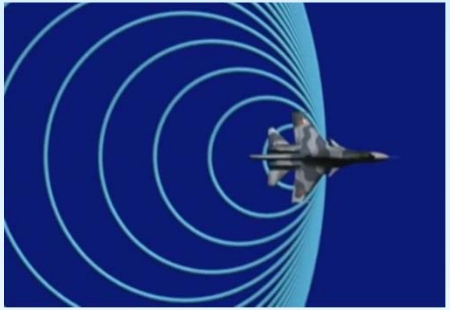
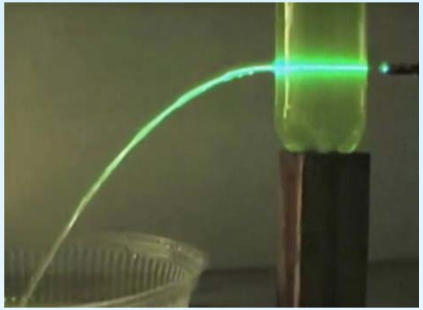
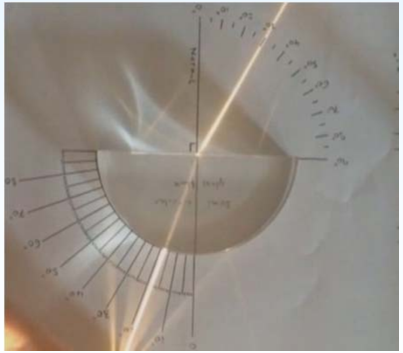
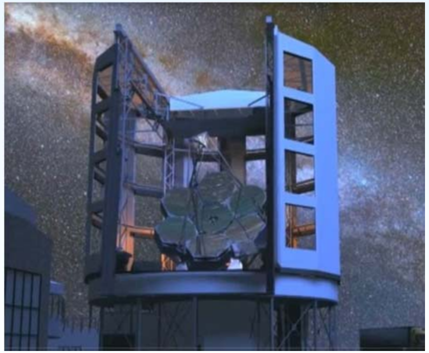
Wave mechanics







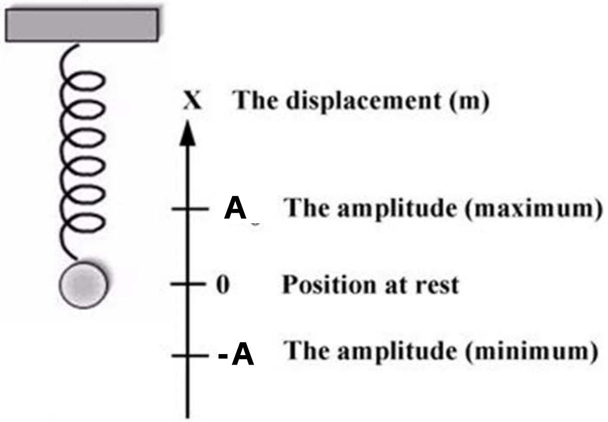


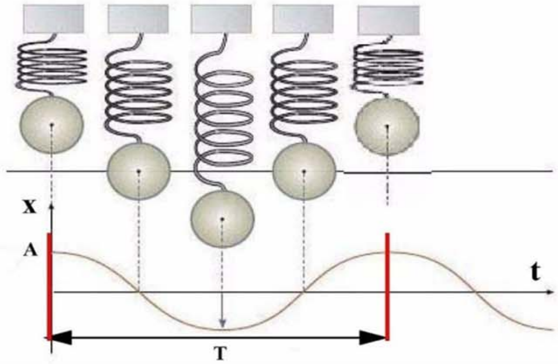
SUMMARY OF ALL SUMMARIES

SUMMARY

Chapter 14
Harmonic oscillation

Harmonic oscillation: Summary





$\phi = \text{acos}(x_0 / A) = \text{acos}(A/A) = 0$

- x The displacement (m)
- A The amplitude (m)
- t Time (s)
- T Period (s)
- f Frequency (Hz) = 1 / T
- ω Angular Frequency (Hz) = 2π / T = 2 π f

$x = A\cos(\omega t+\phi) \rightarrow x_{max} = A$

$v = \frac{dx}{dt} = -\omega A\sin(\omega t+\phi) \rightarrow v_{max} = \omega A$

$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t+\phi) \rightarrow a_{max} = \omega^2 A$

Harmonic oscillation: Summary

Harmonic oscillations in a spring are described by the equation:

$$\frac{\partial^2 x}{\partial t^2} + \frac{k}{m}x = 0$$

if F = -kx

(F: Spring force; k: Spring constant)

which has the solution:

$$x = A\cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Kinetic energy: $E_k = \frac{1}{2}mv^2 = \frac{1}{2}kA^2\sin^2(\omega t)$

Potential energy: $E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t)$

Total energy: $E_t = E_p + E_k = \frac{1}{2}kA^2$

SUMMARY

Chapter 15
Mechanical waves

Mechanical waves: Summary

The sinusoidal oscillations on a string are described by the wave equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

which has the wavefunction as a solution:

$$y(x,t) = A\cos(kx-\omega t)$$

Velocity and acceleration are obtained by derivation:

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$

Wave velocity:

$$v = \lambda/T = \omega/k$$

$$v = \sqrt{\frac{F}{\mu}}$$

Mechanical waves: Summary

Average power:

$$P_{av} = \frac{1}{2}\mu(\omega A)^2 v = \frac{1}{2}\sqrt{\mu F}(\omega A)^2$$

The power function:

$$P(x,t) = 2P_{av}\sin^2(kx-\omega t)$$

Wavefunction for a standing wave:

$$y(x,t) = 2A\sin(kx)\sin(\omega t)$$

Fundamental frequency:

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

$$f_n = nf_1 \quad n = 2, 3, 4\ldots$$

Length of a string

SUMMARY

Chapter 16
Sound

Sound: Summary

Wavefunction:

$$y(x,t) = A\cos(kx - \omega t)$$

Pressure function:

$$p(x,t) = BkA\sin(kx - \omega t) \;\Rightarrow\; p_{\max} = BkA$$

Pressure amplitude

Speed of sound:

$$v = f\cdot\lambda = \frac{\omega}{k} = \sqrt{\frac{B}{\rho}}$$

B: Bulk modulous

ρ: Density

Intensity (average power per unit area)

$$I = P_{av} / Area = \frac{1}{2}B\omega k A^2 = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

The power function:

$$P(x,t) = 2I\sin^2(kx-\omega t)$$

Sound: Summary

The inverse-square law:

$$\frac{I_1}{I_2} = \frac{r_2^{\,2}}{r_1^{\,2}}$$

r_2: Distance to location 2

r_1: Distance to location 1

Intensity level (decibel):

$$\beta = 10\,log\,\frac{I}{I_0}$$

$I_0 = 10^{-12}\ W/m^2$

$$I = I_0 10^{\beta/10}$$

Doppler effect:

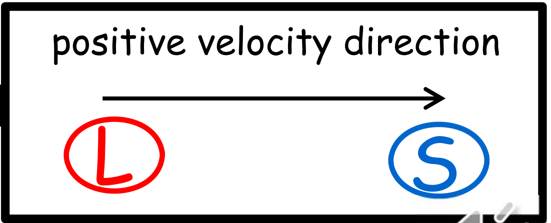
$$f_L = \frac{v+v_L}{v+v_S} f_S$$

v_L: Velocity listener

f_S: Frequency of source

f_L: Frequency for listener

v_S: Velocity source



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SUMMARY

Chapter 32
Electromagnetic waves

Electromagnetic waves: Summary

Wavefunction:

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

$$E = cB$$

Speed of light:

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

c = λ/T = ω/k

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

Energy density:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$u_B = \frac{B^2}{2\mu_0}$$

Permittivity

Permeability

Electromagnetic waves: Summary

Power per unit area:

$$S_x(x,t) = 2S_{av}\cos^2(kx-\omega t)$$

Intensity =
Average power per unit area:

$$S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^{\ 2}}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c E_{max}^{\ \ 2}$$

Comparing all wave functions

String: $y(x,t) = A\cos(kx-\omega t)$ $y(x,t) = 2A\sin(kx)\sin(\omega t)$

Sound: $y(x,t) = A\cos(kx-\omega t)$ $p(x,t) = p_{max}\sin(kx-\omega t)$

EM waves: $E(x,t) = E_{max}\cos(kx-\omega t)$ $B(x,t) = B_{max}\cos(kx-\omega t)$

Comparing all power functions

String: $P(x,t) = 2P_{av}\sin^2(kx-\omega t)$

$$P_{av} = \frac{1}{2}\mu(\omega A)^2 v = \frac{1}{2}\sqrt{\mu F}(\omega A)^2$$

Sound: $P(x,t) = 2I\sin^2(kx-\omega t)$

$$I = \frac{1}{2}\rho(\omega A)^2 v = \frac{1}{2}\sqrt{\rho B}(\omega A)^2 = \frac{p_{max}{}^2}{2\rho v} = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

EM Waves: $S_x(x,t) = 2S_{av}\cos^2(kx-\omega t)$

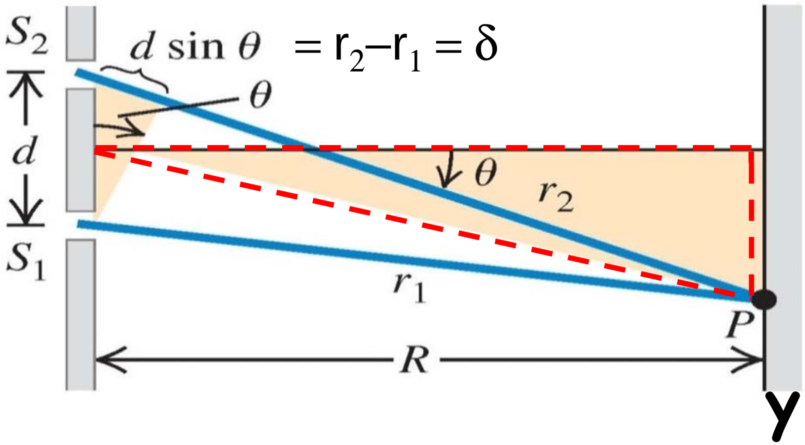
$$S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c E_{max}{}^2$$

SUMMARY

Chapter 35
Interference

Interference: Summary

Young's double-slit experiment



Constructive interference:

$$d\sin\theta = m\lambda \qquad y_m = R\frac{m\lambda}{d} \qquad m = 0, \pm 1 \ldots$$

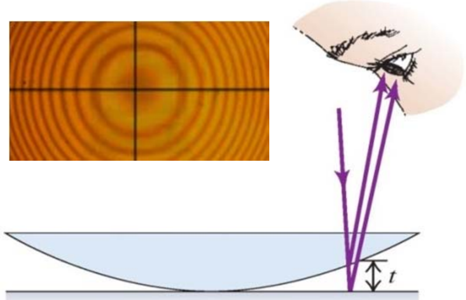
Destructive interference:

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = 0, \pm 1 \ldots.$$

$$I = I_0 \cos^2\frac{\phi}{2}$$

$$\phi = \frac{2\pi\delta}{\lambda} \approx \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d y}{\lambda R}$$

Thin film & Newton's rings



Phase shift = 0 & π

Constructive reflections:

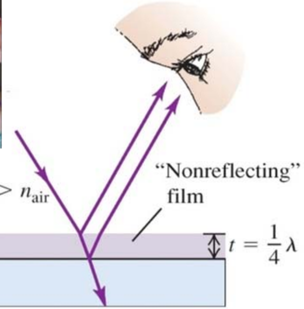
$$2t = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \ldots)$$

Destructive reflections:

$$2t = m\lambda \qquad (m = 0, 1, 2, \ldots)$$

Interference: Summary

Non-reflecting coating

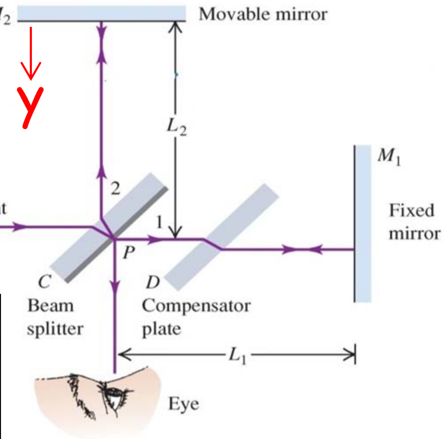


Phase shift = π & π

$$t = \lambda_{film} / 4$$

$$\lambda_{air} = \lambda_{film}\, n_{film}$$

The Michelson Interferometer



$$y = m\frac{\lambda}{2}$$

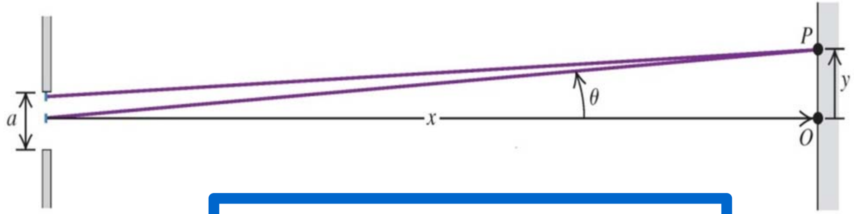
$$\lambda = \frac{2y}{m}$$

SUMMARY

Chapter 36
Diffraction

Diffraction: Summary

One broad slit:



$$\tan(\Theta) = y / x \approx \sin(\Theta)$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

Two broad slits:

$$I = I_0 \cos^2\frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta$$

$$\beta = \frac{2\pi a}{\lambda} \sin\theta$$

Multiple slits:

Path difference for principal maxima: $\delta = d \sin(\theta) = m\lambda$

Chromatic resolving power: $R = \frac{\lambda}{\Delta\lambda} = Nm$