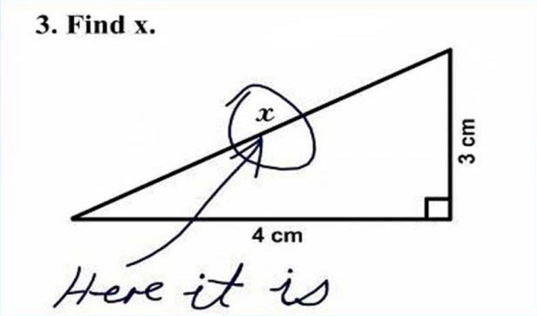
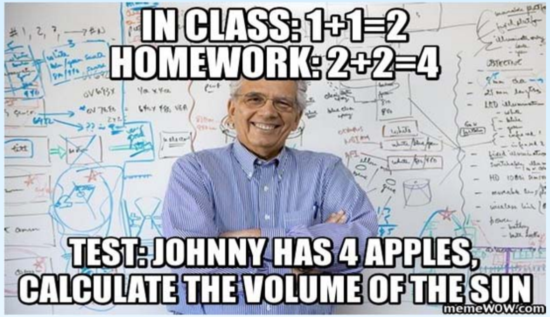
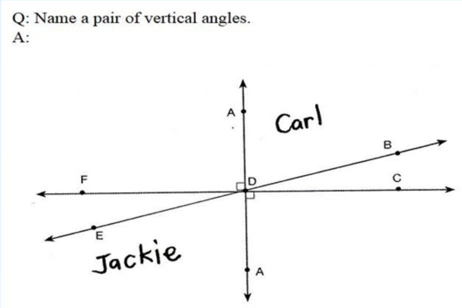
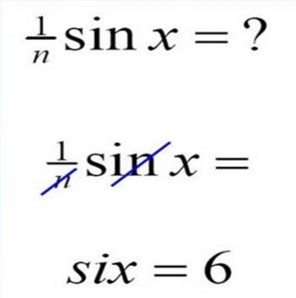
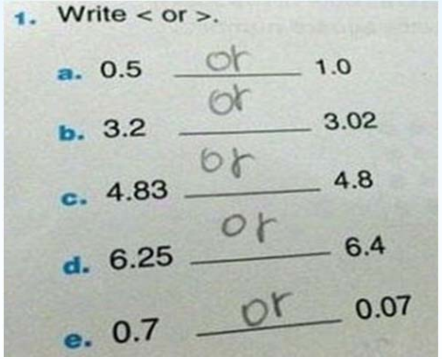
Wavemechanics

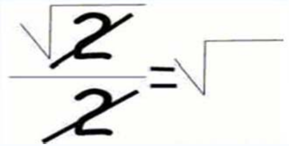


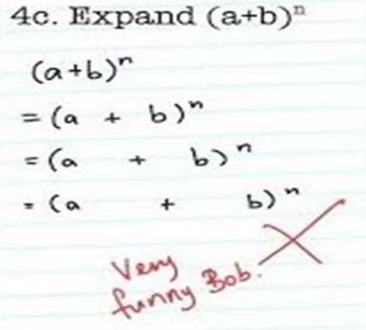


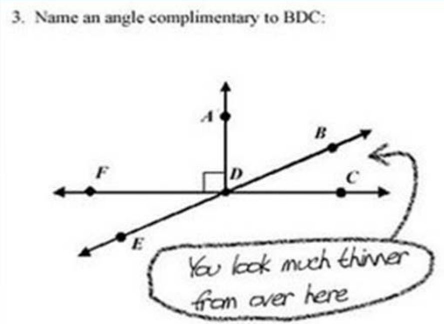


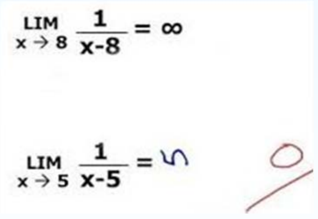












Problem solving

Harmonic oscillation: Problems

An ultrasonic device uses sound at a frequency of 6.7×10^6 Hz.

How long does each oscillation take and what angular frequency does this correspond to ?

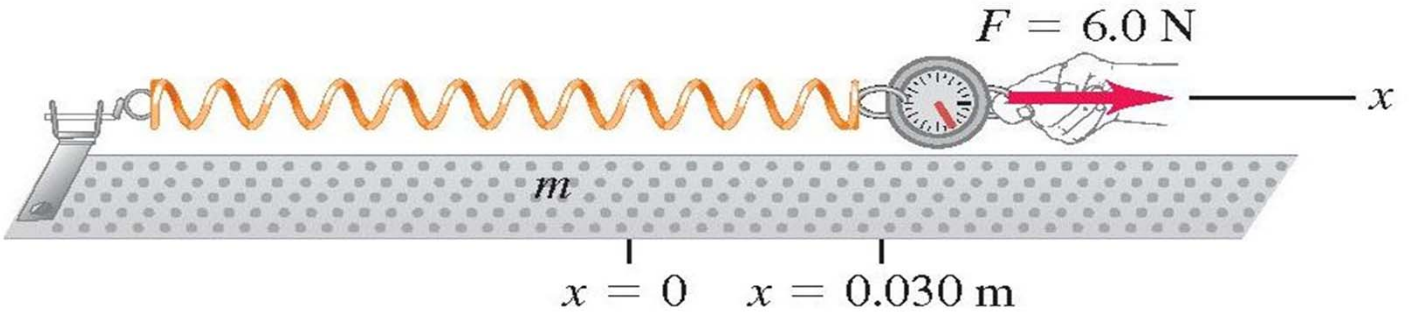
$f = 1/T$

$\omega = 2\pi f$

$$T = \frac{1}{f} = \frac{1}{6.7 \times 10^6\ \text{Hz}} = 1.5 \times 10^{-7}\ \text{s} = 0.15\ \mu\text{s}$$

$$\begin{aligned}\omega &= 2\pi f = 2\pi(6.7 \times 10^6\ \text{Hz}) \\ &= (2\pi\ \text{rad/cycle})(6.7 \times 10^6\ \text{cycle/s}) \\ &= 4.2 \times 10^7\ \text{rad/s}\end{aligned}$$

Harmonic oscillation: Problem



What is the spring constant ?

Note: The spring force is in the negative direction

$F = ma$

$N = kg\ m/s^2$

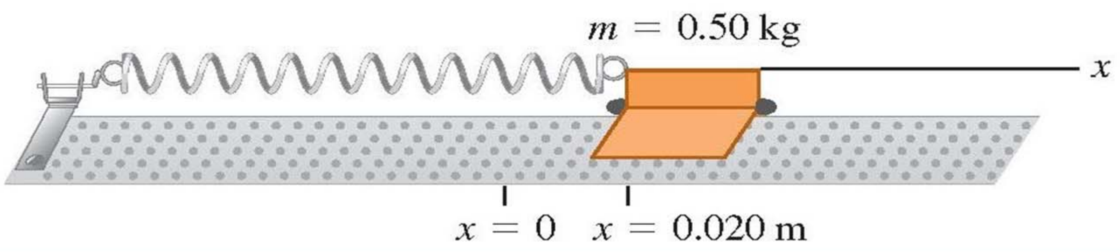
Hooke's law for a spring

$$F = -kX$$

$$k = -\frac{F_x}{x} = -\frac{-6.0\ \mathrm{N}}{0.030\ \mathrm{m}} = 200\ \mathrm{N/m} = 200\ \mathrm{kg/s^2}$$

Harmonic oscillation: Problem

$k = 200\ kg/s^2$



The mass is withdrawn 2 cm and released.
What will be the angular frequency, frequency and period of the oscillations?

$\omega = 2\pi f$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \text{kg/s}^2}{0.50\ \text{kg}}} = 20\ \text{rad/s}$$

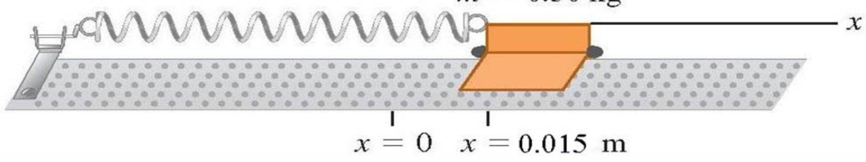
$$f = \frac{\omega}{2\pi} = \frac{20\ \text{rad/s}}{2\pi\ \text{rad/cycle}} = 3.2\ \text{cycle/s} = 3.2\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{3.2\ \text{cycle/s}} = 0.31\ \text{s}$$

Harmonic oscillation: Problem

$k = 200\ kg/s^2$
$\omega = 20\ rad/s$

$t = 0$
$x_0 = 0.015\ m$
$v_0 = +0.40\ m/s$



What is the amplitude and the phase angle ?

$x = A\cos(\omega t+\phi) \longrightarrow x_{max} = A$

$v = \frac{dx}{dt} = -\omega A\sin(\omega t+\phi) \longrightarrow v_{max} = \omega A$

$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a_{max} = \omega^2 A$

t = 0

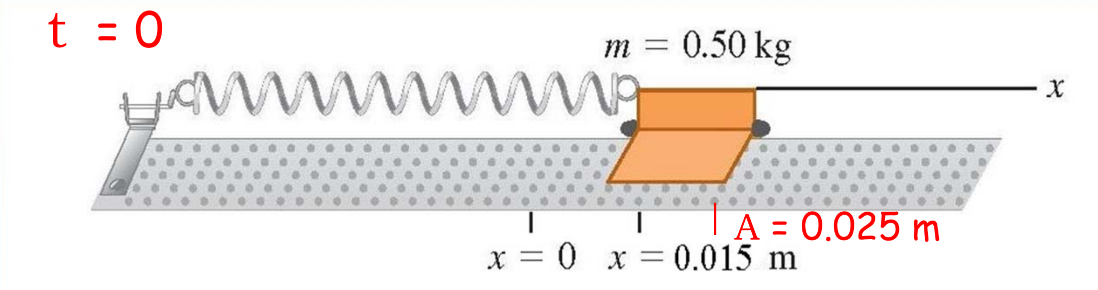
$$x_0 = A\cos\phi$$
$$v_{0x} = -\omega A\sin\phi$$
$$\frac{v_{0x}}{x_0} = \frac{-\omega A\sin\phi}{A\cos\phi} = -\omega\tan\phi$$

$$\phi = \arctan\left(-\frac{v_{0x}}{\omega x_0}\right) = \arctan\left(-\frac{0.40\ \text{m/s}}{(20\ \text{rad/s})(0.015\ \text{m})}\right) = -53^\circ = -0.93\ \text{rad}$$

$$A = x_0 / \cos\phi = 0.015 / \cos(-0.93) = 0.025\ \text{m}$$

Harmonic oscillation: Problem

k = 200 kg/s²
ω = 20 rad/s
ϕ = -0.93 rad
A = 0.025 m



What are the functions for position, velocity and acceleration ?

$$x = A\cos(\omega t+\phi) \longrightarrow x_{max} = A$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t+\phi) \longrightarrow v_{max} = \omega A$$

$$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a_{max} = \omega^2 A$$

$$x = (0.025\ \text{m})\cos[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$

$$v_x = -(0.50\ \text{m/s})\sin[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$

$$a_x = -(10\ \text{m/s}^2)\cos[(20\ \text{rad/s})t - 0.93\ \text{rad}]$$

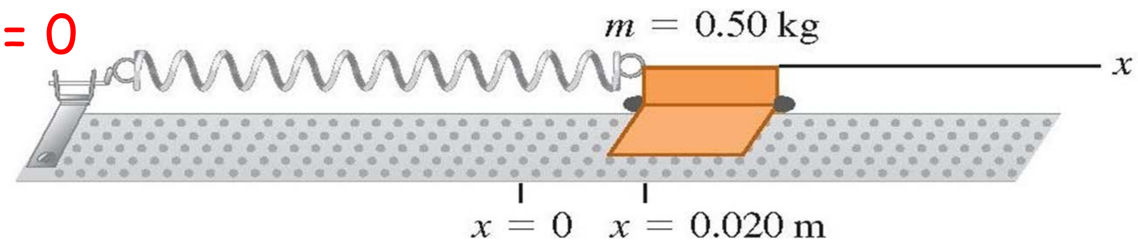
Harmonic oscillation: Problem

$A = 0.020$ m

$k = 200$ N/m

$m = 0.50$ kg

t = 0



What is v_{max}, a_{max} and ω ?

$$x = A\cos(\omega t+\phi) \longrightarrow x_{max} = A$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t+\phi) \longrightarrow v_{max} = \omega A$$

$$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a_{max} = \omega^2 A$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \text{kg/s}^2}{0.50\ \text{kg}}} = 20\ \text{rad/s}$$

$$v_{max} = 20 \cdot 0.020 = 0.40\ \text{m/s}$$

$$a_{max} = 20 \cdot 20 \cdot 0.020 = 8\ \text{m/s}^2$$

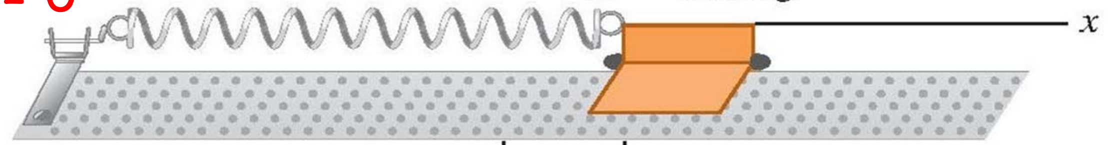
Harmonic oscillation: Problem

$A = 0.020\ \text{m}$

$k = 200\ \text{N/m}$

$m = 0.50\ \text{kg}$

$\omega = 20\ \text{rad/s}$



What is the phase angle ?

$$x = A\cos(\omega t+\phi) \longrightarrow x_{max} = A$$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t+\phi) \longrightarrow v_{max} = \omega A$$

$$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t+\phi) \longrightarrow a_{max} = \omega^2 A$$

Getting the phase angle:

$x = A$ when $t = 0$

$A = A\cos(0 + \phi)$

$\phi = 0$

Harmonic oscillation: Problem

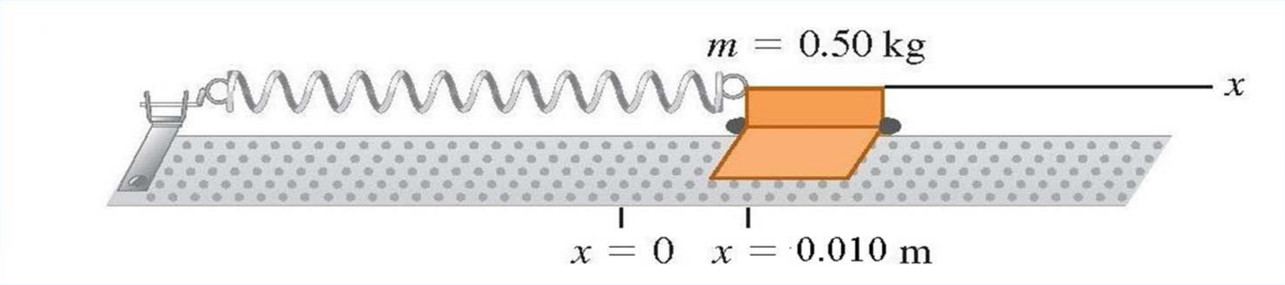
$A = 0.020$ m

$k = 200$ N/m

$m = 0.50$ kg

$\omega = 20$ rad/s

$\phi = 0$



What is v and a when x is halfway in from the maximum position ?

$x = A\cos(\omega t) \longrightarrow x_{max} = A$

$v = \frac{dx}{dt} = -\omega A\sin(\omega t) \longrightarrow v_{max} = \omega A$

$a = \frac{dv}{dt} = -\omega^2 A\cos(\omega t) \longrightarrow a_{max} = \omega^2 A$

$x = A\cos(\omega t)$

$0.010 = 0.020\cos(20t)$

$\omega t = 20t = \mathrm{acos}(0.010/0.020) = 1.047$ rad

$V = -20 \cdot 0.020 \sin(1.047) = -0.35$ m/s

$a = -20^2 \cdot 0.020 \cos(1.047) = -4.0$ m/s^2

Harmonic oscillation: Problem

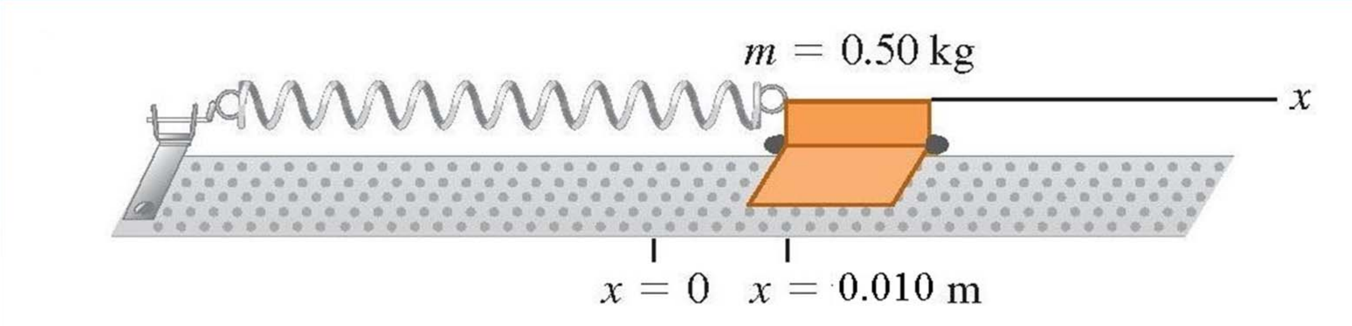
A = 0.020 m

k = 200 N/m

m = 0.50 kg

ω = 20 rad/s

$\phi = 0$
x = 0.010 m
v = -0.35 m/s



What is the kinetic, potential and total energy ?

Kinetic energy: $E_k = \frac{mv^2}{2}$ where $v = -\omega A \sin(\omega t)$

Potential energy: $E_p = \frac{kx^2}{2}$ where $x = A\cos(\omega t)$

Total energy: $E_t = E_k + E_p = \frac{kA^2}{2}$ ($E_k = 0$ for $x = A$)

$$Ep = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200\ \text{N/m})(0.010\ \text{m})^2 = 0.010\ \text{J}$$

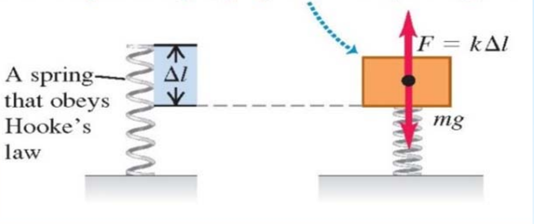
$$Ek = \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}(0.50\ \text{kg})(-0.35\ \text{m/s})^2 = 0.030\ \text{J}$$

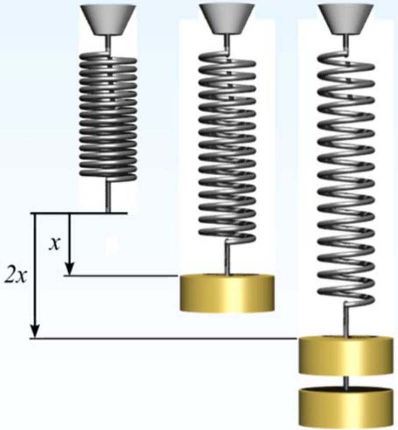
$$E_T = E_p + E_k = 0.040\ J$$

Harmonic oscillation: Problem

Assume the following: A car has a mass of 1000 kg. A drivers weight is F = 980 N and causes the shock absorbers to drop by 2.8 cm. The car drives over a bump and begins to swing by harmonic oscillation.

What will be the period and frequency ?





$F_x = -kx$

$f = 1/T$

$\omega = 2\pi f$

$$\omega = \sqrt{\frac{k}{m}}$$

$$k = -\frac{F_x}{x} = -\frac{980\ \text{N}}{-0.028\ \text{m}} = 3.5 \times 10^4\ \text{kg/s}^2$$

The person's mass is $w/g = (980\ \text{N})/(9.8\ \text{m/s}^2) = 100\ \text{kg}$. The *total* oscillating mass is $m = 1000\ \text{kg} + 100\ \text{kg} = 1100\ \text{kg}$. The period T is

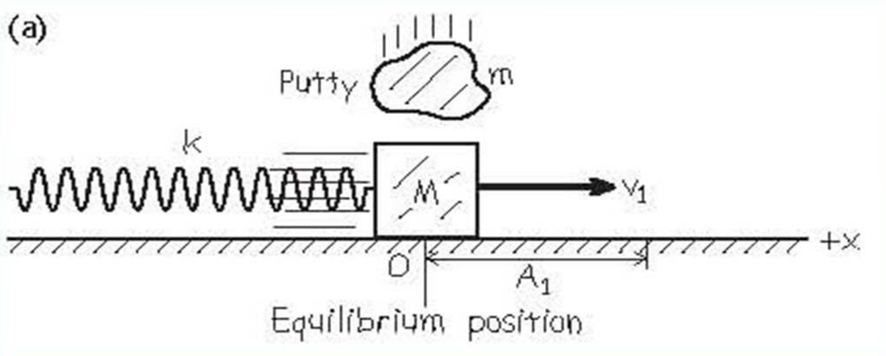
$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1100\ \text{kg}}{3.5 \times 10^4\ \text{kg/s}^2}} = 1.11\ \text{s}$$

The frequency is $f = 1/T = 1/(1.11\ \text{s}) = 0.90\ \text{Hz}$.

Harmonic oscillation: Problem

A lump of clay with the mass m falls on a moving mass M at the equilibrium position.

Calculate the new period T_2 !
Give the result as a function of
k, m, M !



f = 1/T
$\omega = 2\pi f$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

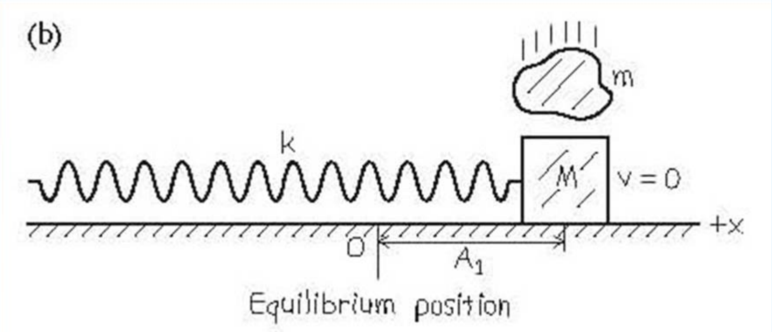
The new period T_2:

$$T_2 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M+m}{k}}$$

Harmonic oscillation: Problem

A lump of clay with the mass m falls on a moving mass M at the maximum position.

Calculate the new period T_2 and the new amplitude A_2!



f = 1/T

ω = 2πf

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$T_2 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M+m}{k}}$$

For x = A the kinetic energy = 0:

$$E_{t1} = 0 + E_{p1} = \frac{1}{2}kA_1^2$$

$$E_{t2} = 0 + E_{p2} = \frac{1}{2}kA_2^2$$

The total energy is conserved: $E_{t1} = E_{t2}$ och $A_2 = A_1$

Mechanical waves: Problems

The speed of sound depends on the temperature and is 344 m/s at 20 degrees.

What, then, is the wavelength of sound with the frequency 262 Hz ?

$$v = f \lambda$$

$$\lambda = \frac{v}{f} = \frac{344\ \text{m/s}}{262\ \text{Hz}} = \frac{344\ \text{m/s}}{262\ \text{s}^{-1}} = 1.31\ \text{m}$$

Mechanical waves: Problems

You wave a rope up and down and create a sine wave with the frequency 2.00 Hz, the amplitude 0.075 m and the wave speed 12.0 m/s.

Calculate the period, the wavelength, the angular frequency and the wave number !

$f = 1/T$
$\omega = 2\pi f$
$v = f\lambda$
$k = 2\pi/\lambda$

Given:

A: Amplitude = 0.075 m

f: Frequency = 1 / T = 2.00 Hz

v: Wave speed = λ / T = 12.0 m/s

To calculate:

T: Period = 1 / f = 0.5 s

λ: Wavelength = v T = 6.00 m

ω: Angular frequency = 2 π f = 4π rad/s

k: Wave number = 2 π / λ = ⅓π rad/m

Mechanical waves: Problems

At t = 0, the rope you hold in your hand (x = 0) is in its highest position (0.075 m).

What is the wave function for the oscillations ?

What will be the wave function at x = 0 and x = 3.00 m ?

Calculated previously:

ω: Angular frequency $= 2\pi f = 4\pi$ rad/s

k: Wave number $= 2\pi / \lambda = \frac{1}{3}\pi$ rad/m

$$y(x,t) = A\cos(kx - \omega t) = 0.075\cos(\tfrac{1}{3}\pi x - 4\pi t)$$

$$y(0,t) = 0.075\cos(-4\pi t) = 0.075\cos(4\pi t)$$

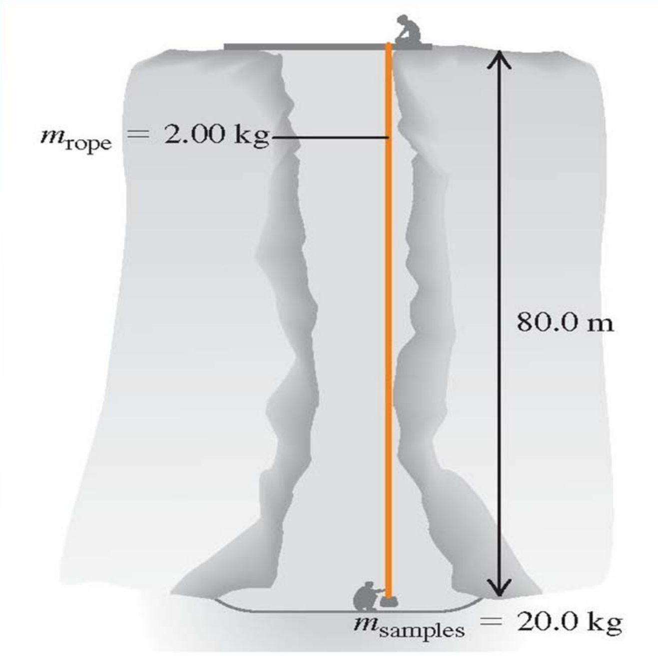
cos(-x) = cos (x)

$$y(3,t) = 0.075\cos(\pi - 4\pi t) = -0.075\cos(-4\pi t) = -0.075\cos(4\pi t)$$

cos(π-x) = -cos (x)

Mechanical waves: Problems

A man in a hole sends a signal by making a wave on a rope at whose end it hangs a weight of 20 kg. What is the speed of the wave in the rope? If the rope is put into sinus oscillation with f = 2Hz, how many wavelengths can fit on the rope ?



The tension in the rope due to the box is

$$F = m_{box}g = (20.0\,\text{kg})(9.80\,\text{m/s}^2) = 196\,\text{N}$$

and the rope's linear mass density is

$$\mu = \frac{m_{rope}}{L} = \frac{2.00\,\text{kg}}{80.0\,\text{m}} = 0.0250\,\text{kg/m}$$

the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{196\,\text{N}}{0.0250\,\text{kg/m}}} = 88.5\,\text{m/s}$$

the wavelength is

$$\lambda = \frac{v}{f} = \frac{88.5\,\text{m/s}}{2.00\,\text{s}^{-1}} = 44.3\,\text{m}$$

There are $(80.0\,\text{m})/(44.3\,\text{m}) = 1.81$ wavelengths (that is, cycles of the wave) in the rope.

Mechanical waves: Problems

A: Amplitude = 0.075 m

f: Frequency = 1 / T = 2.00 Hz

v: Wave speed = λ / T = 12.0 m/s

T: Period = 1 / f = 0.5 s

λ: Wavelength = v T = 6.00 m

ω: Angular frequency = 2 π f = 4π

k: Wave number = 2 π / λ = ⅓π

μ: Linear mass density = 0.250 kg/m

F: Tension = 36.0 N

You swing a rope up and down and create a sine wave with the frequency 2.00 Hz, the amplitude 0.075 m and the wave speed 12.0 m/s. The rope weighs 250 grams per meter and is tensioned with the force of 36.0 N.

Calculate the maximum power and average power needed.

$$P_{\max} = \sqrt{\mu F}\omega^2 A^2$$

$$= \sqrt{(0.250\ \mathrm{kg/m})(36.0\ \mathrm{N})}(4.00\pi\ \mathrm{rad/s})^2(0.075\ \mathrm{m})^2$$

$$= 2.66\ \mathrm{W}$$

$$P_{\mathrm{av}} = \tfrac{1}{2}P_{\max} = \tfrac{1}{2}(2.66\ \mathrm{W}) = 1.33\ \mathrm{W}$$

Mechanical waves: Problems

A sine wave moves in negative x-direction along a guitar string at the speed of 143 m/s.
The amplitude is 0.750 mm and the frequency 440 Hz.
The wave is reflected at x = 0 and forms a standing wave.

What will be the function that describes the movement of the string in the y-direction ?

$$y(x,t) = 2A\sin(kx)\sin(\omega t)$$

$$A = 0.750\text{ mm} = 7.50\times10^{-4}\text{ m}$$

$$\omega = 2\pi f = (2\pi\text{ rad})(440\text{ s}^{-1}) = 2760\text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{2760\text{ rad/s}}{143\text{ m/s}} = 19.3\text{ rad/m}$$

Mechanical waves: Problems

$v = 143$ m/s
$f = 440$ Hz
$A = 0.00075$ m
$\omega = 2760$ rad/s
$k = 19.3$ rad/m

Where will there be nodes on the string ?

There will be nodes for $$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

$f = v / \lambda$ ⟹ $\lambda = v / f = 143 / 440 = 0.325$ m

There will be nodes for $x = 0, 0.163$ m, 0.325 m,

Mechanical waves: Problems

$v = 143$ m/s
$f = 440$ Hz
$A = 0.00075$ m
$\omega = 2760$ rad/s
$k = 19.3$ rad/m

What is the amplitude of the standing wave ?
What will be the maximum speed and the maximum acceleration ?

$$y(x,t) = 2A \sin(kx) \sin(\omega t)$$

→ Amplitude = 2A = 0.0015 m

$$v_y(x,t) = 2A\omega \sin(kx) \cos(\omega t)$$

$$v_y(x,t)_{max} = 2A\omega = 4.14 \text{ m/s}$$

$$a_y(x,t) = -2A\omega^2 \sin(kx) \sin(\omega t)$$

$$a_y(x,t)_{max} = 2A\omega^2 = 11426 \text{ m/s}^2$$

Mechanical waves: Problems

An octobasse has a string that is 2.50 m long and weighs 40.0 grams per meter.

What tension force is needed for the fundamental frequency to be 20.0 Hz ?

$$v = \sqrt{\frac{F}{\mu}} \qquad f_n = \frac{nv}{2L}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

$$F = 4\mu L^2 f_1^{\,2} = 4(40.0 \times 10^{-3}\,\text{kg/m})(2.50\,\text{m})^2(20.0\,\text{s}^{-1})^2 = 400\ \text{N}$$

Mechanical waves: Problems

f_1 = 20.0 Hz
L = 2.50 m
μ = 40.0 g/m
F = 400 N

What will be the frequency and wavelength of the second harmonic frequency ?

What will be the frequency and wavelength of the second overtone ?

$$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots)$$

$$\lambda_n = \frac{2L}{n} \quad (n = 1, 2, 3, \ldots)$$

$$f_2 = 2f_1 = 2(20.0\ \text{Hz}) = 40.0\ \text{Hz}$$

$$\lambda_2 = \frac{2L}{2} = \frac{2(2.50\ \text{m})}{2} = 2.50\ \text{m}$$

The second overtone is the second frequency above the fundamental frequency, i.e. n = 3

$$f_3 = 3f_1 = 3(20.0\ \text{Hz}) = 60.0\ \text{Hz}$$

$$\lambda_3 = \frac{2L}{3} = \frac{2(2.50\ \text{m})}{3} = 1.67\ \text{m}$$

Mechanical waves: Problems

$f_1 = 20.0$ Hz
$L = 2.50$ m
$\mu = 40.0$ g/m
$F = 400$ N
$\lambda_1 = 1.25$ m

The string vibrates at its fundamental frequency.

What is the frequency and wavelength of the sound it emits ?

The speed of sound is 344 m/s.

$$v = \lambda / T = \lambda f$$
$$\lambda = v / f$$

$$f = f_1 = 20.0 \text{ Hz}$$
$$\lambda_{1(\text{sound})} = \frac{v_{\text{sound}}}{f_1} = \frac{344 \text{ m/s}}{20.0 \text{ Hz}} = 17.2 \text{ m}$$

Sound: Problems

A sinusoidal sound wave has a frequency of 1000 Hz and a pressure amplitude of 3.0×10^{-2} Pa.

Air: $v = 344$ m/s, $B = 1.42 \times 10^5$ Pa

What will be the maximum movement of the air due to this sound wave ?

$$p_{max} = BkA$$

$$k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(1000 \text{ Hz})}{344 \text{ m/s}} = 18.3 \text{ rad/m}$$

$$A = \frac{p_{max}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(18.3 \text{ rad/m})} = 1.2 \times 10^{-8} \text{ m}$$

Sound: Problems

A human can hear frequencies between 20 and 20000 Hz. What wavelengths does this correspond to ?

Assume that v = 344 m/s

$$v = f \cdot \lambda = \frac{\omega}{k}$$

$$\lambda = v / f$$

$$\lambda = 344 / 20 = 17 \text{ m} \quad \text{for } f = 20 \text{ Hz}$$

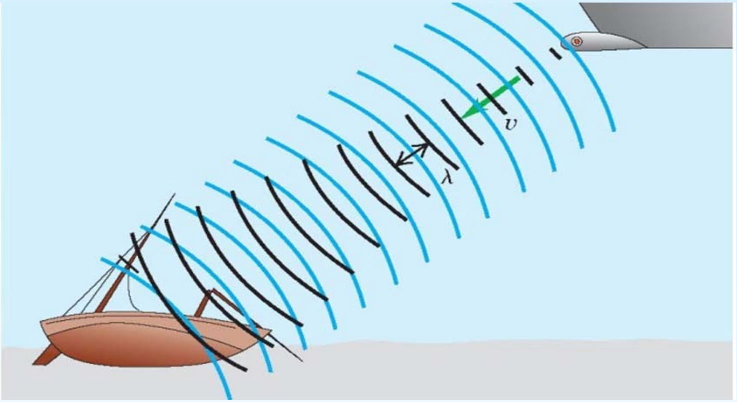
$$\lambda = 344 / 20000 = 1.7 \text{ cm} \quad \text{for } f = 20 \text{ kHz}$$

Sound: Problems

A sonar system sends out sound waves at a frequency of 262 Hz.

What will be the speed and wavelength of this sound wave if $B = 2.18 \times 10^9$ Pa ?

What will be the velocity and wavelength of the wave in air if $B = 1.42 \times 10^5$ Pa and the density 1.225 kg/m^3



$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \times 10^9 \text{ Pa}}{1.00 \times 10^3 \text{ kg/m}^3}} = 1480 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{1480 \text{ m/s}}{262 \text{ s}^{-1}} = 5.65 \text{ m}$$

v = 340 m/s in air

λ = 1.3 m in air

Sound: Problems

A siren sends out sound waves uniformly in all directions. The sound intensity is 0.250 W/m^2 at a distance of 15.0 m.

At what distance is the intensity 0.010 W/m^2 ?

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(inverse-square law for intensity)}$$

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (15.0\ \mathrm{m})\sqrt{\frac{0.250\ \mathrm{W/m^2}}{0.010\ \mathrm{W/m^2}}} = 75.0\ \mathrm{m}$$

Sound: Problems

Calculate the sound intensity if the pressure amplitude is 3.0×10^{-2} Pa, the air density is 1.20 kg/m^3 and the speed of sound is 344 m/s !

$$I = \frac{{p_{max}}^2}{2\sqrt{\rho B}}$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$\sqrt{\rho B} = v\,\rho$$

$$I = \frac{{p_{\max}}^2}{2\rho v} = \frac{(3.0 \times 10^{-2}\ \mathrm{Pa})^2}{2(1.20\ \mathrm{kg/m^3})(344\ \mathrm{m/s})}$$

$$= 1.1 \times 10^{-6}\ \mathrm{J/(s \cdot m^2)} = 1.1 \times 10^{-6}\ \mathrm{W/m^2}$$

Sound: Problems

What is the pressure amplitude of a sound wave with f = 20 Hz if it has the same intensity as a sound wave with f = 1000 Hz, $I = 1.1 \times 10^{-6}$ W/m^2 and $p_{max} = 3.0 \times 10^{-2}$ Pa. Assume that $\rho = 1.20$ kg/m^3 and $v = 344$ m/s

Wave 1: f = 1000 Hz, $p_{max} = 3.0 \times 10^{-2}$ Pa, $\rho = 1.20$ kg/m^3, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m^2

Wave 2: f = 20 Hz, p_{max} = ???????????, $\rho = 1.20$ kg/m^3, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m^2

$$I = \frac{{p_{max}}^2}{2\sqrt{\rho B}}$$

Since ρB = constant and $I_1 = I_2$ then follows that $p_{max2} = p_{max1} = 3.0 \times 10^{-2}$ Pa

Wave 2: f = 20 Hz, $p_{max} = 3.0 \times 10^{-2}$ Pa, $\rho = 1.20$ kg/m^3, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m^2

Sound: Problems

What is the displacement amplitude of Wave 2 in the previous problem ?

Wave 2: $f = 20$ Hz, $p_{max} = 3.0 \times 10^{-2}$ Pa, $\rho = 1.20$ kg/m^3, $v = 344$ m/s, $I = 1.1 \times 10^{-6}$ W/m^2

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$$

$$I = \frac{p_{max}^2}{2\sqrt{\rho B}} \Rightarrow \sqrt{\rho B} = p_{max}^2/2I \Rightarrow I = (p_{max}^2/2I)\,\omega^2 A^2/2$$

$$I = (p_{max}^2/2I)\,\omega^2 A^2/2 \Rightarrow I^2 = p_{max}^2\omega^2 A^2/4 \Rightarrow I = p_{max}\omega A/2$$

$$A = 2I / p_{max}\omega = 2 \times 1.1 \times 10^{-6} / (3.0 \times 10^{-2} \times 2\pi \times 20) = 0.58\ \mu\text{m}$$

Sound: Problems

At a concert you want a sound intensity that is 1 W/m^2 at a distance of 20 m from the speakers.
What output power do the speakers need?

Intensity is the average power per unit area: $I = P_{av} / A_{rea}$

The intensity through a sphere with radius r: $I = \frac{P}{4\pi r^2}$

The intensity through a hemisphere with radius r: $I = \frac{P}{2\pi r^2}$

$$P = 2\,\pi\, r^2\, I = 2.5\ \text{kW}$$

Sound: Problems

After 10 minutes at 120 dB, the human hearing threshold is temporarily changed from 0 dB to 28 dB if f = 1000 Hz.

After 10 years of 92 dB, the limit for human hearing is permanently changed from 0 dB to 28 dB if f = 1000 Hz.

What sound intensity corresponds to 28 dB and 92 dB ?

$$\beta = 10\, log \frac{I}{I_0}$$

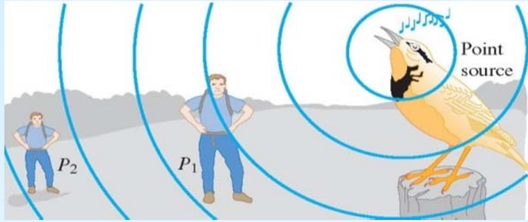
$$\mathrm{I} = I_0 \cdot 10^{\beta/10}$$ with $I_0 = 10^{-12}\ \mathrm{W/m^2}$

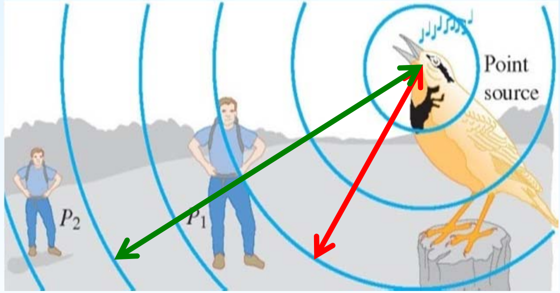
$$I_{28\ \mathrm{dB}} = (10^{-12}\ \mathrm{W/m^2})10^{2.8} = 6.3 \times 10^{-10}\ \mathrm{W/m^2}$$

$$I_{92\ \mathrm{dB}} = (10^{-12}\ \mathrm{W/m^2})10^{9.2} = 1.6 \times 10^{-3}\ \mathrm{W/m^2}$$

Sound: Problems

A bird sings with constant power.
How many decibels does the intensity level go down if the listener doubles the distance to the bird?





$r_2 = 2r_1$ | r_1
β_2 | β_1
I_2 | I_1

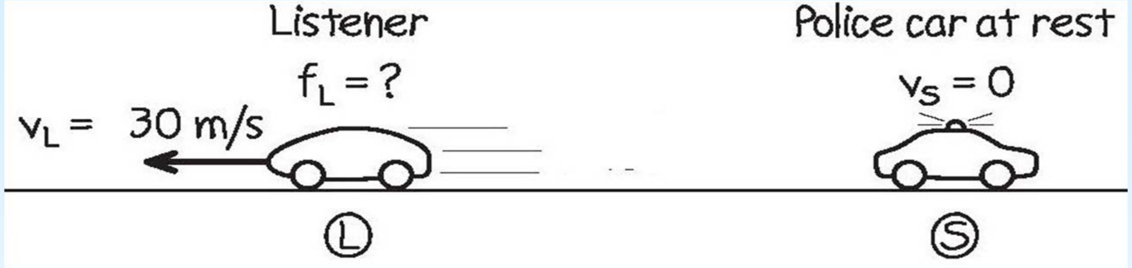
$$\frac{I_1}{I_2} = \frac{r_2^{\,2}}{r_1^{\,2}} = \frac{4r_1^{\,2}}{r_1^{\,2}} = 4$$

$$\beta = (10\text{ dB})\log\frac{I}{I_0}$$

$$\beta_2 - \beta_1 = (10\text{ dB})\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right)$$
$$= (10\text{ dB})[(\log I_2 - \log I_0) - (\log I_1 - \log I_0)]$$
$$= (10\text{ dB})\log\frac{I_2}{I_1}$$

$$\beta_2 - \beta_1 = (10\text{ dB})\log\frac{I_2}{I_1} = (10\text{ dB})\log\tfrac{1}{4} = -6.0\text{ dB}$$

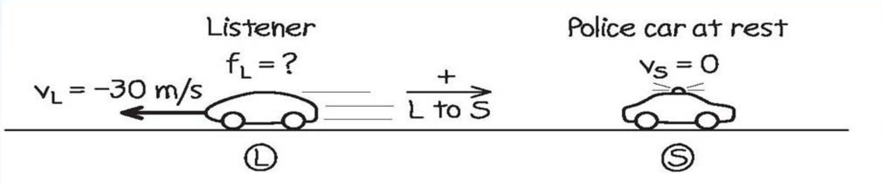
Sound: Problems



f = 300 Hz

Speed of sound = 340 m/s

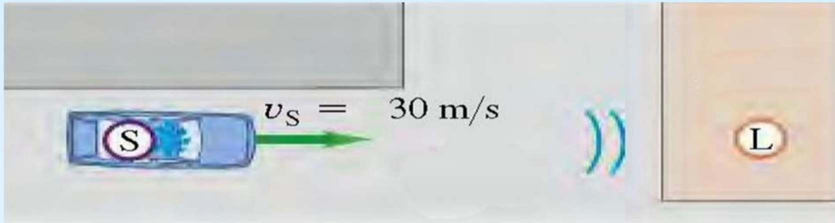
What frequency does the listener hear ?

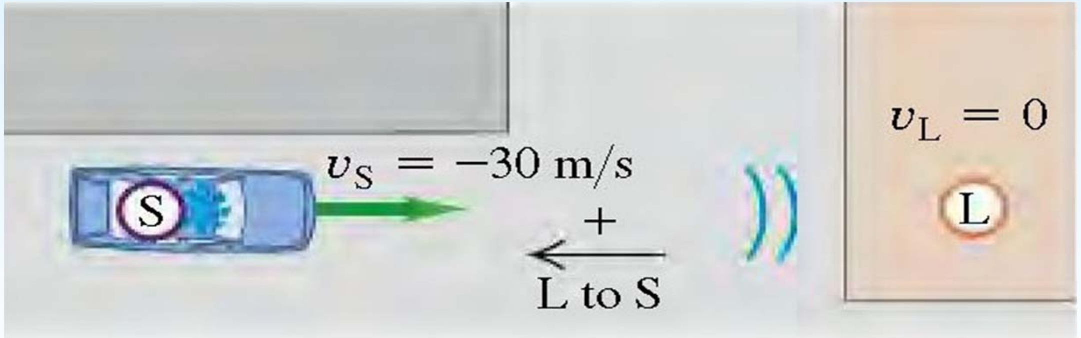


$$f_L = \frac{v + v_L}{v + v_S} f = \frac{340 \text{ m/s} + (-30 \text{ m/s})}{340 \text{ m/s}} (300 \text{ Hz}) = 274 \text{ Hz}$$

Sound: Problems

A police car with a siren of f = 300 Hz drives towards a house at the speed of 30 m/s. What frequency does a listener hear in the house?



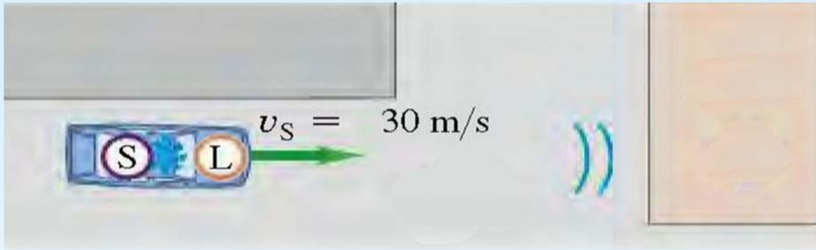


$$f_L = \frac{v + v_L}{v + v_S} f_s$$

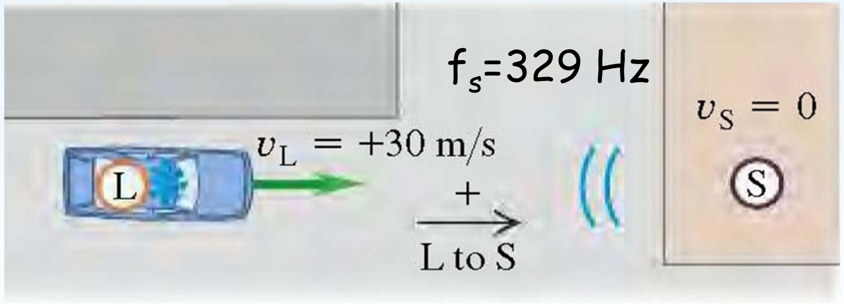
$$f_W = \frac{v}{v + v_S} f_S = \frac{340 \text{ m/s}}{340 \text{ m/s} + (-30 \text{ m/s})} (300 \text{ Hz}) = 329 \text{ Hz}$$

Sound: Problems

A police car with a siren of f = 300 Hz drives towards a house at the speed of 30 m/s. What frequency does a listener hear in the police car if the sound is reflected back to it?



The house becomes a sound source with the frequency 329 Hz as calculated earlier:



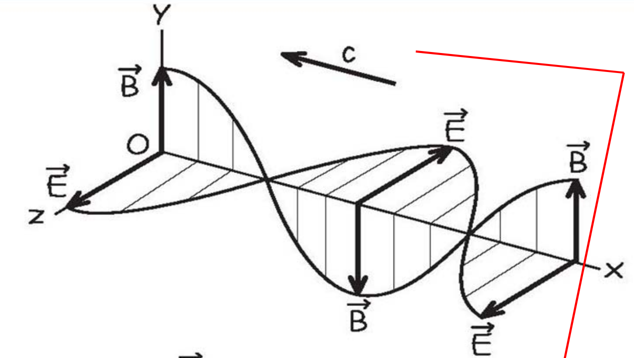
$$f_L = \frac{v+v_L}{v+v_S} f_S$$

$$f_L = \frac{v + v_L}{v} f_S = \frac{340\ \text{m/s} + 30\ \text{m/s}}{340\ \text{m/s}}(329\ \text{Hz}) = 358\ \text{Hz}$$

Electromagnetic waves: Problems

A laser sends out a sinusoidal electromagnetic wave in the negative x-direction with the wavelength 10.6 μm. The E-field is in the z-direction and E_{max} = 1.5 MV/m.

Give the wave function of the laser beam.



$$\vec{E}(x,t) = \hat{k}E_{max}\cos(kx + \omega t)$$
$$\vec{B}(x,t) = \hat{j}B_{max}\cos(kx + \omega t)$$

$E_{max} = c\,B_{max}$

$k = 2\pi/\lambda$

$c = \omega/k$

$$B_{max} = \frac{E_{max}}{c} = \frac{1.5 \times 10^6 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} = 5.0 \times 10^{-3} \text{ T}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{10.6 \times 10^{-6} \text{ m}} = 5.93 \times 10^5 \text{ rad/m}$$

$$\omega = ck = (3.00 \times 10^8 \text{ m/s})(5.93 \times 10^5 \text{ rad/m}) = 1.78 \times 10^{14} \text{ rad/s}$$

$$\vec{E}(x,t) = \hat{k}(1.5 \times 10^6 \text{ V/m}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

$$\vec{B}(x,t) = \hat{j}(5.0 \times 10^{-3} \text{ T}) \times \cos[(5.93 \times 10^5 \text{ rad/m})x + (1.78 \times 10^{14} \text{ rad/s})t]$$

Electromagnetic waves: Problems

Yellow light with f = 5.09x10^{14} Hz goes from vacuum into a diamond.

What is the wavelength in vacuum?
What is the wavelength and wave velocity in the diamond if K = 5.84 & K_m=1.00

$v=c=\lambda/T=\lambda f$

$$v = \frac{c}{\sqrt{KK_m}}$$

Vacuum:

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}} = 5.89 \times 10^{-7} \text{ m} = 589 \text{ nm}$$

Diamond:

$$v_{\text{diamond}} = \frac{c}{\sqrt{KK_m}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(5.84)(1.00)}} = 1.24 \times 10^8 \text{ m/s}$$

$$\lambda_{\text{diamond}} = \frac{v_{\text{diamond}}}{f} = \frac{1.24 \times 10^8 \text{ m/s}}{5.09 \times 10^{14} \text{ Hz}}$$
$$= 2.44 \times 10^{-7} \text{ m} = 244 \text{ nm}$$

Electromagnetic waves: Problems

Radio waves with 90.0 MHz go from vacuum into insulating ferrite.

What is the wavelength in vacuum?
What is the wavelength and wave velocity in the ferrite if K = 10.0 & K_m=1000 ?

$v=\lambda/T=\lambda f=c$

$$v = \frac{c}{\sqrt{KK_m}}$$

$$\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}} = 3.33 \text{ m}$$

$$v_{\text{ferrite}} = \frac{c}{\sqrt{KK_{\text{m}}}} = \frac{3.00 \times 10^8 \text{ m/s}}{\sqrt{(10.0)(1000)}} = 3.00 \times 10^6 \text{ m/s}$$

$$\lambda_{\text{ferrite}} = \frac{v_{\text{ferrite}}}{f} = \frac{3.00 \times 10^6 \text{ m/s}}{90.0 \times 10^6 \text{ Hz}}$$

$$= 3.33 \times 10^{-2} \text{ m} = 3.33 \text{ cm}$$

Electromagnetic waves: problems

A sinusoidal electromagnetic wave has E_{max} = 100 V/m. What is B_{max} ?
What is the maximum energy density?

Given:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{100\ \mathrm{V/m}}{3.00 \times 10^{8}\ \mathrm{m/s}} = 3.33 \times 10^{-7}\ \mathrm{T}$$

$$E(x,t) = E_{max} \cos(kx - \omega t)$$

$$u(x,t) = \varepsilon_0 E^2 = \varepsilon_0 E_{max}{}^2 \cos^2(kx - \omega t)$$

$$u_{max} = \varepsilon_0 E_{max}{}^2 = (8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(100\ \mathrm{N/C})^2 = 8.85 \times 10^{-8}\ \mathrm{N/m^2}$$

Electromagnetic waves: problems

A sinusoidal electromagnetic wave has
E_{max} = 100 V/m and B_{max} = 3.33x10^{-7} T.

What is the intensity?

Given:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

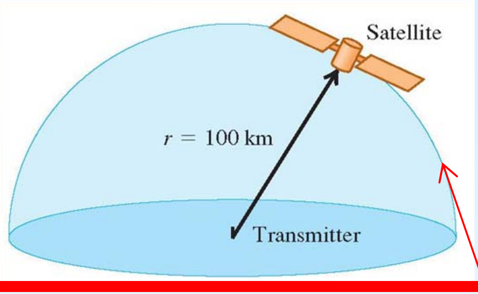
$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

$$S_x(x,t) = \frac{E_{max}B_{max}}{\mu_0}\cos^2(kx - \omega t)$$

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0}$$

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{(100\ \mathrm{V/m})(3.33 \times 10^{-7}\ \mathrm{T})}{2 \times 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}} = 13.2\ \mathrm{W/m^2}$$

Electromagnetic waves: problems



A radio station sends out a sinusoidal wave with an average power of 50 kW. What will be the amplitude of the wave if it is detected by a satellite 100 km away?

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$
$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

Area: $A = 2\pi R^2$

I from method 1: $I = \frac{P}{A} = \frac{P}{2\pi R^2}$

$$B_{max} = \frac{E_{max}}{c}$$

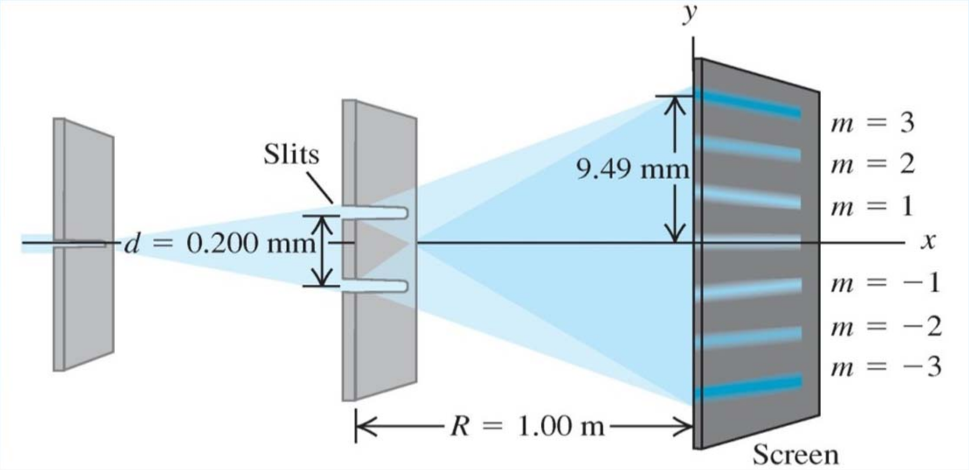
I from method 2: $I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c}$

Amplitude for E: $\frac{E_{max}{}^2}{2\mu_0 c} = \frac{P}{2\pi R^2} \Rightarrow E_{max} = \sqrt{\frac{P\mu_0 c}{\pi R^2}} = 2.45\ x\ 10^{-2}\ V/m$

Amplitude for B: $B_{max} = \frac{E_{max}}{c} = 8.17 \times 10^{-11}\ \mathrm{T}$

Interference: Problem

Y = 9.49 mm for the line with m = 3



What is the wavelength of the light?

$$y_m = R\frac{m\lambda}{d}$$

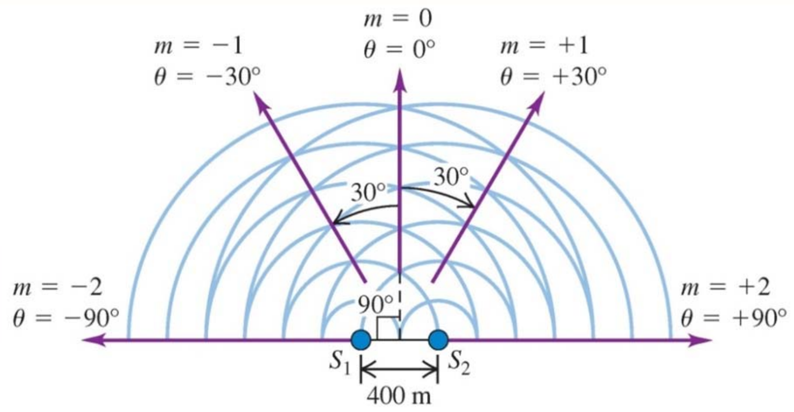
$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3}\ \text{m})(0.200 \times 10^{-3}\ \text{m})}{(3)(1.00\ \text{m})}$$

$$= 633 \times 10^{-9}\ \text{m} = 633\ \text{nm}$$

Interference: Problem

Two antennas send out radio waves with f = 1500 kHz. They sit 400 m apart.

Why is the intensity greatest at 0, 30 and 90 degrees ?



$$d \sin\theta = m\lambda$$

$$d = 400 \text{ m}$$

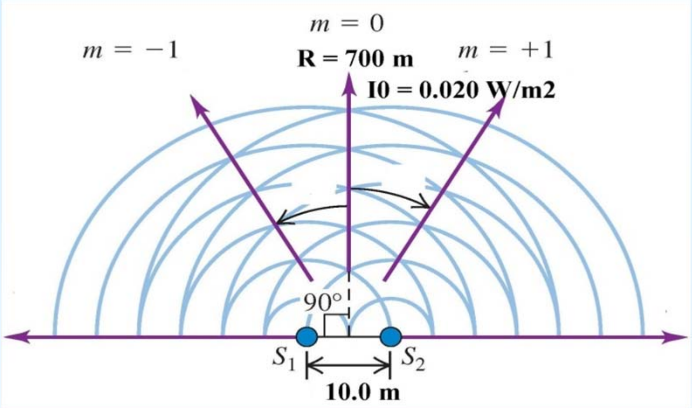
$$\lambda = c/f = 200 \text{ m}$$

$$\sin\theta = \frac{m\lambda}{d} = \frac{m(200 \text{ m})}{400 \text{ m}} = \frac{m}{2}$$

$$\theta = 0, \pm 30°, \pm 90°$$

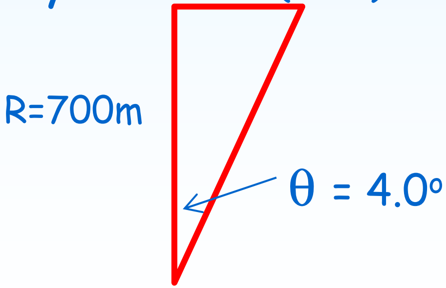
Interference: Problems

Two antennas emit radio waves at f = 60.0 MHz.
They sit 10.0 m apart. The intensity is 0.020 W/m^2 at a distance of 700 m for m = 0.



What is the intensity at the distance 700 m for $\theta = 4.00^\circ$?

y = 700 tan(4.0°) = 48.9 m



$$I = I_0 \cos^2 \frac{\phi}{2} = I_0 \cos^2\left(\frac{\pi d y}{\lambda R}\right)$$

$$\lambda = c/f = 5.00 \text{ m}$$

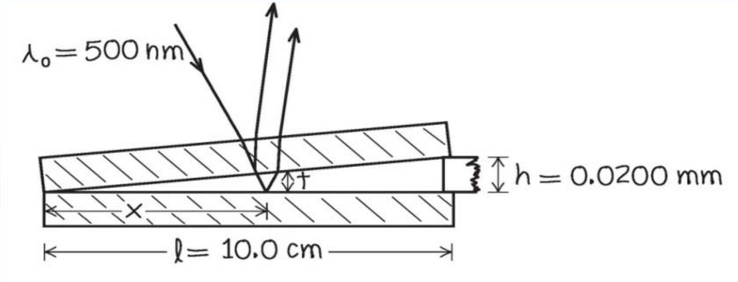
d = 10.0 m

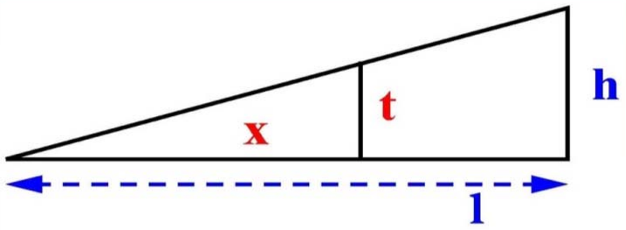
$$I = 0.020 \cos^2(\pi \cdot 10.0 \cdot 48.9 / (5.00 \cdot 700)) = 0.016 \text{ W/m}^2$$

Interference: Problem

Two thin 10.0 cm long glass plates are separated at one end by a 0.02 mm thick paper. Light with a wavelengths of 500 nm creates dark interference lines.

What is the distance between the lines ?





$$\frac{t}{x} = \frac{h}{l}$$

$$2t = \frac{2xh}{l}$$

Destructive reflections: $2t = m\lambda \qquad (m = 0, 1, 2, \ldots)$

$$\frac{2xh}{l} = m\lambda_0$$

$$x = m\frac{l\lambda_0}{2h} = m\frac{(0.100\text{ m})(500 \times 10^{-9}\text{ m})}{(2)(0.0200 \times 10^{-3}\text{ m})} = m(1.25\text{ mm})$$

Successive dark fringes, corresponding to $m = 1, 2, 3, \ldots,$ are spaced 1.25 mm apart.

Interference: Problems

A thin layer of MgF_2 with n=1.38 is coating a lens with n = 1.52 in order to stop reflections of light with wavelength 550 nm.

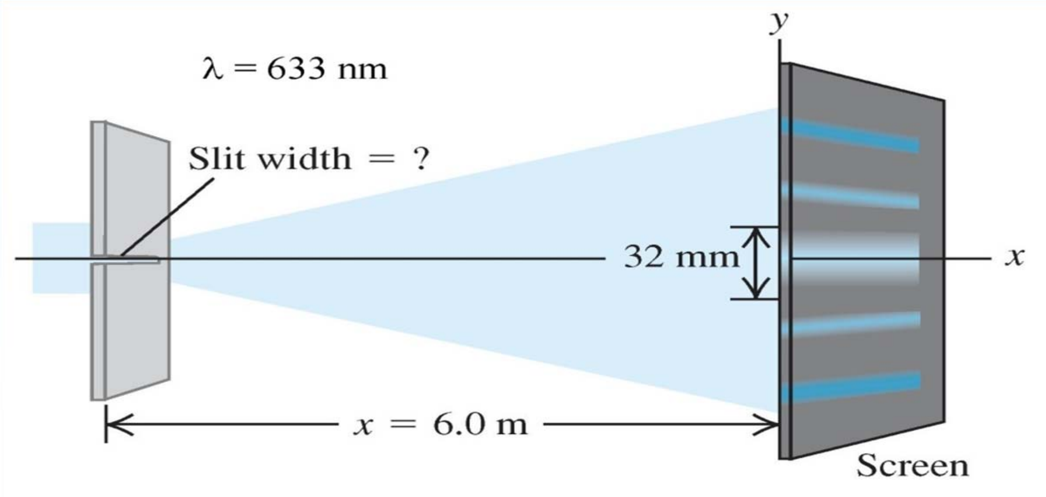
How thick does the MgF_2 layer need to be ?

$$\lambda_{film} = \lambda_{air} / n_{film} = 550 \text{ nm} / 1.38 = 400 \text{ nm}$$

Film thickness: $t = \lambda_{film} / 4 = 400 / 4 = 100 \text{ nm}$

Diffraction: Problem

What is the slit width ?



$$y_m = x\frac{m\lambda}{a}$$

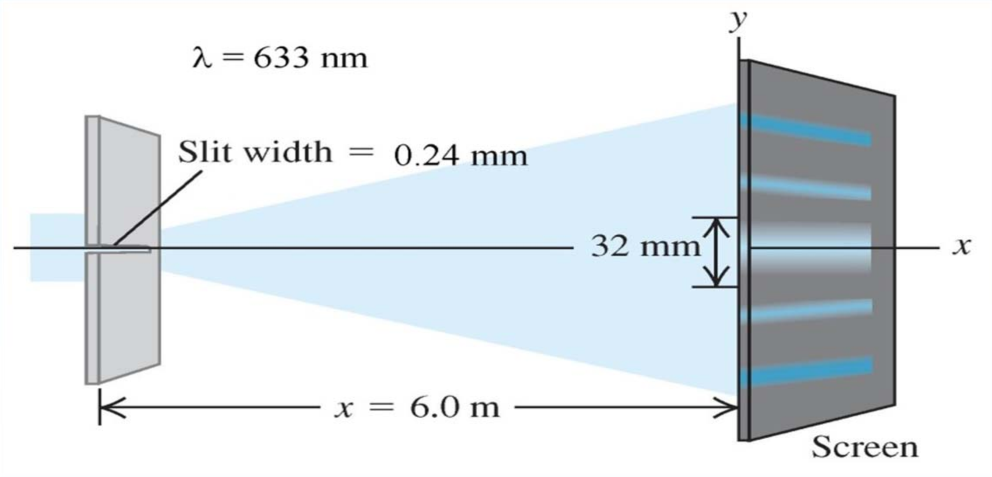
$$y = (32\text{ mm})/2 = 16\text{ mm}$$

$$a = \frac{x\lambda}{y} = \frac{(6.0\text{ m})(633 \times 10^{-9}\text{ m})}{16 \times 10^{-3}\text{ m}} = 2.4 \times 10^{-4}\text{ m} = 0.24\text{ mm}$$

Diffraction: Problem

The intensity in the central peak is I_0.

What is the intensity 3.0 mm away from this peak ?



λ = 633 nm
x = 6.00 m
a = 0.24 mm
y = 3.0 mm

$$\tan\theta = y/x = (3.0 \times 10^{-3}\ \mathrm{m})/(6.0\ \mathrm{m}) = 5 \times 10^{-4} = \sin(\theta)$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta = \frac{2\pi(2.4 \times 10^{-4}\ \mathrm{m})(5.0 \times 10^{-4})}{6.33 \times 10^{-7}\ \mathrm{m}} = 1.20\ \mathrm{rad}$$

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_0\left(\frac{\sin 0.60}{0.60}\right)^2 = 0.89 I_0$$

Diffraction: Problem

The intensity in the central peak in a single slit spectrum is I_0.

What is the intensity at a point where the phase difference between waves from the top and bottom of the gap is 66 radians ?

If this point is 7.0° from the central peak, how many wavelengths wide is the gap ?

β = 66 rad

θ = 7.0°

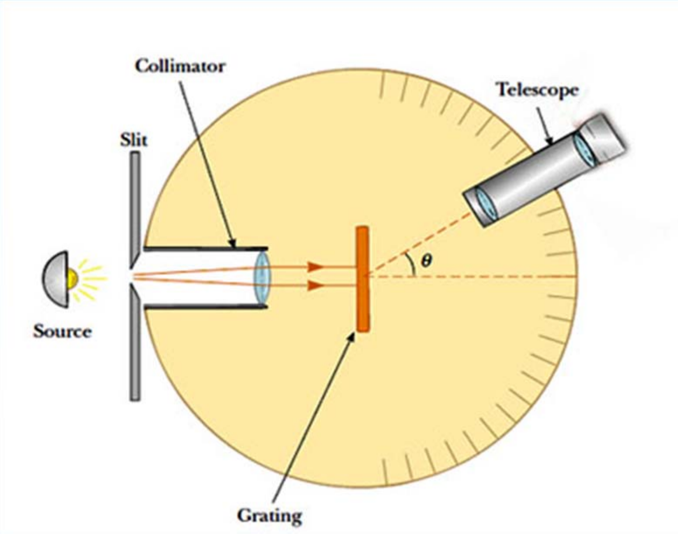
$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$I = I_0\left[\frac{\sin(33\text{ rad})}{33\text{ rad}}\right]^2 = (9.2\times 10^{-4})I_0$$

$$\frac{a}{\lambda} = \frac{\beta}{2\pi\sin\theta} = \frac{66\text{ rad}}{(2\pi\text{ rad})\sin 7.0^\circ} = 86$$

Diffraction: Problem



https://www.youtube.com/watch?v=b85paV77dS8

Grating: 1000 slits per mm   1st order maximum at 24°   What is λ ?

$d \sin\theta = m\lambda$ with   d = 1 mm / 1000 slits = 10^{-6} m
θ = 24°

$\lambda = d \sin(\theta) = 10^{-6} \sin(24^{\circ}) = 0.407 \times 10^{-6} = 407$ nm