ANSWERS WAVES EXAM, FYSA13

V1 Answers

a) 1.33 m/s
b) 1.88 m/s
c) 167 m/s^2
d) 237 m/s^2

V1 Solutions

V2 $y(x,t) = 15.0 \sin(\pi x/8 - 4\pi t)$

a) $v_y(x = 6.00\text{ cm},\ t = 0.250\text{ s}) = ?$

$$v_y = \frac{\partial y}{\partial t} = -4\pi \times 15.0 \cos\left(\frac{\pi x}{8} - 4\pi t\right)$$

$$\boxed{v_y}(x = 6.00\text{ cm},\ t = 0.250\text{ s}) = \underbrace{-4\pi \times 15.0}\times \cos\left(\frac{\pi \times 6.00}{8} - 4\pi \times 0.250\right) =$$

$$= -60\pi \cos\left(\frac{3}{4}\pi - \pi\right) = -60\pi \cos\left(-\frac{\pi}{4}\right) = -60\pi \frac{\sqrt{2}}{2} =$$

$$= 133.286\ \text{cm/s} \approx \boxed{1.33\ \text{m/s}}$$

b) $v_{y\,max} = ?$ coefficient in front of the cosine

$$\boxed{v_{y\,max}} = \left|\overbrace{-4\pi \times 15.0}\right| \frac{\text{cm}}{\text{s}} = 188.496 \frac{\text{cm}}{\text{s}} \approx \boxed{1.88\ \text{m/s}}$$

c) $a_y(x = 6.00\text{ cm},\ t = 0.250\text{ s}) = ?$

$$a_y = \frac{\partial v_y}{\partial t} = -240\pi \sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

$$\boxed{a_y}(x = 6.00\text{ cm},\ t = 0.250\text{ s}) = \underbrace{-240\pi^2}\sin\left(-\frac{\pi}{4}\right) = 1674.9 \frac{\text{cm}}{\text{s}^2} =$$

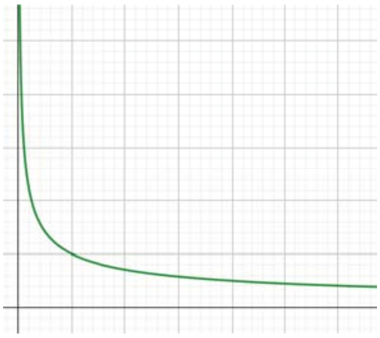
$$= \boxed{167\ \text{m/s}^2}$$

d) $$\boxed{a_{y\,max}} = \left|-240\pi^2\right| = 2369\ \text{cm/s}^2 = \boxed{237\ \text{m/s}^2}$$

V2 Answers

a) 8600 kg/m^3

b) $f = \frac{\text{const}}{\sqrt{\rho}}$ wher const = 590 s^{-1} kg$^{1/2}$ m$^{-3/2}$



V2 Solutions

k

$\rho_1 = 2600 \ \mathrm{kg/m^3}$

f_1

k

$\rho_2 = ?$

$f_2 = 0.55 f_1$

a) $\rho_2 = ?$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} = \frac{1}{2\pi}\sqrt{\frac{k}{\rho_1 V}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{\rho_2 V}} \qquad \left(V = \frac{4}{3}\pi r^3\right)$$

$$\frac{f_2}{f_1} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{\rho_2 V}}}{\frac{1}{2\pi}\sqrt{\frac{k}{\rho_1 V}}} = \sqrt{\frac{\rho_1}{\rho_2}}$$

$$\frac{\rho_1}{\rho_2} = \left(\frac{f_2}{f_1}\right)^2$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{f_1}{f_2}\right)^2 = \left(\frac{f_1}{0.55 f_1}\right)^2$$

$$\rho_2 = \frac{1}{0.55^2}\rho_1 = 8595.04 \ \mathrm{kg/m^3} \approx 8600 \ \mathrm{kg/m^3}$$

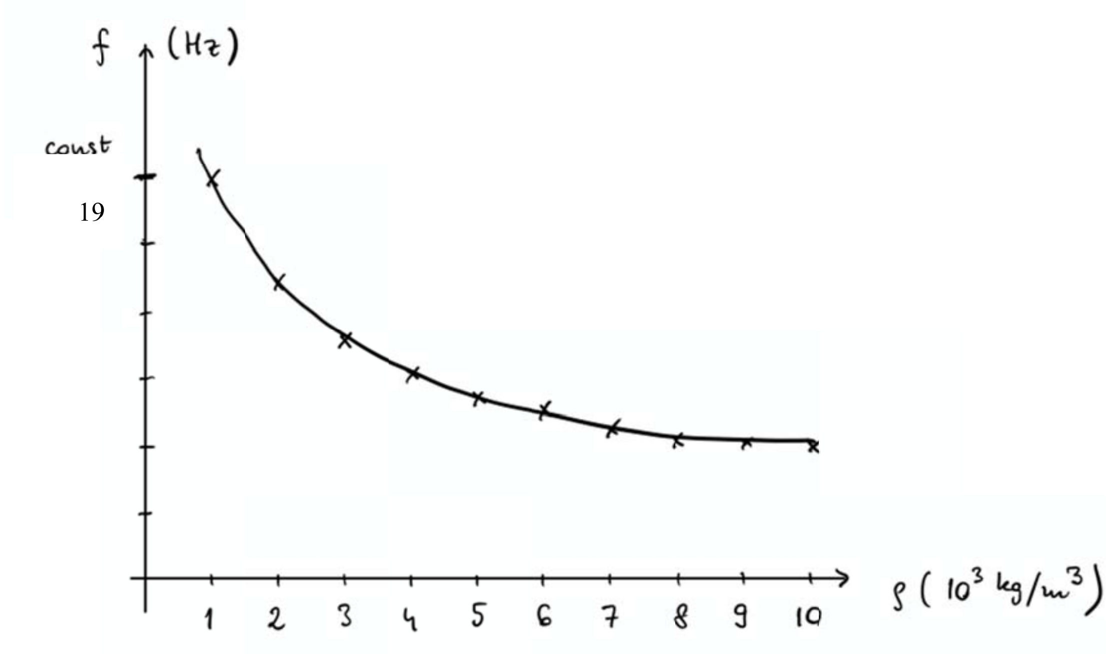
b) $f = \frac{\text{const}}{\sqrt{\rho}} = \frac{590\ \text{s}^{-1}\text{kg}^{1/2}\ \text{m}^{-3/2}}{\sqrt{\rho}}$

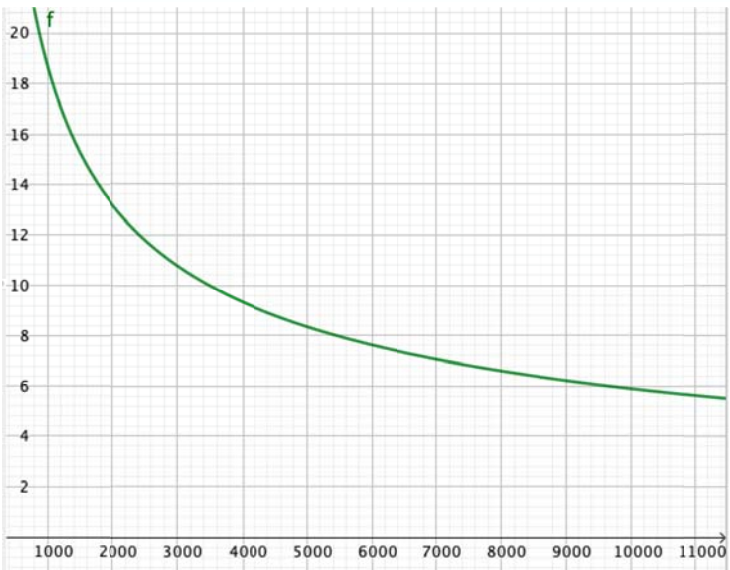
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{V}} \cdot \frac{1}{\sqrt{\rho}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{4}{3}\pi r^3}} \cdot \frac{1}{\sqrt{\rho}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{100}{\frac{4}{3}\pi\, 0{,}012^3}} \cdot \frac{1}{\sqrt{\rho}} = \frac{592}{\sqrt{\rho}}$$

for $\rho = 1000\ \text{kg/m}^3$ we obtain $f = 18.707\ \text{Hz} \approx 19\ \text{Hz}$
etc.





(GeoGebra)

V3 Answers

a) It will be 3.3% lower in Bologna.
b) It is 0.15 dB higher in Malmö.

V3 Solutions

V3a) $I = \frac{1}{2}\sqrt{\rho B}\,(\omega A)^2 = \sqrt{\rho}\,\frac{1}{2}(\omega A)^2 = k\sqrt{\rho}$

$$\frac{I_B - I_M}{I_M} = \frac{k\sqrt{\rho_B} - k\sqrt{\rho_M}}{k\sqrt{\rho_M}} = \sqrt{\frac{\rho_B}{\rho_M}} - 1 = -0{,}033$$

3,3% lower in Bologna.

V3b) $\frac{I_M}{I_B} = \frac{k\sqrt{\rho_M}}{k\sqrt{\rho_B}} = \sqrt{\frac{1{,}225}{1{,}146}} = 1{,}0339 \quad \Rightarrow I_B = 0{,}967\, I_M$

$$\beta_M - \beta_B = 10 \log \frac{I_M}{I_0} - 10 \log \frac{I_B}{I_0}$$

$$\beta_M - \beta_B = 10 \log \frac{I_M}{I_0} - 10 \log \frac{0{,}967\, I_M}{I_0}$$

$$\beta_M - \beta_B = 10 \log I_M - 10 \log I_0 - 10 \log 0{,}967\, I_M + 10 \log I_0$$

$$\beta_M - \beta_B = 10 \log \frac{I_M}{0{,}967\, I_M} = 0{,}15\ \mathrm{dB}$$

V4 Answers

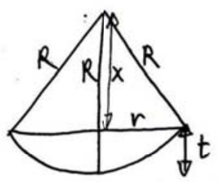
a) $t = 1.47\ \mu\mathrm{m}$
b) $\lambda = 541$ nm

V4 Solutions

V4a) $2t = m\lambda$

$$t = \frac{5\lambda}{2} = \frac{5 \cdot 589 \cdot 10^{-9}}{2} = 1{,}47\ \mu\mathrm{m}$$

V4b)



$$\begin{cases} x^2 + r^2 = R^2 \\ x = R - t \end{cases}$$

$$(R - t)^2 = R^2 - r^2$$

$$R - t = \sqrt{R^2 - r^2}$$

$$t = R - \sqrt{R^2 - r^2} = 0{,}8 - \sqrt{0{,}8^2 - 0{,}00114^2} = 0{,}812\ \mu\mathrm{m}$$

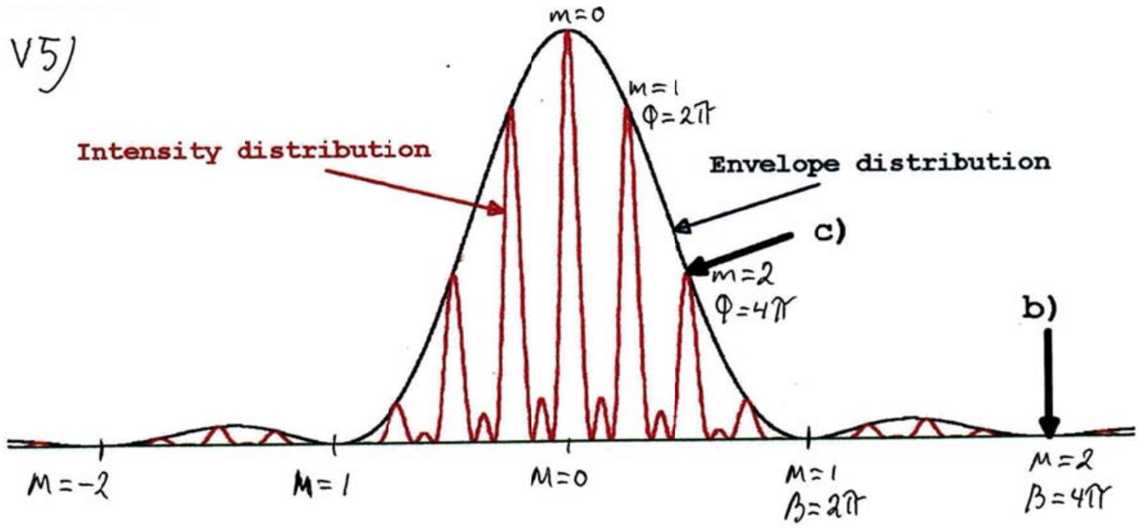
$$2t = m\lambda$$

$$\lambda = \frac{2t}{3} = \frac{2 \cdot 0{,}812\ \mu\mathrm{m}}{3} = 541\ \mathrm{nm}$$

V5 Answers

a) There are two minima between the principle maxima and that means there are three slits.
b) 0.36 degrees
c) 0.81 mW/mm^2

V5 Solutions



a) There are 3 slits because there are 2 minimum between two maximum.

b) $a \sin\theta = m\lambda$

$\sin\theta = \frac{m\lambda}{a} = \frac{2\cdot 632.8\cdot 10^{-9}}{0.2\cdot 10^{-3}}$

$\theta = 0.36^\circ$

c) $\varphi = 4\pi$

$\varphi = \frac{2\pi d}{\lambda}\sin\theta \Rightarrow \sin\theta = \frac{\varphi\lambda}{2\pi d}$

$\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi a}{\lambda}\cdot\frac{\varphi\lambda}{2\pi d} = \frac{a}{d}\varphi = \frac{0.2}{0.8}4\pi = \pi$

$I = I_0\left[\frac{\sin 0.5\pi}{0.5\pi}\right]^2 = 0.405\, I_0 = 0.81\, mW/mm^2$