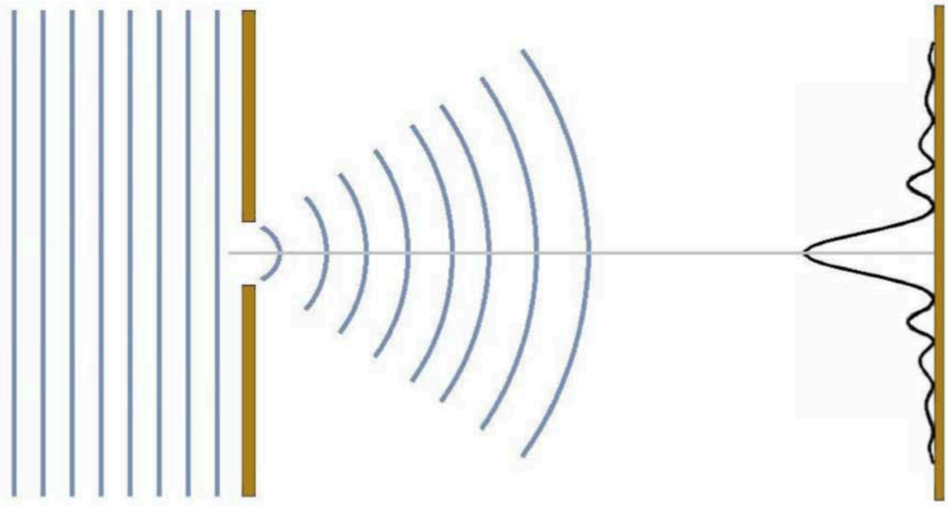
Lund University

O5: Diffraction and Interference

Optics and Waves



Introduction

The phenomenon of interference and diffraction - which comes from the superposition principle - is what really distinguishes a wave motion. Physically, two different concepts are not needed, since everything is fundamentally about interference. But traditionally, the word interference is used when there are two, or rather a small number of waves that meet at one point (eg two-ray interference in thin layers) and diffraction when there are infinitely many waves. It is also generally said that diffraction is the phenomenon that occurs when a wave front is partially blocked by an obstacle, e.g. when a sound wave passes a tree or when a mirror or lens of certain diameter collects a very small portion of the light from a distant star.

In the theoretical description of diffraction, two cases are distinguished. In the general case, called Fresnel diffraction, one takes into account that the wave fronts are bent both before and after the aperture. In many practical situations, however, with good approximation, the wavefront can be considered to be flat, or in other words that the rays are parallel. This case is much easier to handle mathematically and leads to the Fraunhoffer diffraction model. It is important to note that flat fronts can be achieved either approximately, by the distances from the source to the obstacle and then to the viewing point being large, or precisely by using imaging optics.

In this lab you will study diffraction patterns from a variety of apertures both experimentally and theoretically. A detailed lab report is particularly justified in this case to show that you can link theory and experiments together.

WARNING: Never look at the laser beam or its reflections.

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Diffraction and Interference

Preparation questions

Read the textbook chapters 16.4 and 16.6, 35.2 and 35.3, and 36.1 through 36.5. Then solve the following problems and read through the entire instruction. Proper solutions will be checked before you start your lab.

1. A laser beam of wavelength 632.8 nm illuminates a screen through a slit with a high gap of 0.5 mm width. The diffraction pattern is studied on a screen at a 4.0 m distance.

What diameter does the central spot of light get?

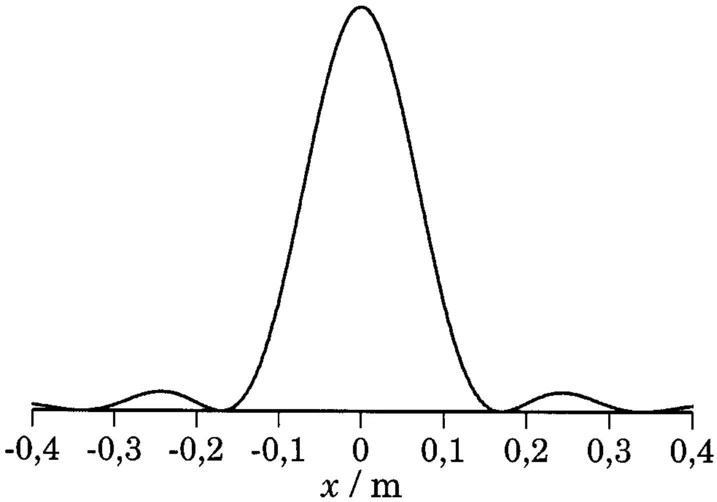

Figure 1. Example of light intensity distribution on a screen after diffraction in a slit.

2. A laser beam of wavelength 632.8 nm illuminates a screen with a hole of 0.5 mm diameter. The diffraction pattern is considered on a screen at 4.0 m distance.

What diameter does the central spot of light get?

3. A laser beam of wavelength 632.8 nm illuminates a screen through a slit with 50µm width.

a. Calculate approximately at what angle the first maximum outside the central maximum occurs.

b. Calculate the ratio between the intensities of the two maxima.

4. Parallel light hits a grating with 600 scratches / mm. The third order of the wavelength λ is observed at an angle of 64.16 degrees. Use the grating formula in Appendix 2 and determine λ.

1 Diffraction experiments with laser light

1.1 Fraunhoffer diffraction

Since the laser light consists of flat wave fronts, it can be used directly to study Fraunhofer diffraction, as long as the diffraction pattern is considered on a sufficiently distant screen.

Single slit

Use an adjustable slit and compare with the theory. Does the diffraction pattern change qualitatively according to the theory when the slit width changes? Determine the slit width for a given slit, which the supervisor distributes, by measuring the distances in the diffraction pattern on the screen. Make a simple error estimate of your result. Which is the most uncertain variable?

Circular hole

Write down the relevant theoretical formula. Compare the pattern with that from the single slit! Use the diffraction pattern to determine the size of a hole.

Rectangular hole

Take a rectangular hole plate and observe the diffraction pattern. Do you see any connection with the single-slit diffraction pattern?

Transmission grating

Qualitatively study the diffraction patterns of a few different grids. Place several gratings one after the other with the slits at different angles. Can you give a simple principle for the resulting pattern?

1.2 Fresnel diffraction

This case can easily be obtained from the previous set up by placing a negative lens between the laser and the object so that the wavefront becomes sufficiently curved. Fresnel diffraction is much more difficult to handle theoretically, so we will only qualitatively look at some interesting observations.

Circular hole

Use an iris aperture, vary the diameter of the hole and observe what happens in the center.

Circular disc (ball)

Use a small ball as an obstacle and study the diffraction pattern carefully. What does it look like in the center?

Thin thread

Use a thin thread as a barrier and study the diffraction pattern carefully. What does it look like in the center? Compare that with what you can hear a person speak even if you are standing right behind a tree but you cannot see the person on the other side!

2 Experiment with a grating spectroscope

2.1 Adjustment

See Appendix 1. This does not normally need to be done.

2.2 Qualitatively

In this laboratory experiment you will see a "primitive" instrument demonstrating the principle of detection and analysis of the wavelength spectrum. If you do a good job you will still measure wavelengths with better than 1% accuracy!

Illuminate the collimator slit with a spectral lamp and use a grating, with only 100 slits per mm. Look at the colorful picture and try to explain how it occurs. Why does one talk about spectral lines and not eg. spectral circles or other shapes? Note that there is a spectrum on each side of the normal to the grating surface, ie, on each side of the zero order. Also note that for large diffraction angles, there is an overlap between light from different orders.

2.3 Wavelength Measurement

Quantitative wavelength determinations are made using the so-called grating format(Appendix 2) which states that the maximum for a given wavelength λ, and the order, m, occurs at an angle, given by:

$$a \cdot \sin(\theta) = m \cdot \lambda$$

Here, a is the so-called grating constant, the distance between two adjacent slits. In our case, a = 1/100 mm. Create a table where you enter the results below.

1. Measure the angle between the zero order line and each visible spectral line in order 1 and 4, respectively .
2. Calculate the wavelength for each spectral line and identify which atomic type the lamp contains by comparing with a table (eg from TeFyMa) .
3. Calculate the deviation in wavelength from table values and compare the size of the wavelength deviation in order 1 with order 4 . Explain the difference.

3 Diffraction in N Slits

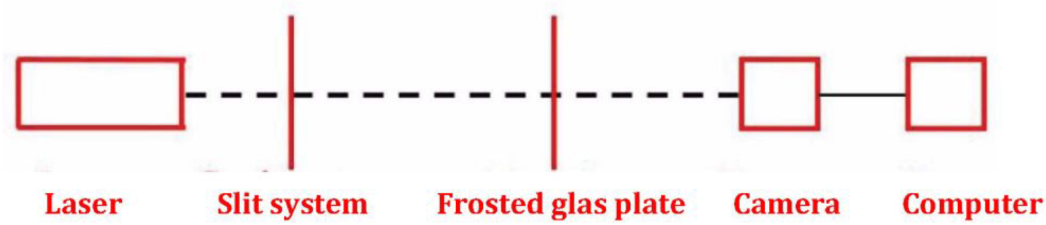

Figure 1 *. Arrangement for N-slit diffraction*

3.1 Preparation

The diffraction object is a glass disc with four objects, each consisting of a number of (N) narrow slits of width b and separated by the distance a . Move the glass plate sideways so that the laser beam passes one of the objects. Measure the distance from the object to the glass screen.

Log in with your STIL identity on the computer. The *Nspalt* folder appears on the desktop and contains links to two programs: *Debut* to view and capture the image from the camera and *Nspalt* to simulate the diffraction pattern from N slits as shown in Appendix 2.

3.2 Register the diffraction pattern

Start the *Debut program* , select " Web Camera " or "Device" (depending on the camera type) as the input source and switch to "**full screen mode**" (**important!**) either by right-clicking in the image or via the "View" menu

Set the focus of the camera to get a sharp image of the ruler and measure the image size on the screen using the fixed ruler on the glass disc.

Minimize stray light by turning off all lights.

Set the aperture so that the picture of the red laser is not overexposed.

Capture an image by pressing the "**Print Scr**" **key** . The screen image is then saved as a bitmap image in the computer's clipboard.

3.3 Analysis of the diffraction pattern

1. Launch the *Nspalt* program and *load* the captured image with the *File / Paste bitmap image (paste)* menu. Select the pattern from the laser as instructed in the program. *Nspalt* creates an intensity profile of the selected two- dimensional pattern by summing the intensity in the vertical direction.

2. Open the *N-column model dialogue.* Provide the data for the image that the program needs to convert the diffraction angle θ to distance along the screen. Data is saved until you exit *Nspalt.*

3. Simulate the diffraction pattern by entering the slit data in the dialog. The red laser has a wavelength of 635 nm. Think about what will happen before trying different values so that you test your understanding of the diffraction problem, rather than how fast you can enter different values!

4. Once you have found good slit data, you can choose to also show the "diffraction factor" and "interference factor" in the same image to clearly see how the total (observable) pattern arises. If you close the dialog, you can use the *File / Save As Bitmap* menu to save the screen image to a file to use in your report.

Repeat these steps for all 4 slit systems

Select the system with the most number of slits, register the pattern from the **green** laser and determine its wavelength .

If you have enough time, you can replace the gap systems with a small hole and study the pattern in the same way as above, but now with the "Circular aperture" dialogue instead. In circular symmetry, the sine function in the single-slit pattern is replaced by a Bessel function ($J_1(x)$). Compare with task 1.1b above and note the big difference between visually viewing the pattern, with the logarithmic sensitivity of the eye, and measuring photoelectrically with a linear sensitivity.

Appendix 1: Spectroscope and its alignment

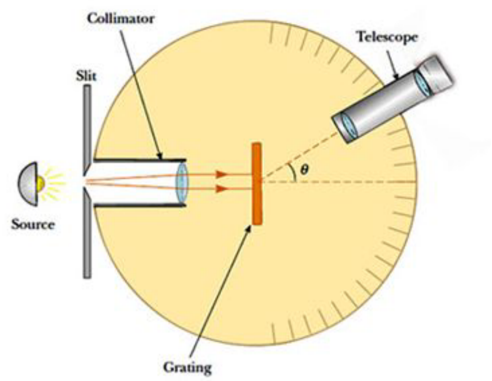

Figure A1-1. Principle sketch of the spectroscope

The task of the **collimator** is to ensure that parallel light (ie, flat waves) from the input slit hits the transmission grating.

The task of the **binoculars** is to focus parallel light from the grating on a crosshair, illuminated by a small lamp, so that in the eyepiece at the same time you see the crosshair and a spectrum sharply. The binoculars are rotatable so that different parts of the diffraction pattern can be viewed.

The rotation can be measured with an accurate angular scale. Note that the scale is graded in degrees and minutes (1 '= 1/60 degree). To determine wavelengths via the angle measurement, first set the crosshair on the zero order spectral line (alternatively you can adjust the scale so that it really shows 0 here) and then measure the diffraction angle for the different spectral lines in the different orders.

Before you begin, there are three setting steps that you must complete.

A1.1 Set the binocular for parallel beams.

Take the binoculars out of its hold. Move the eyepiece until the cross hair is sharply visible with a relaxed eye. Turn the binoculars focus knob until you see a very distant object sharply, ie. so that the middle image falls on the hairline and both are sharply visible.

Put the binoculars back in its position. It should now be set for parallel incident beams and should not be adjusted further.

A1.2 Set the collimator for parallel beams.

Align the binoculars and the collimator with each other. Make the collimator slit opening very small. Turn the collimator's focus knob until you see in the binoculars the image of the collimator slit sharply in the plane of the crosshair. Since the binoculars were already set for parallel beams, this must mean that the collimator now provides parallel light.

A1.3 The opening of the collimator slit.

When you look at a spectrum, you may notice that a reduction in the collimator gap's width gives the image ***greater*** sharpness but ***less*** light. In particular, an excessive opening causes the details of the spectrum to be completely blurred. It is therefore important to optimize the width of the collimator gap.

Appendix 2: Fraunhofer diffraction in a system of slits

The light intensity as a function of the exit *angle* θ for a system of N columns in the Fraunhofer case, ie. with parallel rays is given by:

$$I = I(0) \cdot \left(\frac{\sin \beta}{\beta} \right)^2 \cdot \left(\frac{\sin N\alpha}{N \sin \alpha} \right)^2$$

$$\beta = \frac{1}{2} kb \sin \theta \qquad \alpha = \frac{1}{2} ka \sin \theta \quad \text{where}$$

I (0) = the observed intensity at the center of the pattern (Sigma = 0) .

b = slit width one

a = slit distance (from center to center)

N = number of slits

k = 2/π / λ

Special case :

$$N = 1: \quad I = I(0) \cdot \left(\frac{\sin \beta}{\beta} \right)^2$$

$$N = 2: \quad I = I(0) \cdot \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin 2\alpha}{2 \sin \alpha} \right)^2 = I(0) \cdot \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha$$

The diffraction pattern is thus a product of two factors. It is very instructive to discuss these factors separately.

$(\sin \beta / \beta)^2$ we can call the single-slit factor because it is identical to the result of a single slit.

$(\sin(N\alpha) / N \sin \alpha)^2$ we can call the interference factor because it describes how light from ***different slits*** interferes. See Figure A2-1. Note that we divided by N inside the parentheses to obtain a factor that assumes values between 0 and 1. The fact that the intensity of the maximum points increases with the number of slits we take care of by normalizing with I (0) given by $I_0 \ N^2$ where I_0 is the contribution of a single slit.

*P*rincipal maxima in Figure A2-1 then *arise when a* = *m* π(*m* = 0, ±1, ± 2 , ...) where the interference factor has the limit value 1. To see this for *a* = 0 we do and McLaurin development:

$$\frac{\sin(N\alpha)}{N\sin\alpha} = \frac{1}{N}\cdot\frac{N\alpha - \frac{1}{6}(N\alpha)^3 + \ldots}{\alpha - \frac{1}{6}\alpha^3 + \ldots} = \frac{1 - \frac{1}{6}(N\alpha)^2 + \ldots}{1 - \frac{1}{6}\alpha^2 + \ldots} \rightarrow 1 \text{ när } \alpha \rightarrow 0$$

Here follows the so-called lattice formula which states that the maximum for a given wavelength occurs when:

$$\alpha = m\pi \Rightarrow \frac{1}{2}\frac{2\pi}{\lambda} a\sin\theta = m\pi \Rightarrow \boxed{a \cdot \sin\theta = m \cdot \lambda}$$

We also see that the intensity decreases from the maximum value to zero, ie the width of a main maximum ($2\pi / N$) decreases the more slits we have. We can summarize by saying that the more slits the higher and narrower the peaks become.

Secondary maxima occurs when the numerator $\sin(N\alpha) = 1$ and $\sin(\alpha)$does not equal 0. As Figure A2-1 shows, there are N - 2 secondary maximums between two main maximum. If we study the different cases in Figure A2-1, we also see that the relative intensity of the secondary maxima decreases as N increases.

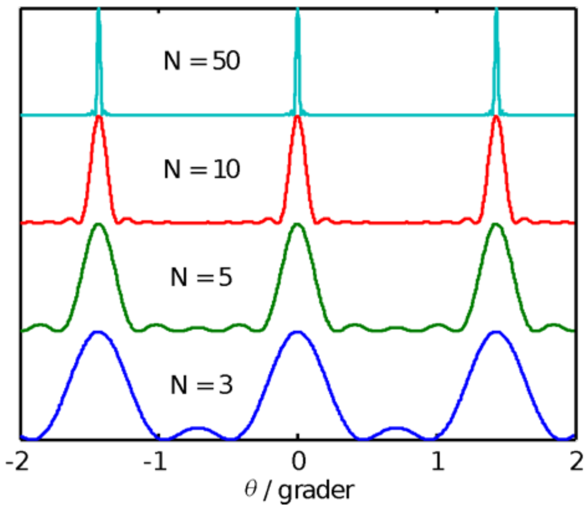

Figure A2-1 *. The interference factor for a system with 3, 5, 10 and 50 columns.*

When we have a lot of slits, we talk about a grating. For gratings, the number of slits per mm is usually specified. Common values are 600 or 1200 per mm, which means that the distance between the slits - the so-called grating constant a - becomes 1666 or 833 nm. If the grid is about 10 cm wide, then there are a total of about 10^5 slits.

Figure A2-2 illustrates how the observable pattern arises as a product of the single gap factor and the interference factor.

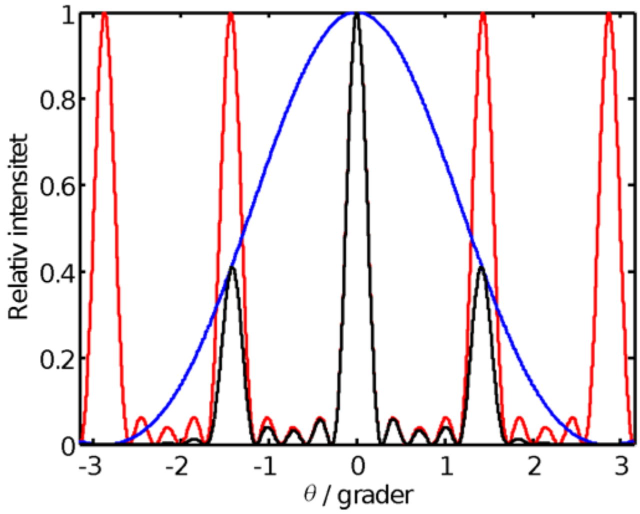

Figure A2-2 *. Diffraction in 5 slits. The figure shows the single gap factor (blue) , the inference factor (red) and the product (black) for a = 20 um and b = 10 u m.*