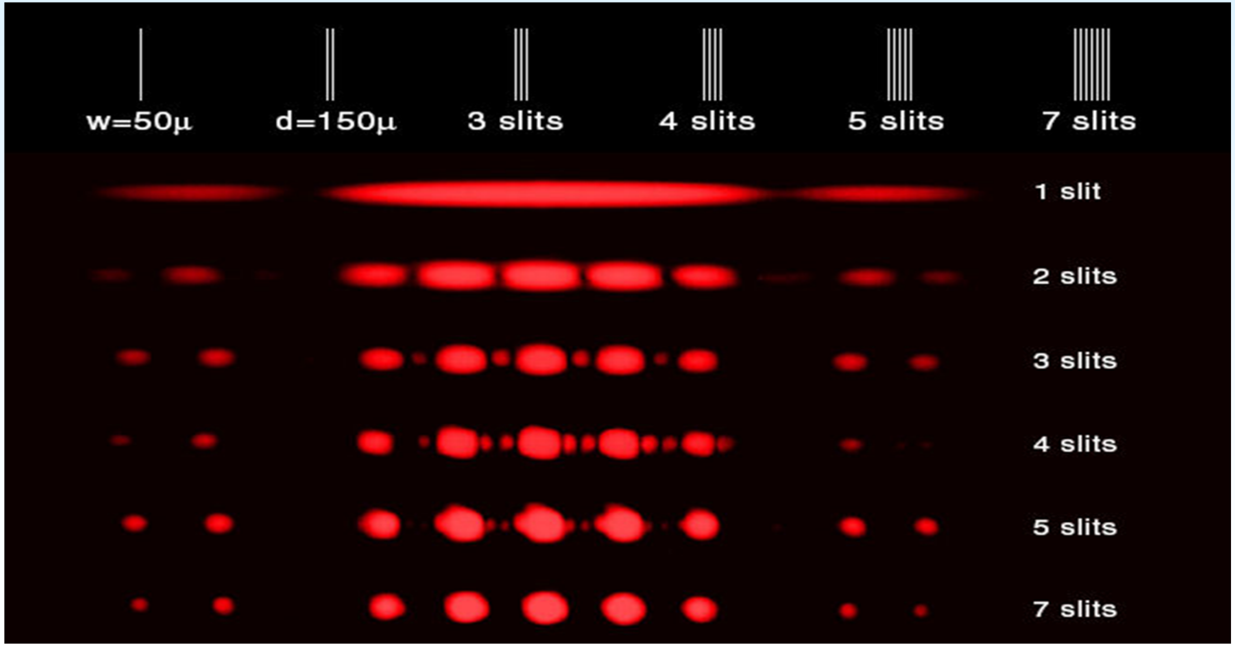


Two broad slits:

$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$



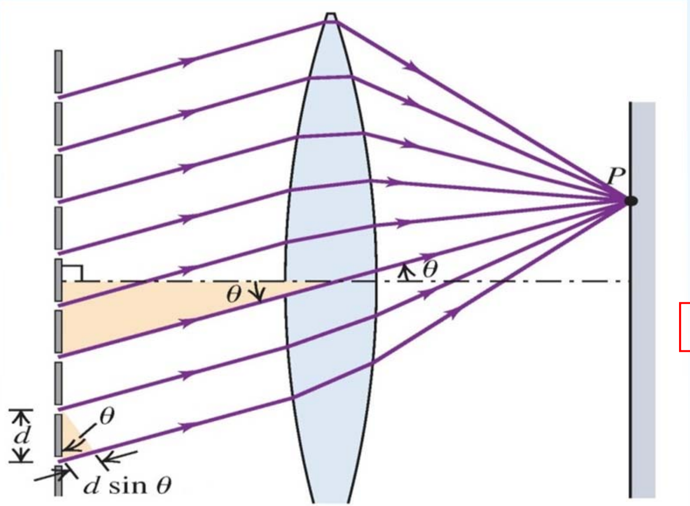
Multiple slits

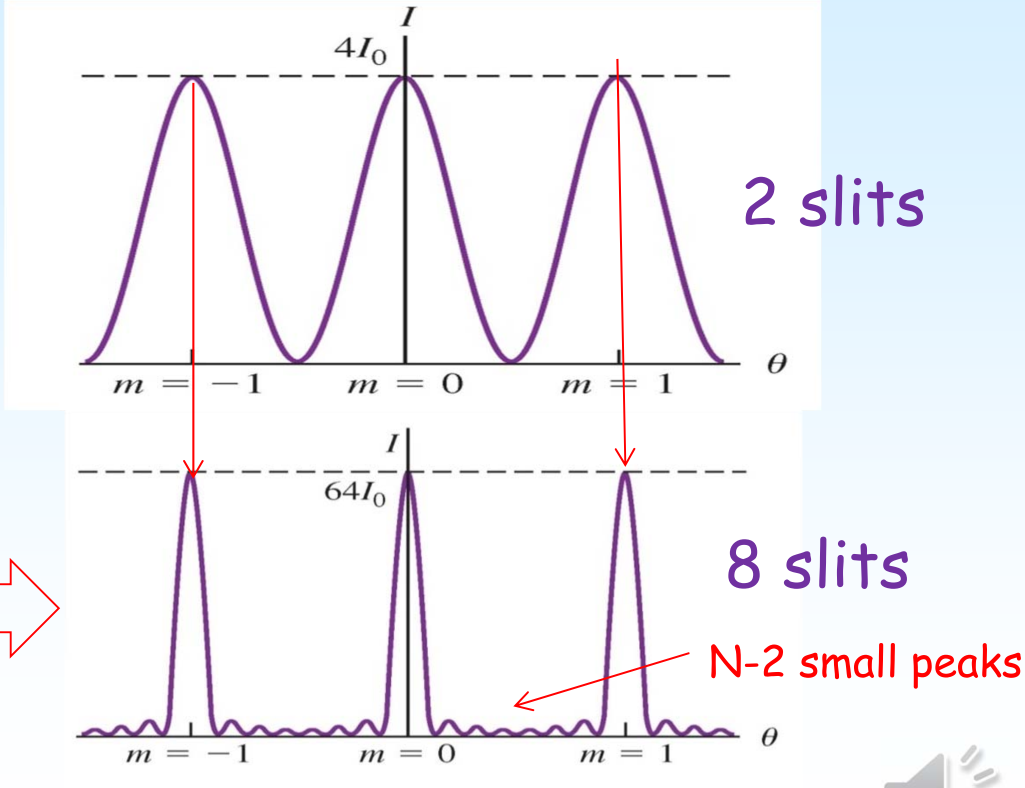


Diffraction: Multiple slits

The path difference between adjacent slits gives the principal peak intensity and is always:

$$d\sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

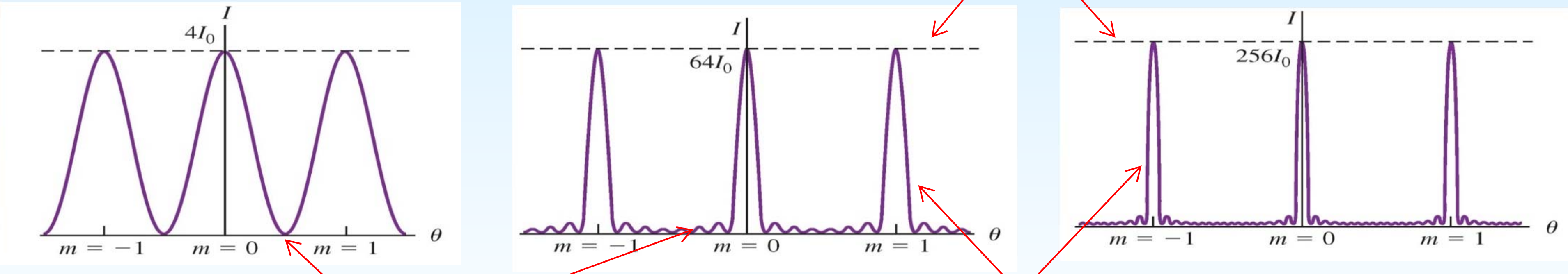




Diffraction: Multiple slits

$I_{max} \sim N^2$

N = 2 N = 8 N = 16



N-1 minima

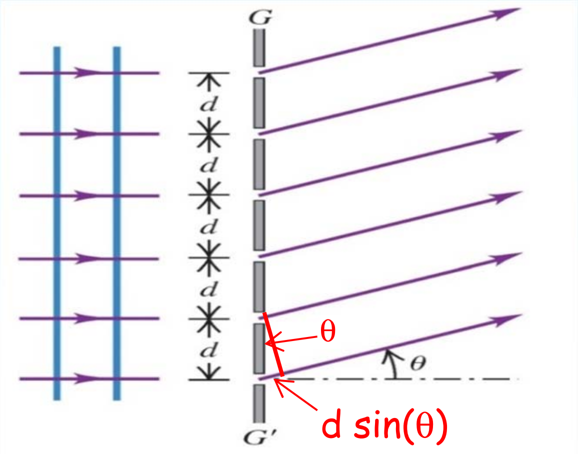
$I_{width} \sim 1 / N$

Principal maxima: $d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \pm 3, \ldots)$

Diffraction: Multiple slits

In diffraction grating one uses devices with thousands of slits or reflecting surfaces. This gives very narrow principal maximum that can be used to determine the wavelength of light.

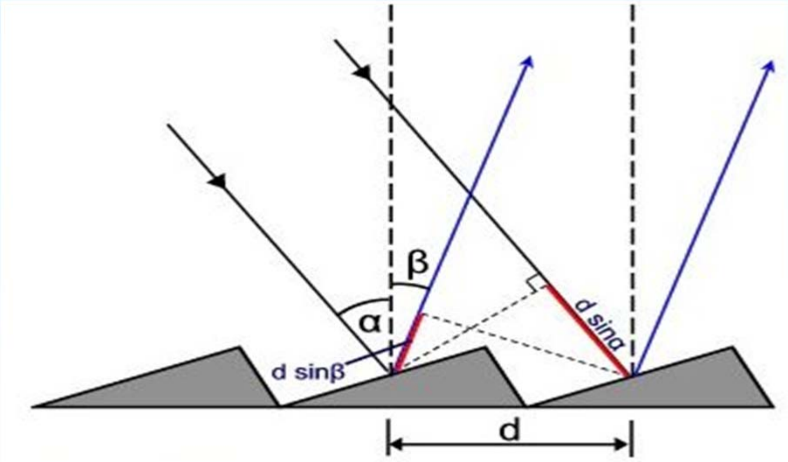
Transmission grating



Path difference for maxima:

$$\delta = d\sin(\theta) = m\lambda$$

Reflection grating



Path difference for maxima:

$$\delta = d\sin(\alpha) - d\sin(\beta) = m\lambda$$

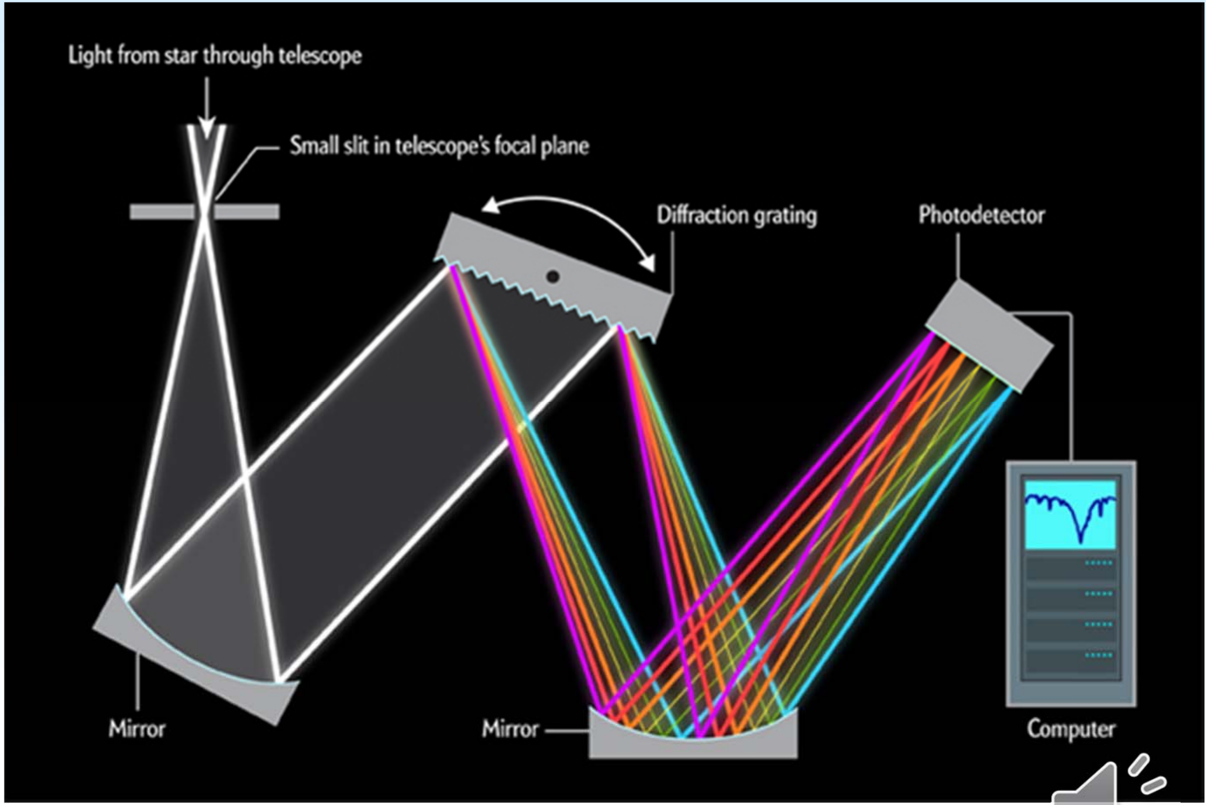
The spectrometer

Diffraction: The spectrometer

A spectrometer for astronomy:

Light incident on a grating is dispursed into a spectrum.

The angles of deviations of the maxima are measured to calculate the wave length.



Diffraction: The spectrometer

Chromatic resolving power (R):

The minimum wavelength difference ($\Delta\lambda$) that can be distinguished by a spectrograph:

$$R = \frac{\lambda}{\Delta\lambda} \qquad \text{(chromatic resolving power)}$$

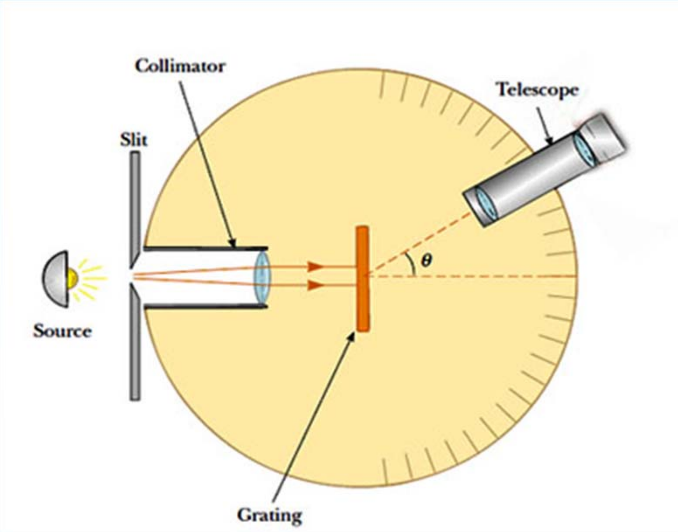
$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

N: Number of slits in the grating

m: The order of the peak in the diffraction spectrum

R is higher for many slits and higher orders !

Diffraction: Problem



https://www.youtube.com/watch?v=b85paV77dS8

Grating: 1000 slits per mm   1st order maximum at 24°   What is λ ?

$$d \sin\theta = m\lambda$$

with d = 1 mm / 1000 slits = 10^{-6} m
θ = 24°

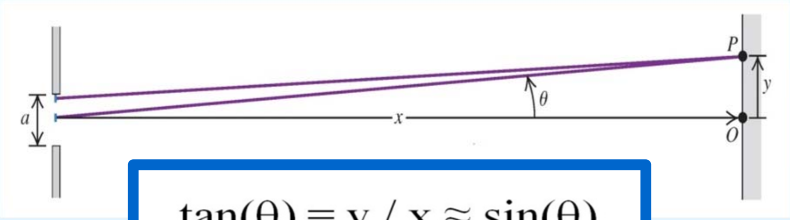
$\lambda = d \sin(\theta) = 10^{-6} \sin(24^{\circ}) = 0.407 \times 10^{-6} = 407$ nm

SUMMARY

Diffraction

Diffraction: Summary

One broad slit:



$$\tan(\Theta) = y / x \approx \sin(\Theta)$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

Two broad slits:

$$I = I_0 \cos^2\frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$

Multiple slits:

Path difference for principal maxima: $\delta = d \sin(\theta) = m\lambda$

Chromatic resolving power: $R = \frac{\lambda}{\Delta\lambda} = Nm$