ANSWERS WAVES, FYSA13

V1 Answers:

a) 55 kg
b) 0.73 m/s

V2 Answer:

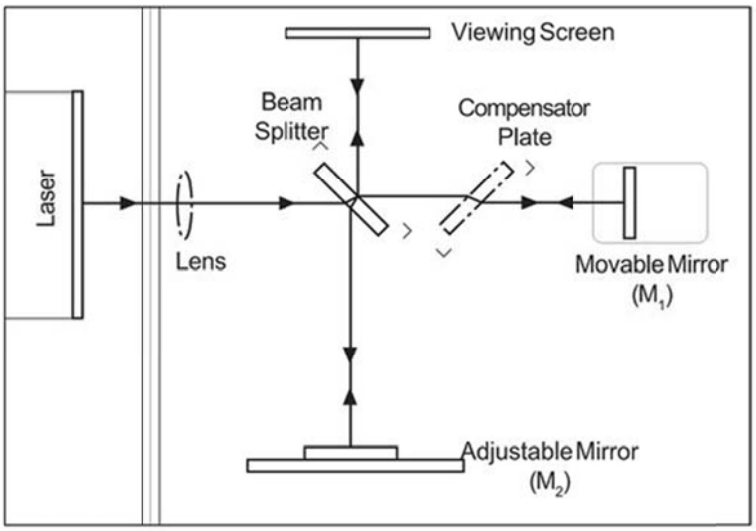
2.4×10^{-4} N

V3 Answers:

a) 109 dB
b) Y(x,t) = 4.9nm cos(73.3 rad/m · x – 25100 rad/s · t)
c) 0.000123 m/s

V4 Answers:

a)



b) 647 nm

V5 Answers:

a) 169 μm
b) 4.20 rad
c) 10.8 mW/mm^2

SOLUTIONS WAVES, FYSA13

V1 Solutions:

V1a) $T = 3s \Rightarrow f = \frac{1}{T} = \frac{1}{3}\,Hz$

$k = 240\,N/m$

$\omega = 2\pi f = \sqrt{\frac{k}{m}}$

$m = \frac{k \cdot T^2}{(2\pi)^2} = 55\,kg$

V1 b) $x = A\cos(\omega t + \varphi)$

$A = A\cos(\varphi) \Rightarrow \varphi = 0$

$v = -\omega A \sin(\omega t)$

$A = 0.4\,m$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

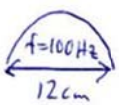
t for $x = 0.2\,m$: $0.2 = 0.4\cos(2.09t)$

$2.09t = 2\cos(0.5) = 1.047$

$t = 0.501\,s$

v for $t = 0.50s$: $v = -2.09 \cdot 0.4 \sin(2.09 \cdot 0.50) = -0.72\,m/s$

V2 Solutions:

V2 

$\Rightarrow \lambda = 24\,cm$

$\lambda = \frac{v}{f} \Rightarrow v = \lambda \cdot f = 24\,m/s$

$v = \sqrt{\frac{F}{\mu}} \Rightarrow F = v^2 \cdot \mu$

what is μ? Calculate mass of 1m long cylindrical string:

$V = \pi r^2 \cdot L = \pi \cdot (10 \cdot 10^{-6})^2 \cdot 1 = \pi \cdot 10^{-10}\,m^3$

$\rho = \frac{m}{V} \Rightarrow \mu = \rho \cdot V = 1300 \cdot \pi \cdot 10^{-10} = 4.08 \cdot 10^{-7}\,kg/m$

$F = v^2 \cdot \mu = 24^2 \cdot 4.08 \cdot 10^{-7} = 2.35 \cdot 10^{-4}\,N$

V3 Solutions:

V3a) $I_2 = I_0 10^{B/10} = 10^{-12} \cdot 10^{65/10} = 10^{-5.5}\ W/m^2$

$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \Rightarrow I_1 = I_2 \cdot \frac{r_2^2}{r_1^2} = 10^{-5.5} \cdot \frac{450^2}{3^2} = 0.0712\ W/m^2$

$\beta_1 = 10 \log \frac{I_1}{I_0} = 10 \log \frac{0.0712}{10^{-12}} = 109\ dB$

b) $y(x,t) = A \cos(kx - \omega t)$

$\omega = 2\pi f = 25.1 \cdot 10^3\ rad/s$

$v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{v} = \frac{25.1 \cdot 10^3}{343} = 73.3\ rad/m$

$I = \frac{1}{2} \rho (\omega A)^2 v \Rightarrow A = \sqrt{\frac{2I}{\rho v}} / \omega$

$A = \sqrt{\frac{2 \cdot 10^{-5.5}}{1.20 \cdot 343}} / 25.1 \cdot 10^3 = 4.9 \cdot 10^{-9}\ m$

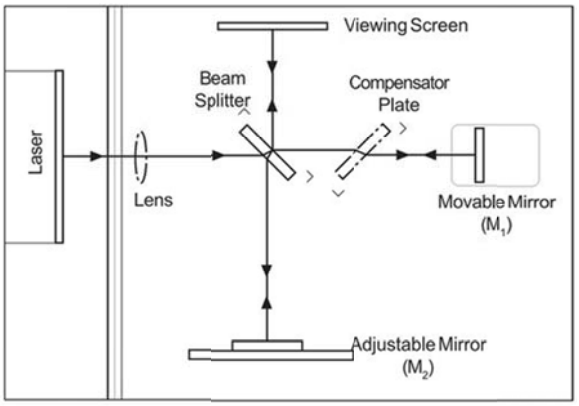
$y(x,t) = 4.9nm \cos(73.3\ rad/m \cdot x - 25.1 \cdot 10^3\ rad/s \cdot t)$

c) $v_y(x,t) = \omega A \sin(kx - \omega t)$

$v_{y\,max} = \omega A = 25.1 \cdot 10^3 \cdot 4.9 \cdot 10^{-9} = 0.000123 m/s$

V4 Solutions:

a)



V4 b) $\lambda = \frac{2 \cdot d}{N} = \frac{2 \cdot 9.7 \cdot 10^{-6}}{30} = 647nm$

V5 Solutions:

V5 a) $y_m = \frac{x_m \lambda}{a} \Rightarrow a = \frac{x \lambda}{y} = \frac{4.00 \cdot 632.8 \cdot 10^{-9}}{0.015}$

$a = 169 \mu m$

b) $\beta = \frac{2\pi}{\lambda} a \frac{y}{x} = \frac{2\pi \cdot 169 \cdot 10^{-6} \cdot 0.010}{632.8 \cdot 10^{-9} \cdot 4.00} = 4.20\ rad = 241°$

c) $I_0 = \frac{I}{\left(\frac{\sin \beta/2}{\beta/2}\right)^2} = \frac{1.83}{\left(\frac{\sin 2.1}{2.1}\right)^2} = 10.8\ mW/mm^2$